

Enforcing conservation of axial angular momentum in
the atmospheric general circulation model CAM6

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Numerical general circulation models of the atmosphere are generally required to conserve mass and energy for their application to climate studies. Here we draw attention to another conserved global integral, viz. the component of angular momentum (AM) along the Earth’s axis of rotation, which tends to receive less consideration. We demonstrate the importance of global AM conservation in climate simulations on the example of the Community Atmosphere Model (CAM) with the finite-volume (FV) dynamical core, which produces a noticeable numerical sink of AM. We use a combination of mathematical analysis and numerical diagnostics to pinpoint the main source of AM non-conservation in CAM-FV. We then present a method to enforce global conservation of AM, and we discuss the results in a hierarchy of numerical simulations of the atmosphere of increasing complexity. In line with theoretical expectations, we show that even a crude, non-local enforcement of AM conservation in the simulations consistently results in the mitigation of certain persistent model biases.

1 Introduction

The atmosphere exchanges angular momentum (AM) with the material bodies at the surface which are, to a good approximation, in a state of motion consisting in uniform rotation about the planetary axis connecting the poles. Per unit of mass, surface AM increases in quadratic proportion to its distance from the planetary axis of rotation, from zero at the poles to a maximum at the Equator. AM is a constant of motion of the dynamical (e.g. Newton’s) equations, so that as air travels meridionally, it carries a specific AM that increasingly differs from that of the Earth’s surface. A variety of mechanisms redistribute atmospheric AM and eventually lead to an exchange of AM between the atmosphere and the surface, mainly as a result of low-level wind shear (“surface stress”) and of small-scale wave motions over steep surface topography (“form drag”).

The importance for the atmospheric circulation of conservation of AM in the free troposphere and of AM exchange of air with the surface was recognised long ago. Already in 1735, George Hadley, Esq, F.R.S., noted that without the Assistance of the diurnal Motion [i.e. rotation] of the Earth, Navigation [...] would be very tedious (Hadley 1735), due to the absence of the trade winds. This insight still lies at the core of modern conceptual models for the atmospheric circulation (Schneider, 1977; Held and Hou, 1980; Lindzen and Hou, 1988; Pauluis, 2004; Walker and Schneider, 2006). In the upper branch of the Hadley Circulation (HC), the advection of planetary angular momentum determines a sharp acceleration of the zonal wind in the mid-latitudes, linked with a front-like drop in air temperatures, marking the location of the subtropical jets (STJs). Partly by baroclinic instability, the mid-latitude circulation redistributes atmospheric

43 AM vertically and produces intense surface westerlies, where the air loses AM to the surface.
44 The equatorward return flow in the surface branch of the HC in turn results in easterly “trade”
45 winds, where surface stresses replenish atmospheric AM until air is lifted in cumulus convection
46 within the inter-tropical convergence zone (ITCZ).

47 This circulation is the object of numerical simulations with general circulation models (GCMs)
48 used in meteorological forecasting and in climate modelling. They describe the atmosphere as
49 a thin, density-stratified, rotating gaseous spherical shell. These properties allow the introduc-
50 tion of a convenient set of approximations in the equations of motion, which result in a system
51 known as the Hydrostatic Primitive Equations (HPEs). The reader is referred to White et al.
52 (2005) for a detailed analysis and discussion. Given suitable boundary conditions, the HPEs
53 guarantee the global conservation of three fundamental physical quantities: mass; energy; and
54 AM along the Earth’s rotation axis. Analytic expressions of these laws can be found e.g. in
55 Laprise and Girard (1990). The three conservation laws determine the fundamental character of
56 the large-scale circulation of the atmosphere, and virtually every climate application of GCMs is
57 sensitive to their enforcement when the continuum equations are discretized in space and time.
58 For example, the effects of changes in radiative forcing of 2 W/m^2 (e.g. IPCC AR5, Chapter
59 8, pg 697) can only be simulated if the model’s energy conservation is significantly better than
60 1%. Estimates based on ECMWF reanalysis data suggest that conservation of AM of a simi-
61 lar precision is desirable for an accurate representation of the annual cycle and of interannual
62 variations of the atmospheric circulation in model simulations (e.g. Egger and Hoinka 2005).

