Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2019-258-AC1, 2020

Geoscientific Model Development Discussions

Interactive comment on "On the numerical integration of the Lorenz-96 model, with scalar additive noise, for benchmark twin experiments" *by* Colin Grudzien et al.

Colin Grudzien et al.

cgrudzien@unr.edu

Received and published: 8 February 2020

1 Introduction

The authors would like to thank the referees for their time and their valuable comments – their reviews have been very useful for clarifying several points in the text, and for improving the robustness of our results. We will highlight the main change to the text in the following, and address smaller points in the section thereafter.

2 Main change

Our largest change to the manuscript is to address the following comments:

- **Referee 1:**

  > In Section 3.1, there should be an explanation on that, in the order tests with different time-steps, how the Taylor scheme in Section 2.5 is implemented. In particular, how the random vectors are added up to consistently approximate the multiple stochastic integral (which leads to the Taylor scheme).

- **Referee 2:**

  > One thing that would have been interesting to see (although perhaps difficult to produce) would be to compare trajectories generated with the various schemes but with the same noise realization (probably requiring Brownian bridges due to the different interior point samples).

In our original manuscript, we included a description on how we used the Brownian motion realizations, discretized on the fine scale for the reference path, to consistently define the coarse discretization methods and the Brownian bridge for the Taylor scheme. However, this earlier exposition was not written in a clear and obvious way, and to account for this issue we have expanded and refined this description in our new Appendix B.

In addition, we have clarified the exposition of the overall experimental setup in Section 3.1 to make this more understandable. This is in conjunction with revising this experimental setup as in the following suggestion:

- **Referee 1:**

> In the convergence order tests, three time-step sizes are used, and this is not very convincing, in particular in Figure 2. It would be more convincing to use about four or five time-step sizes.

We appreciate this suggestion, and we believe our new Figs. 1 - 2 are indeed a nice improvement over the previous versions. We have chosen to use the discretization time steps $\{2^{-q}\}_{q=5}^{9}$ exactly as was suggested. In order to do so, we have also changed the step size of the finely-discretized reference path to 2^{-23} and the time horizon $T = 0.125$. These changes are performed in order to keep the discretization of the Brownian motion realizations consistent between the fine and coarse step-sizes, as well as to consistently define the Brownian Bridges in between the coarse steps. The methods for doing so are detailed in our revised Section 3.1 and our new Appendix B.

Updates are performed to Table 1, for the estimated coefficients for the bounds on the expected discretization errors based on these new experiments. Statements that used calculations based on Table 1 have been updated. Specifically, we have found that the estimated discretization error for the Taylor scheme, using a step size 5×10^{-3}, is bounded by approximately 0.001075 across all difussion regimes. For this reason, we rephrase earlier conclusions stating that the error was bounded by 10^{-3} to state that the error is close to 10^{-3} across all regimes.

The updated figures are included in this comment at the bottom of the text.

3 Minor changes

- **Referee 1:**

  > There might be typos in the indexing of i, j, b in Eq.(15-16).

  Response

  > We believe that the indices are correct, but we note that we suppressed the use of the index b in the terms on the right-hand-side. To make this more clear, we have included text before to indicate that we have suppressed indices, and after the equations to clarify which ones are suppressed.

- **Referee 1:**

  > The conclusion seems weak and too specific (in particular, the time-steps in this specific test setting are presented without further analysis). It might be helpful to explore more on the balance between the level of diffusion and the right-hand-side of the equation, as well as the Lyapunov exponent of the system.

  Response

  > We appreciate this suggestion and to address this we have slightly expaned the conclusion to discuss when the state evolution is drift dominated versus diffusion dominated. However, relating this specifically to the Lyapunov time of the system appears to us to be a subtle question and one that we are not prepared to answer at this stage of the work. We believe that this is a worthwhile question for future investigation, but one that goes beyond our current scope.

