



# Description and validation of the ice-sheet model Yelmo (version 1.0)

Alexander Robinson<sup>1,2,3</sup>, Jorge Alvarez-Solas<sup>1,2</sup>, Marisa Montoya<sup>1,2</sup>, Heiko Goelzer<sup>4,5</sup>, Ralf Greve<sup>6</sup>, and Catherine Ritz<sup>7</sup>

<sup>1</sup>Complutense University of Madrid, Madrid, Spain

<sup>2</sup>Geosciences Institute CSIC-UCM, Madrid, Spain

<sup>3</sup>Potsdam Institute for Climate Impact Research, Potsdam, Germany

<sup>4</sup>Institute for Marine and Atmospheric Research Utrecht, Utrecht, The Netherlands

<sup>5</sup>Laboratoire de Glaciologie, Université Libre de Bruxelles, Brussels, Belgium

<sup>6</sup>Institute of Low Temperature Science, Hokkaido University, Sapporo, Japan

<sup>7</sup>Institute for Geosciences and Environmental Research, Grenoble, France

Correspondence: Alexander Robinson (robinson@ucm.es)

**Abstract.** We describe the physics and features of the ice-sheet model Yelmo, an open-source project intended for collaborative development. Yelmo is a thermomechanical model, solving for the coupled velocity and temperature solutions of an ice sheet simultaneously. The ice dynamics are currently treated via a "hybrid" approach combining the shallow-ice and shallowshelf/shelfy-stream approximations, which makes Yelmo an apt choice for studying a wide variety of problems. Yelmo's main

5 innovations lie in its flexible and user-friendly infrastructure, which promotes portability and facilitates long-term development. In particular, all physics subroutines have been designed to be self-contained, so that they can be easily ported from Yelmo to other models, or easily replaced by improved or alternate methods in the future. Furthermore, hard-coded model choices are eschewed, replaced instead with convenient parameter options that allow the model to be adapted easily to different contexts. We show results for different ice-sheet benchmark tests, and we illustrate Yelmo's performance for the Antarctic ice sheet.

#### 10 1 Introduction

The field of continental-scale, ice-sheet modeling started with a handful of pioneering models (e.g., Huybrechts et al., 1988; Ritz et al., 1997; Greve, 1997a). These models were computationally efficient for the resources available at the time. Typical grid resolutions were on the order of 20-40 km and generally the shallow ice approximation (SIA) was used to solve the ice dynamics. These classic models have been most useful for long time-scale paleo simulations in part because they are fast, but

15 also because they are relatively simple in design, usually relying on low-tech solutions to numerical problems. Most of these models were designed before the era of the high-performance computing cluster, which made it challenging to build models otherwise.

Nowadays, a large number of ice-sheet models exist, supported by a growing and active community of developers. Models today represent a broad spectrum of approaches that incorporate different levels of physical complexity and computational

20 ingenuity. These models include hybrid approaches that heuristically combine the SIA with the shallow shelf approximation (SSA) (e.g., Bueler and Brown, 2009; Winkelmann et al., 2011; Goldberg, 2011; Pollard and DeConto, 2012; Pattyn, 2017;





Quiquet et al., 2018) and higher-order approximations including full Stokes solutions (e.g., Larour et al., 2012; Gagliardini et al., 2013; Hoffman et al., 2018; Lipscomb et al., 2019). Newer models often feature finite-element/finite-volume methods (e.g., Larour et al., 2012; Gagliardini et al., 2013; Hoffman et al., 2018) or adaptive mesh refinement (Cornford et al., 2013), which allows simulation of complex terrain and very high resolution where it is needed (e.g., at the grounding line in Antarc-

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tica). While more complex models are driving advances in our understanding of the physics and relevant processes of ice sheets over a range of time scales, simpler and thus faster methods are still required to understand the evolution of the ice sheets on multi-millennial, paleo time scales.

Here we introduce the ice-sheet model Yelmo<sup>1</sup>, which is intended to provide access to complex and robust model physics through an intuitive model design. It is a hybrid ice-dynamics model that is easy to use and configure. We expect that Yelmo

10 will be useful for long time-scale paleo simulations of the continental ice sheets, coupled climate – ice-sheet modeling, ensemble simulations and uncertainty studies, as well as for teaching. Below, we first describe the model structure, physics and application programming interface (API, Section 2). Then, we present results for several benchmark experiments to validate the model performance (Section 3), and a simulation of the present-day and glacial Antarctic ice sheet (Section 4), followed by the conclusions (Section 5).

### 15 2 Model design

Yelmo has been inspired and largely derived from classical ice-sheet models that have been used successfully for many years – with the most in common with GRISLI (Ritz et al., 1997; Quiquet et al., 2018) and SICOPOLIS (Greve, 1997a, 2019). However, in contrast to many models, Yelmo was designed from scratch to run as a modular library that can be called by other programs rather than as a stand-alone executable. The strict application of this philosophy drove many design choices and

20 allowed us to develop a robust ice-sheet model library with a clear API that would be difficult to develop in an ad-hoc way later.

Yelmo is written in Fortran90, which provides continuity from previous code bases and supports the fact that clarity and readability of the code are important features. Like SICOPOLIS and other models, we have opted for "low tech" solutions whenever possible, meaning that internally coded routines are preferred and, thus, the external dependencies of the model are

25 kept to a minimum. This ensures that the algorithms used remain accessible and easily changeable. Nonetheless, Yelmo has two key dependencies: the NetCDF library for convenient, community-standard input/output capability and the Library of Iterative Solvers for linear systems<sup>2</sup> (Lis, Nishida, 2010), which is used for solving the elliptical SSA equations.

Yelmo has been designed to be user friendly (i.e., straightforward to understand), accessible, portable and adaptable. These features were facilitated by the design choice to separate what we call the "model accounting" from the model physics itself, and
by following an object-oriented approach. There are no global variables in Yelmo (except for a few global constants related to the general physics of the planet being simulated), which means that variables and parameters are saved together in containers

<sup>&</sup>lt;sup>1</sup>The name Yelmo refers to a semi-domed, rocky mountain in the Guadarrama Mountains outside of Madrid, Spain.

<sup>&</sup>lt;sup>2</sup>https://www.ssisc.org/lis/



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(called derived types in Fortran) specific to each of the main Yelmo components, such as dynamics or thermodynamics, as described in the sections below. These containers make up the individual components of the overall Yelmo object (itself a container) that contains all of the variables, parameters and information needed to simulate a given domain of the ice sheet. Multiple instances of the Yelmo object can therefore be defined in a program (e.g., one Yelmo-object instance for Greenland and one for Antarctica), and each one will operate fully in isolation from the others. This is the model accounting, which is of a specific design built into Yelmo and is thus the only part not easily portable to other models.