63 CAM, the Community Atmosphere Model developed and maintained at the National Center
64 for Atmospheric Research (NCAR) in Boulder, Colorado, is one of the Atmospheric General
65 Circulations Models (AGCM) in most widespread use today. It also constitutes the core at-
66 mospheric component of NorESM, the Norwegian Earth System Model. Although it offers a
67 choice of dynamical cores, the finite-volume (FV) dynamical core (Lin 2004) has been, and in
68 many instances still is, the default option. The FV dynamical core is exactly mass and vorticity
69 conserving, and it has been employed in all model integrations submitted by NCAR and by the
70 Norwegian Climate Centre (NCC) for the 5th phase of the Coupled Model Inter-comparison
71 Project (CMIP) contributing to the Assessment Report (AR) of the Intergovernmental Panel
72 for Climate Change (IPCC 2013); it is also expected to be used for phase 6 of CMIP by both
73 institutions. Due to its high numerical efficiency, FV also continues to be the code of choice for
74 all uses where overall availability of supercomputing resources is a limiting factor. This includes
75 long historical or palaeoclimate simulations; studies with coupled chemistry and/or carbon cy-

76 cle; seasonal-to-decadal coupled forecasts; academic research; and all model development efforts
77 currently underway with NorESM.

78 In this paper, we employ CAM with the FV dynamical core at two standard CESM resolution
79 only, a coarser one of $1.9^\circ \times 2.5^\circ$ in latitude and longitude, respectively (“f19” for short), and
80 a finer one of $0.9^\circ \times 1.25^\circ$ (“f09”). In agreement with previous results (Lauritzen et al., 2014;
81 Lebonnois et al., 2012), we find that all existing simulations with CAM FV, from CMIP5 to
82 present development versions of CAM6, have a numerical sink of global AM of a magnitude of
83 about 30% of physical sources at f19 resolution, and about 15% at f09 resolution.

84 Figure 1 shows the spurious AM source in aquaplanet (AP; Neale and Hoskins, 2000; Black-
85 burn et al., 2013) and Held-Suarez (HS; Held and Suarez 1994) simulations with CAM FV, and
86 an otherwise identical simulation, but with using the global spectral dynamical core with T42
87 truncation. Although many other models also do not conserve AM, CAM FV is peculiar in
88 producing a sink nearly everywhere, resulting in a particularly large global non-conservation.

89 First principles (e.g. Held and Hou, 1980; Einstein, 1926) suggest that dissipation of AM,
90 equivalent to a body force acting on the fluid as a sink of zonal momentum, forces a secondary
91 circulation with the same sign as the Hadley circulation. As a result, the simulated Hadley
92 circulation may become too vigorous. Reduced meridional advection of zonal momentum may
93 lead to mid-latitude Westerlies that are too weak or displaced poleward. The zonal momentum
94 lost to the non-physical sink must be balanced by a matching additional eastward torque, for
95 example in an expanded or excessively intense area of tropical easterly surface winds. Model
96 simulations with CAM FV consistently tend to reflect such phenomenology: for example, Feldl
97 and Bordoni (2016) and Lipat et al. (2017) show that among CMIP5 models, those based
98 on the FV dynamical core (GFDL-x, CCSM4 and NorESM-x) simulate both relatively large
99 overturning mass flux in the HC, and a high latitude of its edge.

100 It is useful to illustrate these effects of AM non-conservation by means of idealised AGCM ex-
101 periments that do not include complicating factors such as orographic form drag or parametrised
102 bulk stresses associated with gravity waves. Figure 2 shows the surface torques resulting from
103 four solutions for the mean circulation with CAM in AP mode. One of these is obtained directly
104 from integrations of CAM using the FV dynamical core at f19 resolution (black line). An oth-
105 erwise identical integration with the global spectral-transform dynamical core at T42 spectral
106 truncation (green line) is chosen for comparison as a bone-fide example of an AM conserving
107 simulation (cf Figure 1).