- **Referee 2:**

  > It would be interesting to see if introducing inflation into the data assimilation scheme (artificially increasing the diffusion coefficient used by the filter to compensate for model error) could have compensated for the large errors introduced by the Euler-Maruyama scheme at the coarse time scale.

Response

We appreciate the suggestion, and we believe that this is an interesting question to investigate. However, at the moment we feel that this will go beyond the scope of the work.

Interactive comment on Geosci. Model Dev. Discuss., https://doi.org/10.5194/gmd-2019-258, 2019.

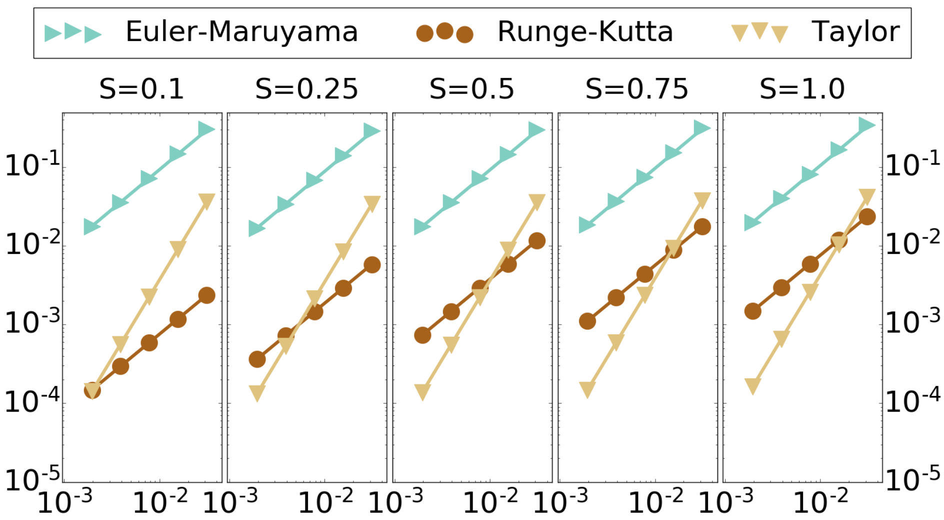

Fig. 1. Strong Convergence Benchmark - Revised

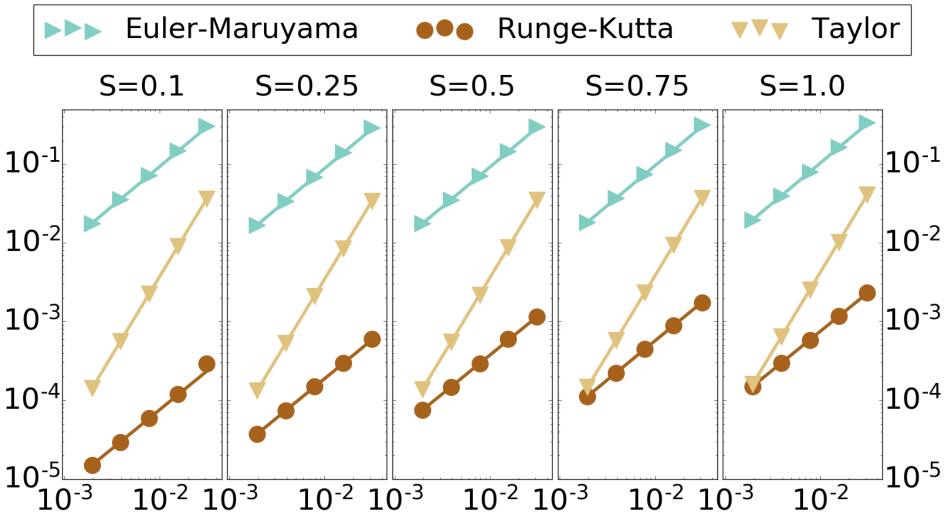

Fig. 2. Weak Convergence Benchmark - Revised