The model physics, meanwhile, consists of subroutines that are fully portable and, whenever possible, only rely on native data types (e.g., scalar, vector, arrays). In other words, the specific, non-portable design structure of the Yelmo object does not contaminate the physics subroutines, since the necessary variables and parameters are always passed as arguments. In

- general, all input and output to subroutines must be defined as arguments. Each argument must further always be given an 10 intent characteristic (in Fortran, the intent of an argument can be one of IN, OUT or INOUT), which ensures that only the variables destined for output from the routine can be modified inside it. This approach not only aides debugging and provides programmatic safety, but provides a clear blueprint to users of what each subroutine does. Most importantly, the subroutines are thus fully self-contained and can be used in other programs and contexts, as long as the correct arguments can be provided.
- 15 Concerning the model accounting, the Yelmo object contains all parameters and variables needed to run a given domain. For clarity and convenience, it has been divided into four components: topography, dynamics, material properties and thermodynamics (Fig. 1). Each component has an associated set of functions to load parameters, allocate and initialize the variables, update the variables (i.e., the actual physics calculation step), and finally to terminate the instance of the component at the end of the program. This pattern is followed for all four components and represents the component-level API.
- 20 Each component contains variables and parameters necessary for the calculation of its specific physics, however each component also relies on the variables defined in other components since the ice sheet is a highly-coupled and nonlinear system. The benefit of the somewhat artificial division of components made here is that the use of INTENT statements ensures that variables of a given component can only be modified in the corresponding module. For example, when the update subroutine of the topography module is called, only the object containing topography variables is defined as INTENT=INOUT, while the objects containing dynamics, material and thermodynamics variables are all defined as INTENT=IN. Analogous to the design 25 of the physics subroutines, the use of intent statements here makes the model blueprint clear, but also enforces consistency with the overall design of the Yelmo structure. The hope is that this will not only make the model more user friendly, but it will also naturally lead to more disciplined model development in the future.

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In addition to the four components that contain prognostic and diagnostic model variables, the Yelmo object includes a boundary component, which defines all fields that Yelmo requires as input from external sources (Fig. 1). These fields can be obtained from other coupled models, or simply by loading data, however Yelmo does not make any assumptions about their source. The boundary component is defined as INTENT=IN in all modules, so that Yelmo does not have the right to modify them internally. This conceptual isolation of the ice-sheet model serves to ensure that coupling with other models is as straightforward as possible.





Yelmo also makes use of a working precision variable, which allows for the model to be compiled with any real precision. For most applications, single precision (32 bits) is sufficient. Double precision (64 bits) gives equivalent results for the tests we have made. This is in part due to the fact that the units of all time variables in Yelmo are cast in years instead of seconds and thus very small numbers are avoided. Nonetheless, this choice is left open to the user.

- In terms of model physics, each component of Yelmo was built to work independently, in the sense that a given component is agnostic to the methods used to calculate variables from other components. For example, the temperature and velocity fields, which are taken as input to calculate the viscosity and rate factor in the material component, are used without any knowledge about the physics and numerical approximations used to calculate them. This means that sometimes simplifying assumptions cannot be used, even though they may be valid in some cases (such as assuming that the strain rate is only due to SIA terms
- 10 where the ice sheet is frozen to the bed). However, the benefit is that typically the most general solutions possible have been implemented for each component. Thus, when the physics of one component is changed or upgraded, it is likely that the other modules will not require any modification.

Grid information is also stored in the main Yelmo object, and a single grid is defined for use with all components. Like many ice-sheet models, Yelmo uses the Arakawa-C grid staggering approach (Arakawa and Lamb, 1977) extended to 3D, as shown

- 15 in Fig. 2. Scalar variables, such as temperature, are defined at the cell centers, which in Yelmo are designated as "aa-nodes". Velocity components and gradients are calculated on cell edges ("ac-nodes") and scalar coefficients, like diffusivity in the SIA approach, are calculated on cell corners ("ab-nodes"). The specific numerical discretization of the finite difference equations largely follows the approach of Macayeal (1997). The advantage of this approach is that it benefits from the natural staggering that occurs when calculating gradients (e.g., the surface slope is naturally defined on the ac-nodes), but it also results in greater
- 20 numerical stability of the model (Macayeal, 1997).

Yelmo requires an evenly-spaced, Cartesian grid in the horizontal direction, while the vertical component follows a classic sigma-coordinate system (Greve and Blatter, 2009). The vertical axis  $\zeta$  represents the relative height within the ice sheet, running from  $\zeta = 0$  at the ice-sheet base to  $\zeta = 1$  at the ice-sheet surface:

$$\zeta(z) = H_z(z)/H \tag{1}$$

- 25 where z is the elevation relative to present-day sea level,  $H_z(z)$  is the ice thickness up to the elevation z within the ice sheet and H is the total ice thickness. Yelmo can be defined with any specified number of vertical grid points, which can be unevenly spaced. Typically, we have set  $n_z = 20$  and the  $\zeta$  axis is defined with higher resolution near the base and surface of the ice sheet, which is important for resolving thermodynamics and ages accurately. Use of the sigma-coordinate system simplifies the numerics of an evolving domain in the vertical direction and inherently results in higher resolution for grid points with less ice
- 30 thickness (Greve and Blatter, 2009). Vertical velocities are calculated on ac-nodes in the vertical and aa-nodes in the horizontal, while horizontal velocities are calculated on ac-nodes in the horizontal and aa-nodes in the vertical. Boundary conditions in a vertical column are applied directly at the ice base and ice surface, which correspond to ac-nodes (see Fig. 2).

Yelmo solves for two prognostic variables using coupled equations of mass and energy conservation: the ice thickness (2D field) and ice temperature (3D field). Velocity (3D vector field) is diagnosed from approximations of ice flow assuming a





nonlinear flow law. These equations are described in the subsections below, along with additional considerations related to each component. For more details on the derivation of the equations, thorough explanations can be found in various references (Greve and Blatter, 2009; Cuffey and Paterson, 2010), and thus are not repeated here.

# 2.1 Topography

5 The evolution of the ice thickness in the model is determined from mass conservation:

$$\frac{\partial H}{\partial t} = -\nabla \cdot H\bar{\mathbf{u}} + \dot{a} + \dot{b}_g + \dot{b}_f - \dot{c}$$
<sup>(2)</sup>

where *H* is the ice thickness,  $\bar{\mathbf{u}} = (\bar{u}, \bar{v})$  is the depth-averaged horizontal velocity,  $\dot{a}$  is the surface mass balance,  $\dot{b}_g$  and  $\dot{b}_f$  are the basal mass balance for grounded and floating ice, respectively, and  $\dot{c}$  is the calving rate at floating ice margins. In Yelmo, in order to obtain more accurate mass balance accounting, the advection of ice and source contributions are treated

- 10 separately as follows. First, a forward Euler explicit method (or optionally an upwind implicit method) is used to solve for the ice thickness at each timestep without accounting for  $\dot{a}$ ,  $\dot{b}_g$ ,  $\dot{b}_f$  or  $\dot{c}$ . The depth-averaged horizontal velocity is obtained from the dynamics component from the previous timestep. Note that dynamics is normally updated with the same timestep frequency as the topography component (see timestepping below). Next the mass balance terms  $\dot{a}$ ,  $\dot{b}_g$  and  $\dot{b}_f$  are applied. It should be noted that the basal mass balance of floating ice is a boundary variable for Yelmo (i.e., it is obtained externally and passed to
- 15 Yelmo), while the basal mass balance of the grounded ice is calculated internally as part of the thermodynamics solver (see Thermodynamics section below).