108 The other two integrations, represented by the blue and red lines, are perturbed in identical,

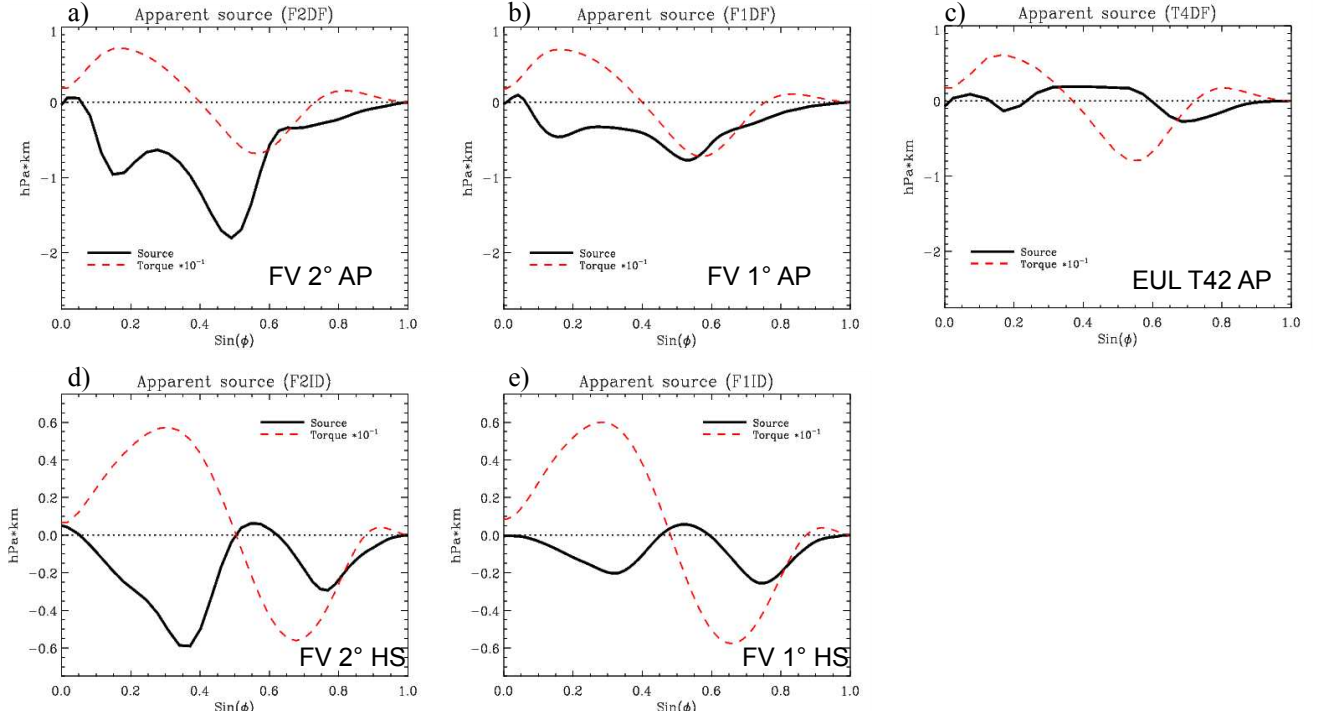


Figure 1: Numerical torque in idealised CAM simulations. The vertically and zonally integrated apparent numerical torque is shown as a function of latitude for CAM simulations in Aquaplanet (AP; panels a), b) and c) in the top row) and Held-Suarez (HS; panels d) and e) at the bottom) configurations. The numerical torque here is obtained as a time-average residual of the tendency of angular momentum in each cylindrical shell of constant latitude of the model’s domain, after subtracting the contributions from meridional convergence and from the surface stress torque. The details of the calculation are in Appendix A. Two simulations with the FV dynamical core are shown for each configuration, one at f19 resolution (i.e. on a regular latitude-longitude grid with spacing of $1.9^\circ \times 2.5^\circ$; panels a) and d)), and one at f09 (i.e. with twice that resolution; panels b) and e)). For comparison, also a CAM simulation in AP configuration with the global spectral dynamical core at quadratic triangular truncation T42 (roughly comparable to FV at f19 resolution) is shown in panel c). The dashed red line in each panel indicate the physical torque from surface stresses, scaled by a factor 0.1. Positive values indicate an eastward torque acting on the atmosphere, and negative values indicate a westward torque acting on the atmosphere.

109 but opposite manner. First, the global-total numerical torque due to the FV dynamical core was
110 diagnosed at every time-step of the reference FV simulation, and averaged in time afterwards.
111 This was converted into a solid-body axial rotation tendency that was applied continuously
112 everywhere as a constant sink of AM in a new integration with the spectral dynamical core,
113 resulting in the simulation represented by the red curve. Vice-versa, the opposite additional
114 solid-body rotation tendency was applied to a new FV integration, thus compensating its internal
115 numerical sink. This integration produced the physical torque represented by the blue curve.

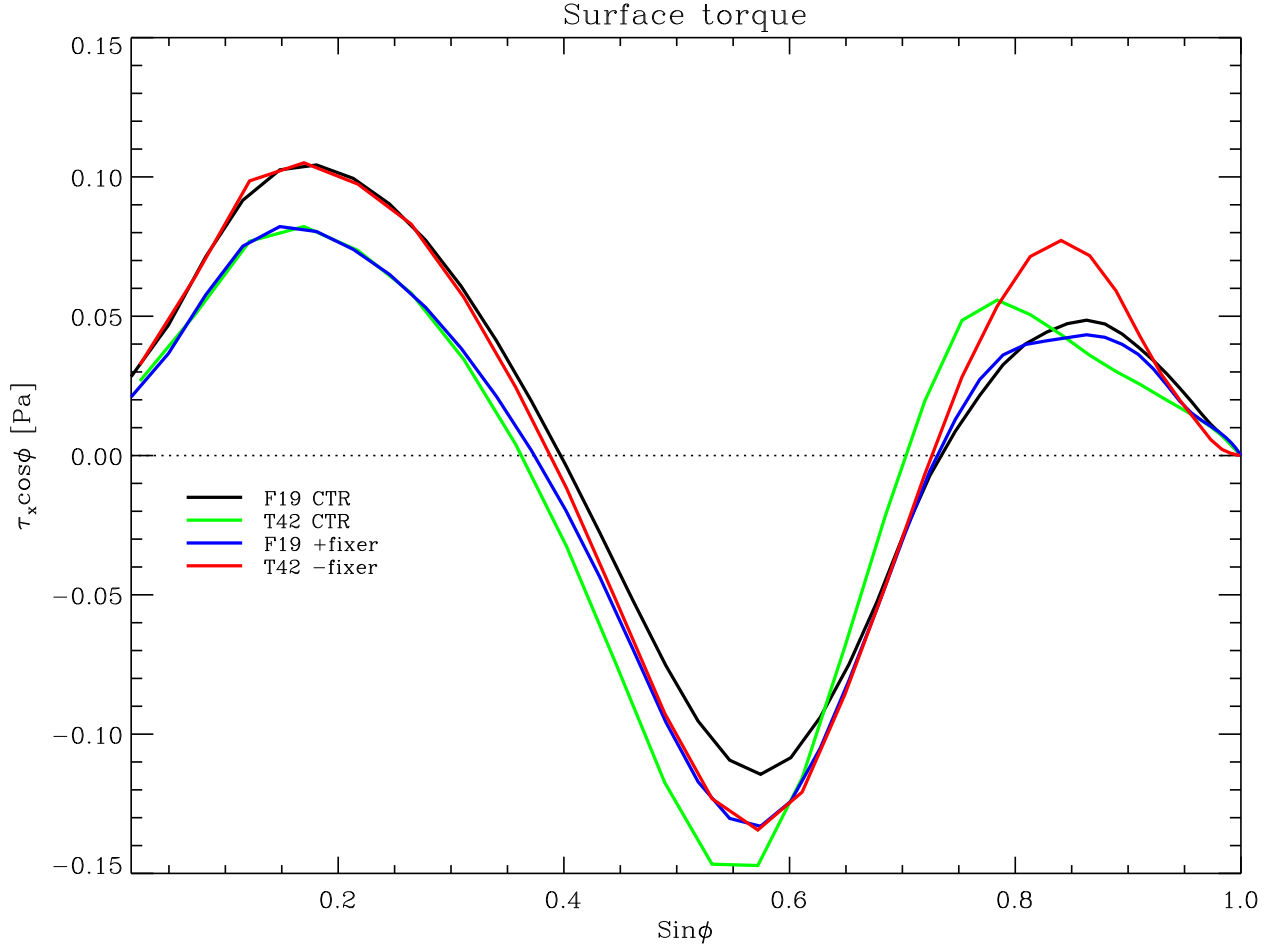


Figure 2: Impact of AM sink in CAM-FV integrations. Meridional distribution of the surface stress torque (analogous to the dashed red lines in Figure 1) in CAM simulations in AP configuration. Two integrations with the FV dynamical core (black and blue lines), and two simulations with the global spectral dynamical core (green and red lines) are shown. One of each pair of integrations is a control case (black and green lines), the other (blue and red lines) is an experiment where an additional solid-body angular acceleration is applied to the entire atmosphere at each time-step of the integration. The acceleration is diagnosed as the time mean of the ratio between the global total numerical torque in the FV control integration and the moment of inertia of the atmosphere. That acceleration is then applied with a negative sign in the FV experiment (blue curve), with the effect of compensating for the numerical torque and achieving approximate global AM conservation in that integration. For the experiment with the spectral dynamical core (red curve), the acceleration is applied with unchanged sign, causing a sink of AM approximately equal to that of the control FV integration. The numerical sink of the control spectral integration is nearly vanishing.