Yelmo also includes special treatment of grid points at the margin of the ice sheet, by making a distinction between icecovered grid points that are totally and partially filled following Albrecht et al. (2011) and Lipscomb et al. (2019). This is done in a relatively simple, yet effective way to avoid artificially thin ice thickness at the ice margin. For each ice-covered grid point

- 20 that has an ice-free neighbor, the reference ice thickness of the margin point  $(H_{ref})$  is defined as the minimum thickness of the ice-covered neighbors, or half of that in the case of grounded ice. This represents the minimum ice thickness for which the cell can be considered completely ice covered. The fraction of ice cover is then defined as  $f_{ice} = \min(H/H_{ref}, 1)$ . Whenever  $f_{ice} < 1$ , the grid cell is considered dynamically inactive and the ice thickness is stored in a buffer field during dynamic and thermodynamic calculations (i.e., these modules see the ice thickness as zero at these points). Since the horizontal velocity
- components are calculated on ac-nodes, this method ensures zero ice flux through the downstream edge of a partially filled margin cell. During mass conservation, however, this buffer is added to the ice thickness field and the steps outlined above are applied. In this way, the ice cell can be filled with ice from upstream and when the threshold of  $f_{ice} = 1$ , the ice sheet can advance.

In the final mass conservation step, calving c is treated at the floating ice margins. Currently, a simple threshold method has
been implemented, as well as a threshold+flux method (Peyaud et al., 2007). In both methods, the calving rate applied to the ice sheet is defined following Lipscomb et al. (2019):

$$\dot{c} = \frac{H_{\rm ref} - H}{\tau_c} \tag{3}$$





where  $\tau_c$  is the characteristic calving time, usually set to 1-10 years, and  $H_{ref}$  is the margin ice thickness as defined above. Setting  $\tau_c$  to higher values facilitates ice-shelf growth and thus grounding-line advance in transient, glacial simulations, but has little impact on the steady-state distribution of ice shelves for present day. This calving rate is applied only when the ice thickness of totally-filled ice margin points is below a threshold value (simple threshold method), or when the ice thickness is

- below a threshold value and the upstream flux is not sufficient to return the ice thickness to above the threshold (threshold+flux 5 method). For paleo simulations the latter is our preferred method, as it allows for more robust ice shelf advance (Peyaud et al., 2007). As mentioned above, calving is only applied to floating margin points with  $f_{ice} = 1$ . However, a final check is also applied to both floating and grounded margin buffer points. If no neighbors are fully ice covered with  $f_{ice} = 1$ , then this buffer ice will also be calved and ice-margin retreat happens in a natural way.
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Once the ice thickness has been completely updated, Yelmo diagnoses whether the ice should be grounded or floating. To facilitate this step, the distance from flotation as measured in ice thickness, i.e., how close a grid point is to the Archimedes flotation criterion, is calculated on each aa-node:

$$H_{\rm g} = H - \frac{\rho_{\rm sw}}{\rho} (z_{\rm sl} - z_{\rm b}) \tag{4}$$

where  $\rho$  is the ice density and  $\rho_{sw}$  the seawater density, and  $z_{sl}$  and  $z_b$  are the boundary fields of sea level and bedrock elevation, respectively.  $H_{\rm g}$  can thus be positive, zero or negative. When  $H_{\rm g}$  is positive, the ice thickness exceeds the flotation criterion, 15 and is considered grounded, while when  $H_{\rm g}$  is zero or negative, the ice is considered floating.

Yelmo also calculates the grounded fraction of each grid point,  $f_g$ . On aa-nodes,  $f_g$  is only assigned binary values to maintain consistency with the overall grid definition: zero when  $H_{\rm g} \leq 0$  or one when  $H_{\rm g} > 0$ . However, on cell edges, the values of  $f_{\rm g,acx}$ and  $f_{g,acy}$  are determined by linearly interpolating  $H_g$  from the two bounding aa-nodes. When both bounding aa-nodes are positive  $f_{g,ac} = 1$ , and when both are negative  $f_{g,ac} = 0$ . When one aa-node is positive  $(H_{g,pos})$  and one aa-node is negative 20  $(H_{\rm g,neg})$ , the grounded fraction on the ac-node is determined from linear interpolation:

$$f_{\rm g,ac} = \frac{-H_{\rm g,pos}}{(H_{\rm g,neg} - H_{\rm g,pos})} \tag{5}$$

Alternatively, it is possible to calculate  $f_g$  via subgrid bilinear interpolation of  $H_g$  to intermediate points to determine the grounded area fraction. However, this operation is more computationally intensive, and we find that in practice, the simple linear interpolation method is sufficient. 25

The surface elevation  $(z_s)$  is calculated following Pattyn (2017) as

$$z_{\rm s} = \max\left[z_{\rm b} + H, z_{\rm sl} + (1 - \frac{\rho}{\rho_{\rm sw}})H\right],\tag{6}$$

This approach ensures that the surface elevation solution is consistent with the Archimedes flotation criterion on aa-nodes.

The remaining tasks of the topography component are to diagnose other useful topographic characteristics, such as surface 30 and ice thickness gradients (on ac-nodes) and topographic masks.





# 2.2 Material

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The material component of Yelmo handles the calculation of the rate factor, the strain rate tensor and effective strain rate, the effective viscosity and, optionally, the age of the ice. Essentially, the material variables make the link between thermodynamics and dynamics, since the rate factor depends on temperature and the strain rate depends on velocity. No distinction is made be-

5 tween the type of approximation used to solve the dynamics here, rather all equations follow from the more general hydrostatic approximation (Greve and Blatter, 2009).

The effective viscosity, used to determine strain heating in the thermodynamics component, is calculated as

$$\eta = \frac{1}{2} \left( \dot{\varepsilon}^2 \right)^{\frac{1-n}{2n}} \left( A^{-1/n} \right),\tag{7}$$

where  $\dot{\varepsilon}$  is the effective strain rate, *n* is the Glen's Flow law exponent (Glen, 1955; Greve and Blatter, 2009), and *A* is the rate 10 factor. The effective strain rate is given by the second invariant of the strain rate tensor ( $\dot{\varepsilon}_{ij}$ ):

$$\dot{\varepsilon} = \left(\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{\frac{1}{2}} \tag{8}$$

and the strain rate tensor itself, following index notation, is

$$\dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad i, j = 1, 2, 3.$$
(9)

The rate factor, A(x, y, z), can be prescribed to a constant value, or calculated as a function of ice temperature following an 15 Arrhenius equation:

$$A(T') = E_f A_0 e^{-Q_a/RT'}$$
(10)

Here R is the ideal gas constant,  $A_0$  and  $Q_a$  are the temperature-dependent rate factor coefficient and activation energy, respectively (see Greve and Blatter, 2009).  $E_f$  is a so-called enhancement factor, which is used to approximate the effect of anisotropic flow. In Yelmo, it is possible to specify different values of the enhancement factor for different flow regimes (shear, stream and shelf). The shelf value is prescribed anywhere ice is floating, while the inland value of  $E_f$  is a weighted average

between the shear and stream value with the weighting given by a diagnosis of the vertical shearing fraction at any given point:

$$f_{\text{shear}} = \frac{\left(\dot{\varepsilon}_{\text{xz}}^2 + \dot{\varepsilon}_{\text{yz}}^2\right)}{\dot{\varepsilon}^2}.$$
(11)

Typical values of the enhancement factor for the shearing, streaming and shelf regime are  $E_f = (3.0, 1.0, 0.7)$ , respectively 25 (Ma et al., 2010).