116 Comparing the different curves, it may be seen that Equatorward of about 23 degrees of latitude
 117 the simulated physical torque depends primarily on the global budget of atmospheric AM. In

118 particular, notwithstanding the complications of interactive moist physics and the different
119 spatial and temporal discretisations used in the two integrations, the stronger trade winds
120 (in terms of surface stress) in the FV simulation compared with the T42 simulation can be
121 explained entirely with the non-physical, numerical torque of the FV dynamical core. The
122 result is insensitive to how that torque is in fact applied. Even at subtropical and middle
123 latitudes, half of the difference between the two simulations, in terms of surface stresses, can be
124 explained in this way. Similar results are found for the zonal-mean meridional circulation and
125 for the surface pressure in the HC (Figure S1 in the Supplementary Information), confirming
126 the strength and robustness of the Einstein (1926) “tea-leaves” mechanism.

127 These results motivate us to address the issue of AM conservation in the CAM’s FV dynamical
128 core. One may speculate that systematic biases in surface stresses due to the numerical sink
129 of AM must also impact coupled ocean-atmosphere climate simulations, with excessive Ekman
130 and Sverdrup forcing of the subtropical gyres. The northward displacement of the mid-latitude
131 westerlies may also result in excessive mechanical and thermal forcing of the subpolar gyres with
132 possible implications for the Atlantic meridional overturning circulation.

133 In this paper, we propose ways to address numerical dissipation of AM in CAM-FV sim-
134 ulations. Section 2 describes our main hypotheses as to the root cause of the error, and our
135 approaches towards rectification. Section 3 presents the result of our corrections in a set of
136 idealised simulations. The impact on realistic simulations of the atmospheric circulation is
137 discussed in Section 4. Conclusions are finally offered in Section 5.

138 **2 Analysis of potential causes and approaches to correc-** 139 **tion.**

140 The FV dynamical core (Lin 2004) solves the HPE by updating first the advective (C-grid)
141 and then the prognostic (D-grid) winds in two steps. The first step represents pure advection,
142 i.e. the increments associated with transport, including geometric and Coriolis terms. In this
143 step, the scheme conserves absolute vorticity exactly for the D-grid winds (Lin and Rood 1997;
144 hereafter LR97). The second step calculates the wind increments associated with hydrostatic
145 pressure forces. These are computed in a special way (Lin 1997) that differs from most Arakawa
146 and Lamb (1980) type schemes. Violations of AM conservation may occur in either sub-step.

147 2.1 Pressure-gradient force

148 We first analysed the Lin's (1997) treatment of the pressure-gradient terms for conservation. A
 149 general discussion is given by Simmons and Burridge (1980), who introduce a set of hybrid-level
 150 dimensionless variables, a_k , defined as $a_k := (\phi_k - \phi_{k+1/2})/2(\alpha p)_k$ (in Simmons and Burridge
 151 these variables are denoted by α_k ; we change the notation here to avoid confusion), where ϕ is
 152 the geopotential, p the pressure, $\alpha := -\partial_\eta\phi/\partial_\eta p$ the specific volume, and η is the generalised
 153 or hybrid vertical coordinate. Here and in the following, the index k refers to the vertical
 154 level, or to half-levels as appropriate, and subscripts to the partial derivative symbol indicate
 155 differentiation with respect to the variable in subscript, $\partial_X \equiv \partial/\partial X$. The variables a_k need
 156 not be constants. Simmons and Burridge (1980) derive the discrete form that pressure and
 157 geopotential terms must take in general vertical coordinates in order to ensure conservation of
 158 axial angular momentum. Their Equation (3.8) can be generalised to:

$$(\alpha \partial_\lambda p + \partial_\lambda \phi)_k = - \left(\frac{\Delta\phi}{\Delta p} \right)_k \partial_\lambda p_{k-1/2} + \partial_\lambda \phi_{k+1/2} + \frac{1}{\Delta p_k} \partial_\lambda [a_k(\alpha p)_k \Delta p_k] , \quad (1)$$

159 where the symbol Δ is employed to represent a difference between vertical levels, $\Delta p_k := p_{k+1/2} -$
 160 $p_{k-1/2}$ (and similarly for ϕ), and λ is the longitude.

161 Performing Lin's (1997) path integration around the finite-volume element on this expression
 162 yields the following form for the body force:

$$\oint \phi dp = \delta_\lambda \{ [\phi_{k+1/2} + a_k(\alpha p)_k] \Delta p_k \} - \Delta (\overline{\phi} \delta_\lambda p)_k \quad (2)$$

163 where δ_λ is the finite-difference operator in the zonal direction, and $\overline{\phi_{k\pm 1/2}}$ is an average over λ .
 164 An expression identical in form to Lin's (1997) Equation (11) is then recovered if the choices

$$a_k = \frac{\Delta\phi_k}{2(\alpha p)_k} , \quad \overline{\phi} = \frac{\phi_{i+1/2} + \phi_{i-1/2}}{2} , \quad (3)$$

165 are made, where i is the index corresponding to the longitude λ .

166 In other words, Lin's (1997) expression for the pressure-gradient term is consistent with
 167 Simmons and Burridge (1980) prescription for AM conservation, provided that the physical
 168 pressure variable p is used in the integration in place of the general pressure function indicated
 169 by the symbol π in Lin (1997). This can be directly verified algebraically by summing all
 170 expressions of the form of the numerator in the right-hand side of Equation (11) in Lin (1997)
 171 along all longitudes and levels. Provided ϕ is constant at one model boundary, and p at the
 172 other, it always returns zero. This is the required result provided that the denominator on the

173 right-hand side of Eq.(11) in Lin (1997) represent the inertial mass associated with the velocity
174 points. They do so if π is the hydrostatic pressure.

175 Accordingly, we performed tests in which the integration variable in the relevant section of
176 CAM-FV’s dynamical core was replaced with true interface pressure. The effect was generally
177 seen to be very small on the dynamical core’s momentum conservation properties.

178 We note however that in the CAM implementation there may be an additional problem, asso-
179 ciated with the use of the D-grid. The application of Lin’s (1997) method would strictly require
180 a C-grid, with zonal velocity points interleaving pressure (scalar) points along the same latitude.
181 Thus, in CAM pressure is interpolated to the grid-cell corners before use. While the formal ex-
182 pressions for the pressure forces do not change, thus ensuring S&B’s total torque constraints,
183 the inertial mass associated with each D-grid U -point is in fact averaged over six scalar point
184 surrounding it, with 1-2-1 weights along the zonal direction. This additional zonal smoothing
185 effectively adds spurious terms to the zonal momentum equation, of the form $-u\partial_x^2\Delta p$. This is
186 a potential source of non-conservation. However, it is not expected to be systematic.

187 **2.2 Geometry, polar filtering, and FFSL extension**

188 AM conservation may be affected by the treatment of geometric terms in latitude-longitude
189 coordinates, especially near the poles where such terms become large. Furthermore, convergence
190 of the meridians forces filtering of the solution, and additional approximations to be made. In
191 particular, LR97 implement a flux-form semi-Lagrangian extension of Colella and Woodward’s
192 (1984) PPM algorithm which is used near the poles where CFL numbers become large during
193 the time integration. We performed several sensitivity tests on each of these aspects, without
194 being able to notice significant impacts on AM conservation.

195 Particularly compelling is the comparison with the performance of a prototype implementa-
196 tion in CAM of the FV scheme on a cubed-sphere grid (“FV3”), which lacks any poles and does
197 not require or use any of these special formulation (and is, in particular, run in pure Eulerian
198 mode, i.e. without the flux-form semi-Lagrangian extension described in Lin and Rood, 1996).
199 We ran an AP simulation on the C48 grid, viz. six pseudo-cubic faces with 48x48 grid-cells each,
200 for total number of grid-points identical to the standard 2-degree FV configuration, but a 25%
201 higher resolution at the Equator. The AM sink (Figure S2 in the Supplementary Information)
202 is nevertheless comparable, i.e. about 25% smaller, consistently with the scaling with the res-
203 olution of simulations with standard FV. We conclude that FV and FV3 suffer from the same

204 problem, independent of geometry or the FFSL extension of LR97.

205 In order to minimise the impact of other minor (and partly intentional) numerical sources
 206 and sinks of AM, in all idealised numerical tests presented in this paper we applied the following
 207 modifications: 1. the order of the advection scheme is kept the same (4th) for all model layers,
 208 instead of reducing it to 1st in the top layer and to 2nd up to the 8-th layer; 2. an additional
 209 conservation check is applied in the vertical remapping of zonal wind and column momentum
 210 is conserved in the moist-mass adjustment at the end of physics; 3. the surface-stress residual
 211 resulting from closure of the diffusion operator (in physics) is applied in full rather than partially.

212 **2.3 Discretisation of the kinetic-energy term**

213 The evidence from our theoretical and diagnostic analysis points at the advective, shallow-water
 214 part of the implementation of LR97 in CAM-FV as the root of the AM conservation error. Its
 215 "vector-invariant" formulation (Arakawa and Lamb 1981) allows for different forms of the diver-
 216 gence to be used in the momentum and in the mass and tracer equations, resulting in inconsistent
 217 values for the divergence of the flux of planetary AM (associated with mass divergence) and of
 218 the flux of relative AM (associated with momentum divergence). In the momentum equations,
 219 the divergence is contained in a kinetic-energy (KE) gradient term, which due to the presence of
 220 a numerical symmetric instability (Hollingworth et al., 1983) is expressed as the local gradient
 221 of a Lagrangian-average KE. Its form violates the finite-volume approximations used for other
 222 quantities, e.g. vorticity. This feature is intrinsic to the LR97 numerical discretisation scheme
 223 and cannot be eliminated.