In addition, it is possible to track the deposition time (i.e., age) of the ice using the online Eulerian tracer advection model. The general 3D advection equation of a conservative variable X,

$$\frac{\partial X}{\partial t} = -u\frac{\partial X}{\partial x} - v\frac{\partial X}{\partial y} - w\frac{\partial X}{\partial z},\tag{12}$$





is solved with a second-order, upwind explicit method. When tracing the ice deposition time, the ice surface boundary condition is X(t) = t. At the ice base, an initial deposition time is prescribed to be several thousand years before the start of the simulation, however this plays little role in the resulting vertical profile of deposition times. When ice is melting at the base  $(\dot{b} < 0)$ , the following flux boundary condition is defined (Rybak and Huybrechts, 2003):

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$$\frac{\partial X}{\partial t} = -\dot{b}\frac{\partial X}{\partial z}.$$
 (13)

Basal freeze-on is assumed to be negligible. It is well known that Eulerian solvers lose accuracy towards the base of the ice sheet, and therefore this method can only be considered to give a first-order estimate of age (Greve et al., 2002; Rybak and Huybrechts, 2003). It can nonetheless be useful for diagnosing the age of ice, in order to know the timescale of different dynamic properties or to, e.g., impose an age-dependent enhancement factor (Greve, 1997b).

#### 10 2.3 Dynamics

The Yelmo dynamics component is currently representative of a "hybrid" class of ice-sheet model, treating different modes of ice deformation via a combination of the simplifying shallow-ice and shallow-shelf approximations (SIA and SSA, respectively). In the following, the description of the dynamics equations follows closely the notation and definitions of Greve and Blatter (2009) and Pollard and DeConto (2012).

Yelmo treats the horizontal velocity u(x, y, z) and v(x, y, z) as the sum of transport via internal shear  $(u_i, v_i)$  and basal sliding  $(u_b, v_b)$ :

$$u = u_i + u_b \tag{14}$$

$$v = v_i + v_b.$$

Here, and analogously for v,  $u_b(x, y)$  represents a depth-averaged velocity, so it is vertically constant, and  $u_i(x, y, z_b) = 0$ , where the subindex "b" here represents the basal boundary of the ice sheet. It also holds that in the vertical average (denoted by a bar),  $\bar{u} = \bar{u}_i + u_b$ . To calculate  $u_i$  and  $v_i$ , Yelmo uses zero-order SIA equations:

$$u_{i}(z) = -\left[2(\rho g)^{n} |\nabla z_{s}|^{(n-1)} \int_{z_{b}}^{z} A(z_{s} - z')^{n} dz'\right] \frac{\partial z_{s}}{\partial x}$$

$$v_{i}(z) = -\left[2(\rho g)^{n} |\nabla z_{s}|^{(n-1)} \int_{z_{b}}^{z} A(z_{s} - z')^{n} dz'\right] \frac{\partial z_{s}}{\partial y},$$
(15)

where  $u_i(z)$  and  $v_i(z)$  are the horizontal components of the SIA velocity as a function of depth at a given location, A is the material rate factor of the ice, which is obtained from the material component, and n is the Glen's Flow law exponent (Glen, 1955; Greve and Blatter, 2009). In the horizontal plane, the term in brackets is calculated on the ab-nodes for stability and mass conservation (Type-I discretization), and then it is staggered onto the ac-nodes where it is multiplied with the surface gradient. In the vertical plane, the horizontal velocities are calculated at the vertical center of each grid point (aa-nodes). Following

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Bueler and Brown (2009), we use the SSA solution to calculate the transport implied by sliding at the base (i.e., in regions of ice streams and floating ice shelves):

$$\frac{\partial}{\partial x} \left[ \eta_d \left( 4 \frac{\partial u_b}{\partial x} + 2 \frac{\partial v_b}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[ \eta_d \left( \frac{\partial u_b}{\partial y} + \frac{\partial v_b}{\partial x} \right) \right] = \rho g H \frac{\partial z_s}{\partial x} - \tau_{\rm b,x}$$

$$\frac{\partial}{\partial y} \left[ \eta_d \left( 4 \frac{\partial v_b}{\partial y} + 2 \frac{\partial u_b}{\partial x} \right) \right] + \frac{\partial}{\partial x} \left[ \eta_d \left( \frac{\partial u_b}{\partial y} + \frac{\partial v_b}{\partial x} \right) \right] = \rho g H \frac{\partial z_s}{\partial y} - \tau_{\rm b,y}.$$
(16)

where  $(\tau_{b,x}, \tau_{b,y}) = -\beta(u_b, v_b)$  (or in vector notation  $\tau_b = -\beta u_b$ ) is the basal stress due to friction. The basal friction co-5 efficient  $\beta$  is set to zero for floating ice shelves, and can otherwise be set to a constant value or follow another user-defined formulation (power law, regularized Coulomb, etc.), depending on the context. See the subsection on basal friction below for details. The depth-integrated (2D) effective viscosity, which is only used for solving the SSA dynamics, is defined as

$$\eta_d = \left[\frac{1}{2} \left(\bar{A}^{-1/n}\right) \left(\dot{\varepsilon}_d^2 + \dot{\varepsilon}_0^2\right)^{\frac{1-n}{2n}}\right] H \tag{17}$$

where  $\overline{A}$  is the vertically-averaged rate factor,  $\dot{\varepsilon}_d$  is the 2D effective strain rate and  $\dot{\varepsilon}_0^2$  is a small regularization factor for 10 avoiding a potential singularity when velocity gradients are zero. The 2D effective strain rate is calculated as a reduced form 16 of the second invariant of the strain rate tensor (Eq. 9) that does not include vertical shear terms:

$$\dot{\varepsilon}_d^2 = \left(\frac{\partial u_b}{\partial x}\right)^2 + \left(\frac{\partial v_b}{\partial y}\right)^2 + \frac{\partial u_b}{\partial x}\frac{\partial v_b}{\partial y} + \frac{1}{4}\left(\frac{\partial u_b}{\partial y} + \frac{\partial v_b}{\partial x}\right)^2.$$
(18)

In Yelmo,  $\dot{\varepsilon}_d$  is only used for calculating  $\eta_d$ , while the 3D effective strain rate is calculated from the full strain rate tensor in the material component (see Material section above). Calculating the full tensor during the iterative SSA solution procedure would be much more computationally expensive, while the 2D effective strain rate is already sufficient for the vertically integrated SSA equations (Pollard and DeConto, 2012).

The stress boundary condition imposed at the floating ice front, following Winkelmann et al. (2011) and Greve and Blatter (2009), is

$$\eta_d \left( 4 \frac{\partial u}{\partial x} + 2 \frac{\partial v}{\partial y} \right) n_x + \eta_d \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) n_y = \left( \frac{1}{2} \rho g H^2 - \frac{1}{2} \rho_{\rm sw} g H_{\rm o}^2 \right) n_x$$

$$\eta_d \left( 4 \frac{\partial v}{\partial y} + 2 \frac{\partial u}{\partial x} \right) n_y + \eta_d \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) n_x = \left( \frac{1}{2} \rho g H^2 - \frac{1}{2} \rho_{\rm sw} g H_{\rm o}^2 \right) n_y.$$
(19)

20 The depth of the seawater up to the flotation depth,  $H_o$ , is defined as:  $H_o = \min\left(z_{sl} - z_b, \frac{\rho}{\rho_{sw}}H\right)$ . This is the depth of the ocean directly adjacent to the ice sheet, which acts to reduce the outward pressure at the floating ice margin. In constrast to Winkelmann et al. (2011), this boundary condition is not currently used in Yelmo for grounded ice, where Eq. 16 applies.

The SSA equations are nonlinear, elliptical, partial differential equations with non-local solutions. Yelmo uses Lis for the numerical solution using the biconjugate gradient method. The subroutine to discretize the equations and to call Lis was ported

from the latest SICOPOLIS version 5-dev (Greve, 2019; Rückamp et al., 2019) and subsequently modified for model design choices in Yelmo. We use a Picard iteration method to account for the nonlinear dependence of the effective viscosity ( $\eta_d$ ), and





potentially the basal friction coefficient ( $\beta$ ), on velocity. Convergence of the SSA solution is tested using the  $L^2$  relative error norm (Gagliardini et al., 2013):

$$\delta_{u,v} = \frac{2\sqrt{\sum (u_1 - u_0)^2 + \sum (v_1 - v_0)^2}}{\sqrt{\sum (u_1 + u_0)^2 + \sum (v_1 + v_0)^2}},$$
(20)

where  $(u_1, v_1)$  and  $(u_0, v_0)$  are the velocity solutions for the current and previous iterations, respectively, and the sum is made 5 over all grid points with non-zero velocity. By default, we consider a convergence limit of  $\delta_{u,v} = 10^{-2}$ , which is typically achieved within 1-10 iterations, depending on the context. This limit can be specified by the user.

The result of solving the above equations is the hybrid, 3D horizontal velocity field (u, v). The vertical velocity w can then be diagnosed by applying a kinematic boundary condition at the base, and integrating the continuity equation for incompressible flow (Greve and Blatter, 2009), from  $z_b$  to z,

10 
$$w(z) = \dot{b} - \left(u_b \frac{\partial z_b}{\partial x} + v_b \frac{\partial z_b}{\partial y}\right) - \int_{z_b}^{\tilde{c}} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz'.$$
 (21)

The vertical velocity is naturally defined on the ac-nodes in the vertical plane, analogous to the horizontal velocity in the horizontal plane. The above dynamics update results in a 3D hybrid velocity field (u, v, w) that is consistent with the current state of the topography.

#### 2.3.1 Basal friction

15 Basal frictional stress, as it appears in the SSA elliptical equations, is defined as

$$\tau_b = -\beta \mathbf{u}_b \tag{22}$$

where  $\beta$  represents the basal friction coefficient, with units of [Pa yr m<sup>-1</sup>], which can be defined in several ways.  $\beta$  is prescribed to be zero for floating ice, and otherwise can be set to a constant or a spatially varying field and, depending on the formulation used, it can depend on velocity itself. For this reason, we also define  $c_b$  as the bed friction coefficient, which we consider to only provide information about conditions at the physical bed (e.g., the nature of basal sediments, basal hydrology, effective

20 only provide information about conditions at the physical bed (e.g., the nature of basal sediments, basal hydrology, eff pressure, etc.), independent of velocity. In the model, therefore,  $\beta$  is defined as:

$$\beta = c_b f\left(\mathbf{u}_b\right). \tag{23}$$

Thus in all formulations implemented in Yelmo, the term  $f(\mathbf{u}_b)$  has units of [yr m<sup>-1</sup>] and the coefficient  $c_b$  has units of [Pa], which helps to facilitate its physical interpretation.

Most commonly,  $\beta$  is defined using a linear (e.g. Quiquet et al., 2018), power-law (e.g. Pattyn, 2017), pseudo-plastic powerlaw (e.g. Aschwanden et al., 2013) or regularized-Coulomb (Joughin et al., 2019) formulation. The linear and power-law formulations are contained within the pseudo-plastic power-law formulation, so only the latter and the regularized-Coulomb formulation are needed to represent all four cases.



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The pseudo-plastic power-law formulation (Schoof, 2010; Aschwanden et al., 2013) is

$$\tau_b = -c_b \left(\frac{|\mathbf{u}_b|}{u_0}\right)^q \frac{\mathbf{u}_b}{|\mathbf{u}_b|} \tag{24}$$

and thus  $\beta = c_b u_0^{-q} |\mathbf{u}_b|^{q-1}$ , with the pseudo-plastic exponent  $q \in (0,1)$  and threshold speed  $u_0$ . This expression results in purely plastic friction for q = 0, linear friction for q = 1 and power-law friction for 0 < q < 1. With q = 1 and  $u_0 = 1$ , for example,  $\beta = c_b$  and friction scales linearly with velocity. To obtain the power-law formulation used in the original MISMIP experiments (Pattyn et al., 2012), the following parameter values can be prescribed: q = 1/3,  $u_0 = 1 \text{ m yr}^{-1}$  and  $c_b = 3.165176 \times 10^4 \text{ Pa}.$ 

Alternatively, the regularized Coulomb law (Schoof, 2005; Brondex et al., 2019; Joughin et al., 2019) is defined as

$$\tau_b = -c_b \left(\frac{|\mathbf{u}_b|}{|\mathbf{u}_b| + u_0}\right)^q \frac{\mathbf{u}_b}{|\mathbf{u}_b|} \tag{25}$$

10 and thus  $\beta = c_b(|\mathbf{u}_b| + u_0)^{-q} |\mathbf{u}_b|^{q-1}$ . Again q is the non-linear exponent and  $u_0$  is an empirical threshold speed that dictates the transition from Coulomb friction when cavitation effects dominate at the base (typically for a hard bed) to Coulomb-plastic friction, when friction saturates (typically for weak till). When  $u_0 = 0$  or q = 0, purely plastic friction is recovered.

The merits and physical basis of the different possible friction formulations and non-linear exponents are still part of active debate (Aschwanden et al., 2013; Stearns and van der Veen, 2018; Brondex et al., 2019; Joughin et al., 2019), and all of

- 15 the above formulations are used in ice-sheet modeling today. However, given the large uncertainty in boundary conditions provided to an ice-sheet model, which include bedrock topography, sediment composition and distribution, basal hydrology and its temporal evolution, etc., it is clear that the use of any formulation will rely on empirical tuning. Also, as noted above, different choices for the friction exponents or threshold values can reduce a given formulation to another. Although modeling studies have shown that all four cases above can produce realistic velocity fields of the present-day ice sheets (e.g. Goelzer
- et al., 2018; Joughin et al., 2019), it remains to be seen how the choice of friction formulation may impact transient changes in the ice sheet.

For these reasons, we have chosen to implement the friction formulations in the most general way possible in the code, with essentially two free parameters: q as a non-linear exponent and  $u_0$  as a threshold speed. Meanwhile,  $c_b$  is a 2D field that can be set to a constant value, or a spatially and/or temporally varying field based on e.g., whether the ice is frozen to the bed or temperate, on till strength (Bueler and van Pelt, 2015), effective pressure, or other user-defined criteria.

 $\beta$  and  $c_b$  are initially defined on aa-nodes.  $c_b$  is naturally defined on the grid center, but when  $\beta = f(\mathbf{u}_b)$ , the velocity components that are defined on ac-nodes must be staggered to the grid center. Once  $\beta$  has been calculated using one of the above formulations, it must be staggered to the ac-nodes for use in the elliptical solver. For purely floating points (i.e.,  $f_g = 0$  at both bounding aa-nodes)  $\beta_{ac} = 0$ , and for purely grounded points,  $\beta_{ac}$  is the average of the two neighbors. At the grounding

30 line, Yelmo allows several options to handle staggering. These include simple averaging, taking the upstream value of  $\beta$ , taking the downstream value of  $\beta$  or taking the weighted average based on the grounded fraction of the ac-node.