224 To address the resulting violation of AM conservation, we first note that even in AM-
 225 conserving schemes, conservation can only be guaranteed in the zonal average (Simmons and
 226 Burridge, 1980). We therefore do not attempt a local correction to the scheme, which is li-
 227 able to numerical instabilities (Hollingworth et al., 1983), and instead formulate a zonal-mean
 228 correction as follows. We enforce the AM conservation law:

$$\int d\lambda \partial_t (\Delta p u a \cos^2 \varphi) = - \int d\lambda \partial_\varphi (\Delta p u v \cos^2 \varphi) + \int d\lambda \Delta p f v a \cos^2 \varphi \quad (4)$$

229 by adding a zonal-mean zonal-wind tendency term to the "vector-invariant" form:

$$\begin{aligned} \partial_{t,c} \bar{u} &= \frac{1}{\int d\lambda \Delta p} \\ &\times \left\{ \int d\lambda \Delta p \left(\frac{1}{a \cos \varphi} \partial_\lambda K - \zeta v \right) - \int d\lambda \frac{1}{a \cos^2 \varphi} \partial_\varphi (\Delta p u v \cos^2 \varphi) - \int d\lambda u \partial_t \Delta p \right\}. \end{aligned} \quad (5)$$

230 Here, K is the KE plus the contribution from explicit divergence damping used in FV. In the
 231 continuum limit the expression on the right-hand side reduces simply to the mass-weighted zonal
 232 average of the zonal gradient of $K - (u^2 + v^2)/2$.

233 In discrete form, the last two terms must be approximated. In the C-D grid formulation of
 234 the LR97 scheme the second one is especially problematic. Various possibilities were explored,
 235 which resulted in various degrees of accuracy and stability. The best compromise is to discretise
 236 it as

$$\frac{1}{a \cos^2 \varphi} \partial_\varphi (\Delta p uv \cos^2 \varphi) = \frac{1}{a \cos^2 \varphi} [\Delta p v \partial_\varphi (u \cos \varphi) + u \partial_\varphi (v \Delta p \cos \varphi)] , \quad (6)$$

237 allowing some confusion between prognostic D-grid winds and time-centred advective (C-grid)
 238 winds. The details of the derivation are given in Appendix B. Using the mass conservation
 239 equation, this approximation allows us to discretize the two last terms together and write the
 240 zonal-wind correction increment in a form consistent with LR97:

$$\delta_c \bar{u} = \frac{1}{\int d\lambda \overline{\Delta p_{t+\delta t}}} \left\{ \int d\lambda \overline{\Delta p} \left[\frac{\delta t}{a \cos \varphi \delta \lambda} \delta_\lambda K - \overline{\mathcal{Y}(v^*, \delta t; \zeta_\lambda)} \right] + \bar{u}^t \mathcal{F}(u^*, \delta t; \overline{\Delta p}) + O(\delta t^2) \right\} . \quad (7)$$

241 Here, $\zeta_\lambda := \frac{1}{a \cos \varphi} \partial_\lambda v$, and the notation of LR97 is used for the discrete transport operators \mathcal{Y}
 242 and \mathcal{F} , for the meridional transport of ζ_λ and the zonal transport of mass, respectively. The first
 243 three terms in the integrand of Eq.(7) thus correspond to the first three terms on the right-and
 244 side of Eq.(A11) in Appendix B. The last symbol on the right-hand side of Eq.(7) represents
 245 higher-order terms (also detailed in Eq.(A11)). We will refer to this modification of the LR97
 246 scheme as the ‘‘correction’’.

247 2.4 Diagnostic tools and global conservation

248 Irrespective of whether the correction, as described above, is applied or not, for diagnostic pur-
 249 poses we calculate the apparent non-physical torque associated with the FV dynamical core
 250 advective tendencies only, i.e. excluding the increments associated with pressure gradients.
 251 These tendencies are diagnosed separately for each layer at every advective sub-step, and inte-
 252 grated horizontally to yield the apparent numerical global-total torque during the sub-step. At
 253 the same time, the layer effective moment of inertia over the sub-step is also computed.