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#### 2.4 Thermodynamics

Thermodynamics in Yelmo is treated in the classical way by solving the following energy conservation equation:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c} \frac{\partial^2 T}{\partial z^2} - u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - w \frac{\partial T}{\partial z} + \frac{\Phi}{\rho c}$$
(26)

where k and c are the ice thermal conductivity and specific heat capacity, respectively. The ice temperature T evolution is driven by vertical diffusion, horizontal and vertical advection, and internal strain heating due to ice shearing,  $\Phi$ , where

$$\Phi = 4\eta \dot{\varepsilon}^2. \tag{27}$$

Horizontal diffusion is considered negligible (Greve and Blatter, 2009). At the air-ice interface (i.e., the ice surface), the ice temperature is prescribed via the input boundary temperature field  $T_s$ , limited to a maximum value of  $T_0 = 273.15K$ . At the base of floating ice, the ice temperature is prescribed to the expected freezing temperature of seawater as a function of depth

10 (Jenkins, 1991). At the base of grounded ice, when the ice temperature is below the pressure melting point, the vertical gradient of temperature is prescribed as  $\partial T/\partial z = -Q_{\text{geo}}/k$ , where the geothermal heat flux ( $Q_{\text{geo}}$ ) is provided as a boundary field to Yelmo. If the temperature at the ice base reaches the pressure melting point, then the temperature is prescribed to the pressure melting point, and the basal mass balance is diagnosed as (Cuffey and Paterson, 2010):

$$\dot{b}_{\rm g} = -\frac{1}{\rho L} \left( Q_b + k \frac{\partial T}{\partial z} \bigg|_b + Q_{\rm geo} \right) \tag{28}$$

15 where  $\dot{b}_{g}$  is the basal mass balance of grounded ice (negative for melting), L is the latent heat of fusion for ice,  $Q_{b}$  is the basal heat production to due sliding and  $\frac{\partial T}{\partial z}\Big|_{b}$  is the ice temperature gradient at the base. Yelmo provides  $\dot{b}_{g}$  as a model output, and does not update the basal water layer thickness  $H_{w}$  internally, which can be achieved via external coupling with a basal hydrology model. Once the ice base is temperate (i.e., at the pressure melting point), it will remain so as long as  $H_{w} - \left(\frac{\rho_{w}}{\rho}\dot{b}_{g}\right)dt > 0$ , where  $\dot{b}_{g}$  is used from the previous timestep. In other words, if it is expected that an energy deficit will 20 result in freeze-on of the total available liquid water at the ice base, then the point is treated as a non-temperate ice point.

Eq. 26 is solved with an implicit method in the vertical direction, while the horizontal advection is solved separately applying an explicit, second-order upwind forward Euler method. This separation allows the energy conservation in the vertical to be solved as a 1D column model. The discretization of vertical diffusion follows the form presented by Hoffman et al. (2018), while the discretization of vertical advection follows a second-order central difference scheme. A given column of grid points

25 consists of temperatures defined on the grid-centers (aa-nodes) and boundary values defined directly at the surface and base of the ice sheet.

#### 2.5 Model interface

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The Yelmo model interface is designed to be clear and simple, but also flexible. In its essence, there are three main model functions: yelmo\_init to initialize the model variables, yelmo\_update to perform the ice-sheet model calculations for a given timestep and yelmo\_end to terminate the Yelmo object (free it from memory).





The first subroutine, yelmo\_init, is used to load parameters initialize variables in memory (i.e., allocate arrays) and, optionally, to initialize the topographic state variables (ice thickness, masks, etc.). No other variables are initialized here in the sense of being populated with data values, which is left to the user. An additional, optional helper function can be used, yelmo\_init\_state, which populates the remaining model variables in the material, thermodynamics and dynamics

- 5 components. This initialization step is separated from that of topography because in practice, sometimes boundary variables (e.g., surface temperature) need the surface elevation as input in order to be determined. In contrast, the remaining variables, namely dynamics and thermodynamics, often rely on boundary variables to be initialized. Thus, a typical initialization sequence for a stand-alone ice-sheet model simulation could first call yelmo\_init, then load or calculate boundary variables and then call yelmo\_init\_state to finalize the initialization of all Yelmo variables. After this sequence, the Yelmo state should 10 be consistent with running the model for one timestep with the prescribed boundary conditions and a fixed topography. If the
- model will be initialized from a restart file, then these data are loaded in each case based on parameter choices.

The next subroutine, yelmo\_update, is used to advance the model state to a new timestep. Any modifications to boundary variables are left to the user externally, and Yelmo expects that the boundary conditions are valid for this timestep. The sub-routine does not take any arguments to modify the model behavior – rather, all model configuration choices are specified in the

- 15 parameters of the Yelmo object itself. These are initially loaded from a parameter file in the call to yelmo\_init, however, it is possible to modify any parameter values during simulations, allowing for changing model configuration depending on the experimental setup. An additional optional subroutine, yelmo\_update\_equil, is available to facilitate equilibration. This routine effectively calls yelmo\_update for a specified time window with unchanging boundary conditions, and allowing for the temporary modification of some key model parameters (such as the maximum allowed adaptive time step and the maximum
- 20 allowed SSA velocity).

The last subroutine, yelmo\_end, simply removes the Yelmo object from memory (i.e., all domain variables are deallocated). After calling yelmo\_end, it is possible to reinitialize the Yelmo object via yelmo\_init, for example, in order to test a different grid resolution or other configuration.

- There are several input/output routines defined for Yelmo. yelmo\_write\_init can be used to initialize a NetCDF model output file with the axes of model dimensions defined from the Yelmo object and writing of static fields like domain masks. The writing of model output for inidividual timesteps is left to the user to maintain flexibility, as most programs require specific fields to be written (examples can be found in the test programs included with the code - see further below). In addition, yelmo\_restart\_write will create a NetCDF file and write all Yelmo fields as a snapshot, which can be used to restart the model (loading of a restart file can be activated with parameter choices).
- 30 As mentioned above, given the object-oriented approach, it is possible to run multiple Yelmo domains in one program. Each domain must be initialized separately via <code>yelmo\_init</code>, and the variable fields populated with initial values, then separate calls to <code>yelmo\_update</code> are needed during timestepping, and finally each object should be terminated at the end of the program via <code>yelmo\_end</code>. With this structure, minimum modification of another model, like a global climate model is needed, to incorporate online ice-sheet evolution, or to simulate an ensemble of ice sheets in one program. Furthermore, all fields
- 35 are directly accessible within the main program to facilitate coupling. For example, the 2D array of surface elevation of the





topography component of the Yelmo Antarctica domain could be referenced as yelmo ant%tpo%now%z srf. While it is clear that the nesting of several containers (derived types) results in a rather long variable reference, it is unambiguous and straightforward to use.

## 3 Model validation and benchmarks

- Yelmo has been tested against several ice-sheet model validation tests and benchmarks in wide use today. These include the 5 EISMINT1 (Huybrechts et al., 1996) and EISMINT2 (Payne et al., 2000) model intercomparison experiments that test uncoupled and coupled dynamics-thermodynamics, respectively, and MISMIP (Pattyn et al., 2012) for ice-shelf dynamics, among others. By design, many of these experiments allow isolation of specific model features for testing. When the model passes more complex benchmark tests, the simpler experiments are somewhat redundant (if the model passes a coupled thermodynamics-
- dynamics benchmark, the model should necessarily also be able to pass a dynamics-only benchmark). However, it should be 10 noted that in the process of model development, all tests prove to be extremely useful. The results of all tests will not be reported here, but several are highlighted below to demonstrate that Yelmo performs well.

The EISMINT1 moving margin experiment tests the ice-sheet model dynamics with an imposed constant rate factor and diagnosed thermodynamics (i.e., thermodynamics do not impact the ice-sheet configuration). Radial steady-state surface mass

- 15 balance and background surface temperature fields are imposed as boundary conditions. Starting from ice-free conditions, the ice sheet simulated by Yelmo grows to dynamic and thermodynamic equilibrium within 25 kyr and 100 kyr, respectively. The steady-state summit elevation of Yelmo is 2992.4 m compared to the reported range of 2997.5±7.4 m for "Type-I" discretization models like Yelmo (where diffusivity is staggered to the ab-nodes). The basal temperature relative to the pressure melting point (i.e., homologous temperature) at the summit simulated by Yelmo is -12.85 °C, which lies within the EISMINT1 range of  $-13.40\pm0.56$  °C. Other relevant statistics are given in Table 1. 20

The EISMINT2 benchmark experiments A and F are useful for testing the thermodynamically coupled ice-sheet model. The experiments are identical to the EISMINT1 moving margin experiment, except the resolution is doubled (25 km) and the surface temperature is prescribed to be independent of ice thickness. Experiment A prescribes a summit temperature of 238.15 K, while experiment F is 15 K colder, which promotes an increase in the region of ice frozen to the bedrock. The

- statistics for these experiments are listed in Table 1 as well. Figure 3 shows the basal homologous temperature distribution 25 for experiments A and F. Yelmo produces symmetrical temperature patterns in both experiments, which are consistent with both the benchmark results (Payne et al., 2000) and other more recent models (e.g., Bueler et al., 2007; Hoffman et al., 2018). Yelmo also produces the so-called "cold spokes", which have been shown to be related to internal strain heating in regions of steep gradients in ice thickness, and largely numerical in nature (Bueler et al., 2007). Yelmo includes an option for smoothing
- 30 the strain heating field as suggested by Bueler et al. (2007), however its effectiveness has not been tested yet in detail.

We also test the capability of the SSA solver and grounding-line treatment by running the MISMIP protocol experiments (Pattyn et al., 2012). Particularly, MISMIP EXP 1 (advance) and EXP 2 (retreat) are useful for testing the reversibility of grounding line advance, given the bedrock is defined as a linear downward sloping bed. The rate factor is prescribed according



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to steps that first decrease, allowing grounding line advance, then increase back to the original value. According to theory (Weertman, 1974; Schoof, 2007), only one steady-state grounding line position should exist for each step – i.e., the ice sheet should advance and retreat symmetrically without showing hysteresis. It is now well known, however, that ice-sheet models at coarse resolutions (1 km and greater) are unable to capture proper grounding-line migration, even when subgrid parameterizations to mimic higher resolution are applied (Seroussi et al., 2014; Gladstone et al., 2017).

In the MISMIP experiment performed here, the linear, downward sloping bedrock is defined in the x-direction as  $z_b = 720 - 778.5 (x/750)$  km. The bedrock elevation does not change in the y-direction, which extends to  $\pm 50$  km to allow the simulation of a symmetric ice stream flowing in the positive x-direction. The power-law formulation of Eq. 24 is used with the parameter values q = 1/3,  $u_0 = 1$  m yr<sup>-1</sup> and  $c_b = 3.165176 \times 10^4$  Pa. The rate factor is initially prescribed to  $A = 1 \times 10^{-16}$  Pa<sup>-3</sup> yr<sup>-1</sup> and the simulation is run for 25 kyr to equilibrate. Next, the rate factor is stepped evenly in log-space every 10 kyr until reaching  $A = 1 \times 10^{-19}$  Pa<sup>-3</sup> yr<sup>-1</sup>, and then the rate factor is increased in the same way until returning to the original value.

Figure 4 shows results for this MISMIP experiment with Yelmo at different grid resolutions, ranging from 20 km down to 2.5 km, and with different treatments of basal friction near the grounding line. When the default model setup is used, with no special treatment at the grounding line, the grounding line advance is consistent for all resolutions. However, none of the

- 15 lowest resolution simulations show grounding line retreat as the rate factor increases again. At a resolution of 5.0 km, some minor grounding line retreat can be seen, and for 2.5 km, the model is more successful at retreating though it remains 400 km from the target. In contrast, when the basal friction  $\beta$  is scaled at the grounding line by the grounded fraction of the ac-node ( $f_{g,ac}$ ), the hysteresis is greatly reduced. The 5 km simulation retreats to within 200 km of the original position and the 2.5 km simulation retreats to within 100 km of the original position, thus showing convergence to the correct solution with resolution.
- 20 With this setup, even the 10 km and 20 km simulations retreat significantly. In a third case, the basal friction is also linearly scaled to zero as the ice sheet approaches flotation (Leguy et al., 2014; Gladstone et al., 2017). In this case, the hysteresis and differences between different resolution simulations are further reduced, however, the system also tends to advance much less given all other conditions are the same.

Yelmo's Eulerian ice-age model is validated against the analytical solution at the summit presented by Rybak and Huybrechts (2003). In this case, horizontal advection is neglected, and the vertical velocity is assumed to decrease linearly with depth. Fig. 5 shows the solution with Yelmo as compared to the analytical result. For a nominal vertical resolution of  $n_z = 30$  points and single or double precision, the age tracer gives errors of around 0.2% over the entire depth of the ice sheet. Increasing the vertical resolution to  $n_z = 50$  points decreases the error by an order of magnitude and reducing the vertical resolution to  $n_z = 10$  points increases the error to about 1%. For Eemian-age ice in such simplified conditions, this gives an uncertainty of

30 about 1 kyr. It is expected that the error would increase for more realistic domains (Rybak and Huybrechts, 2003), however the Eulerian age solver can be used for a first-order estimate of the age-depth profile in the ice sheet.





#### 4 Antarctica

As further validation of the model's performance, we ran steady-state simulations of the present-day and glacial Antarctic ice sheet. These simulations, run at 32 km resolution, have been deliberately simplified to include the minimum complexity necessary to simulate the ice sheet without additional external components. There was no active isostasy model and geothermal
heat flux was set to 50 mW m<sup>-2</sup> everywhere. The bedrock topography and initial ice thickness were prescribed from the RTOPO2.1 dataset (Schaffer et al., 2016). Basal friction followed a linear law, where β = c<sub>f</sub>λ<sub>b</sub> (ρgH). We prescribed c<sub>f</sub> = 2 × 10<sup>-3</sup> for most of the domain, except for ad hoc adjustments in specific regions to improve the match with observations. This was additionally scaled by an exponential function of bedrock elevation: λ<sub>b</sub> = min[1.0, exp((z<sub>b</sub> - z<sub>1</sub>)/(z<sub>1</sub> - z<sub>0</sub>))], where z<sub>0</sub> = -200 m and z<sub>1</sub> = 0 m, which ensures that the basal friction decays to low values for marine-based sectors of the ice sheet
(Martin et al., 2011). Friction was scaled by the grounded fraction at the grounding line, but no additional scaling is applied.

For the simulation of the present-day state, surface mass balance (SMB) and surface temperature boundary fields were prescribed from a RACMO2.3 simulation driven by ERA-INTERIM data and averaged over 1981-2010 (van Wessem et al., 2018). The ice-shelf basal mass balance was set to a spatially constant value of  $-0.1 \text{ m a}^{-1}$ . Figure 7 shows a comparison of the simulations with the observed topography (RTOPO2.1) and the present-day observed velocity (Rignot et al., 2011). With

- this relatively simple model setup, it is nonetheless possible to obtain reasonable agreement with observations. The root mean square errors (RMSEs) in ice thickness, velocity and log(velocity) are 290 m,  $230 \text{ m yr}^{-1}$  and  $1.8 \log[\text{m yr}^{-1}]$ , respectively, which fall in the range of other models in the initMIP-Antarctica intercomparison project (Seroussi et al., 2019). The dome configuration, slow deformational speeds and even most ice streams as they penetrate inland are well represented by the model. The margins of the ice sheet are the most difficult to match and, in particular, the large ice shelves. This is likely due in large
- 20 part to the use of a spatially constant value for ice-shelf basal mass balance, but also reflects the simple approach used to represent  $\beta$ . Nonetheless, the results are quite consistent with other models (e.g., Martin et al., 2011; Quiquet et al., 2018; Seroussi et al., 2019).

We use the same setup with modified boundary conditions to simulate a configuration resembling that of a deep glacial period like the Last Glacial Maximum. The surface temperature was set to  $10^{\circ}$  C colder and the present-day SMB was maintained,

- 25 except for points with a low or negative SMB were prescribed with a minimum value of  $0.1 \text{ m a}^{-1}$ . The ice shelf basal mass balance was set to a spatially constant value of  $0.0 \text{ m a}^{-1}$  and sea level was lowered by 120 m. In this case, the grounded ice sheet advances until the continental shelf break and thickens inland (Fig. 7). A similar structure of ice streams can be seen, due to the topographic dependence of  $\beta$ , but their speed is greatly reduced. We do not expect this configuration to be realistic, given that isostasy plays no role and a present-day-like SMB has been imposed. However, this test demonstrates that Yelmo is
- 30 capable of resolving continental-scale changes in the ice sheet configuration in a plausible way.

#### 5 Conclusions and future work

We have described the features and physics of the hybrid ice-sheet-shelf model Yelmo. Yelmo includes the physics to simulate continental-scale ice sheets and floating ice shelves using "shallow" approximations of the ice dynamics. The fully coupled



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thermomechanical ice-sheet model has been validated against several benchmark tests, and has been shown to simulate the dynamic configuration of the Antarctic ice sheet well.

Yelmo is expected to be useful for long-time scale simulations and/or ensembles. It is particularly suited for easy coupling with other models. For example, the simulation of multiple ice-sheet domains with independent parameter configurations coupled to a global climate model can be achieved in a simple and straightforward way. Also, given that the subroutines representing the physics of the model have been isolated from the "model accounting", it is possible to test individual model components in different contexts easily. This should facilitate future model development and comparison of different methods.

The model framework has been designed to facilitate the incorporation of new and different physics. Thus, this initial release of Yelmo lays the foundation for several future developments. These may include more advanced calving and basal friction

10 schemes, as well as improved treatment of the grounding line. We also plan to transition to an enthalpy-based thermodynamics solver, and to implement a variationally-derived "depth-integrated-viscosity approximation" solver (following e.g., Goldberg, 2011; Pollard and DeConto, 2012; Lipscomb et al., 2019) in the near future.

*Code availability.* Yelmo is maintained as a git repository hosted at https://github.com/palma-ice/yelmo under the licence GPL-3.0. Model documentation can be found at https://palma-ice.github.io/yelmo-docs/. The exact version of the model, along with the necessary input data, used to produce the results used in this paper is archived on Zenodo (https://www.doi.org/10.5281/zenodo.3443654).

*Author contributions.* A.R., J.A.S. and M.M. conceived the model design and features. A.R. wrote the model code with contributions from the remaining authors. All authors contributed to the model testing and writing the manuscript.

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**Figure 1.** Overview of the Yelmo model structure highlighting state variables in the four components: topography, dynamics, material and thermodynamics, as well as the boundary conditions required to run the model. The thick black border for boundary variables indicates that these fields are never modified internally by Yelmo, while the components with a thin black border or dashed line are allowed to be modified depending on the context. When, for example, the topography is updated (dashed line), no other components are allowed to be modified (solid lines).







**Figure 2.** Yelmo staggered grid definition and nomenclature. The horizontal grid (left) assumes constant resolution in the x- and y-directions, while in the vertical (right) variable resolution is allowed. With any given cell defined as a 3D box, scalar variables are calculated on cell centers (aa-nodes), velocities are calculated on cell faces (ac-nodes, edges in 2D), and scalar coefficients are calculated on cell edges (ab-nodes, corners in 2D).







**Figure 3.** Steady-state, basal homologous temperature ( $^{\circ}$  C) distribution after 100 kyr obtained by Yelmo in EISMINT2 test A (left) and test F (right). Areas that have reached the pressure-melting point have been shaded grey.







**Figure 4.** Yelmo performance in the MISMIP bedrock advance and retreat simulations on a linear sloping bed. The top panel shows the imposed rate factor *A*, with 10 kyr steps of decreasing and then increasing values. The three lower panels show the grounding line position evolution for each of three model configurations, respectively: "Default" is the standard model setup, with no special treatment of friction at or near the grounding line, "Subgrid" uses the grounded fraction at the grounding line to scale the basal friction, "Scaling" applies both the grounded fraction, and imposes a linear reduction in basal friction as the ice sheet approaches flotation. Separate simulations were run for resolutions ranging from 20 km down to 2.5 km.







Figure 5. Age-depth profile for idealized summit (left) with an analytical solution (thick grey line) and Yelmo (black line) with a vertical resolution of  $n_z = 30$  and double precision, and the associated error (right, %). The error for additional resolutions of  $n_z = 10$  and  $n_z = 50$  are also shown for comparison (thin dark grey lines, smaller points) and for the same resolution, but single precision (thin, light grey line, smaller points).







**Figure 6.** Antarctica present-day ice sheet configuration and surface velocities from observations (left), compared to a steady-state simulation with Yelmo (middle). In contrast, Antarctica glacial configuration and surface velocities simulated by Yelmo (right). Simulations were performed at 32 km resolution. The colors show surface velocity in  $m yr^{-1}$  and the dark grey contours show surface elevation in 500 m intervals (thick lines correspond to 0, 1000, 2000 and 3000 m above sea level). The black line shows the grounding line position.





**Table 1.** Yelmo performance in the EISMINT1 moving margin experiment ("Moving"), as well as in the EISMINT2 experiments A and F. Where available, metrics with the ensemble mean and standard deviation from the original benchmark experiments are also provided for comparison.

Experiment	Model	Volume [10 <sup>6</sup> km <sup>3</sup> ]	Area [10 <sup>6</sup> km <sup>2</sup> ]	Melt fraction	Divide thick- ness [m]	Divide basal temperature [K]	Divide homologous basal temperature [C]
Moving	EISMINT1	_	-	-	$2997.5\pm7.4$	_	$-13.40 \pm 0.56$
	Yelmo	1.939	1.003	0.58	2992.6	256.9	-13.61
А	EISMINT2	$2.128\pm0.073$	$1.034\pm0.043$	$0.72\pm0.15$	$3688\pm48$	$255.6 \pm 1.4$	_
	Yelmo	2.205	1.031	0.71	3736	254.9	-15.03
F	EISMINT2	_	-	-	_	_	_
	Yelmo	2.458	1.040	0.36	4374	240.9	-28.50