254 The opposite of the ratio of these quantities gives an angular acceleration that, applied to
 255 the zonal wind in each layer at every advective sub-step, enforces conservation of AM of that
 256 layer under advection. The application of this solid-body rotation increment at each dynamical

257 time-step and for each layer independently is what we call the “level” fixer. The details of the
258 computation are given in Appendix C.

259 Irrespective of whether they are actually applied, the fixer’s velocity increments, Eq.(A13),
260 are vertically interpolated and accumulated over the entire dynamic time-step, and written out
261 diagnostically. In addition to the fixer, partial wind and pressure tendencies arising from the
262 dynamical core are separately diagnosed and written to the standard output streams, providing
263 additional diagnostic tools for cross-checking.

264 A variant of the fixer was tested in CAM simulations. This variant is a “global” fixer,
265 which still acts by applying an increment to the zonal wind at each time-step. In this fixer, the
266 apparent torque and the moment of inertia are integrated over all levels within the domain over
267 which strict overall angular momentum conservation is desired. The zonal wind increments are
268 then applied as a single solid-body rotational acceleration within this domain. Experimentation
269 showed that such acceleration should not be applied in the stratosphere, where conservation
270 errors are small and the impact of unphysical zonal accelerations large. The necessary limitation
271 of the domain for the global fixer however introduces a certain degree of arbitrariness in its
272 application. Although sometimes used for diagnostic purposes, we do not discuss this global
273 fixer variant any further.

274 Lin’s (2004) FV scheme conserves mass and absolute vorticity exactly. The AM modifica-
275 tions, described above, were explicitly designed not to alter the mass flux calculations, and
276 intervene only on the rotational component only of the flow in the momentum equations. Other
277 choices, involving alterations to the calculation for the divergent flow, would have been possible.
278 However, we judged exact mass conservation more important for climate simulations than exact
279 vorticity conservation. The AM modifications also change the kinetic energy of the flow, and
280 thus change the total energy budget of the model. However, the unmodified FV scheme does not
281 conserve energy. CAM-FV therefore employs an energy “fixer” (analogous to our AM fixer), de-
282 scribed e.g. in Williamson et al. (2015). The fixer diagnoses the energy non-conservation at each
283 time-step. This allowed us to monitor the impact of the AM mods on energy non-conservation
284 in all our experiments. We found no systematic effect, either in sign or in magnitude, of the AM
285 modifications on the energy non-conservation of the model.

3 Numerical Simulations and Results

3.1 Dry baroclinic wave tests

Initial tests were carried out for adiabatic dynamics and flat bottom topography, from baroclinically unstable initial conditions as defined in Jablonowsky and Williamson (2006; “JW06”). Figure 3 shows the result in terms of conservation of global AM for CAM-FV integrations at f19 resolution (1.9×2.5 degree of latitude and longitude) and 30 hybrid levels.

It may be seen that both the correction and the fixer are effective in reducing the systematic numerical sink of AM in these integrations. In particular, the fixer appears to remove it almost completely; in other words, the integration with the fixer conserves global AM in the time mean. This result is central to this paper, and it proves its two main conclusions. The first is that the systematic non-conservation of global AM in the FV dynamical core indeed resides in the advective wind increments of the shallow-water part of the dynamical core. The second is that, by virtue of its effectiveness, and its formulation that is entirely independent of the model configuration or parametrisations (topography, physical momentum sources, etc), the fixer is a useful and accurate general diagnostic tool that allows us to quantify the numerical torque in any CAM-FV integration. By virtue of this quality, the diagnosed time-averaged fixer tendencies were for example used for the perturbations in the experiments shown in Figures 2 and S2.

The impact of the correction on conservation is generally smaller, and different dynamical regimes may be seen when the size and quality of that impact changes. In the baroclinic instability tests of Figure 3, the correction achieves good results in the linear and non-linear stages of baroclinic growth (up to day 30; cf JW06), but is not able to correct the slow drift that sets in after zonalisation of the global flow, then wind speed decreases everywhere as a result of numerical dissipation (there are no external sources or sinks of either momentum or energy in these adiabatic simulations). This is a partly desirable behaviour, as the action of the correction should not change the dissipation properties of the scheme.

Aside from the conservation properties they are designed for, both the correction and the fixer represent a perturbation of the numerical solutions of the FV dynamical core. By arbitrarily modifying the relative vorticity associated with the zonal wind, both destroy one of the fundamental numerical properties of the LR97 formulation, viz. the conservation of absolute vorticity under advection. (In the case of the fixer, the vorticity input has a rigid dependency on latitude, $\sin\varphi$). Figure 4a shows their impact on the accuracy of the JW06 baroclinic wave test in terms of root-mean-square (RMS) of the differences in surface pressure from a nominal

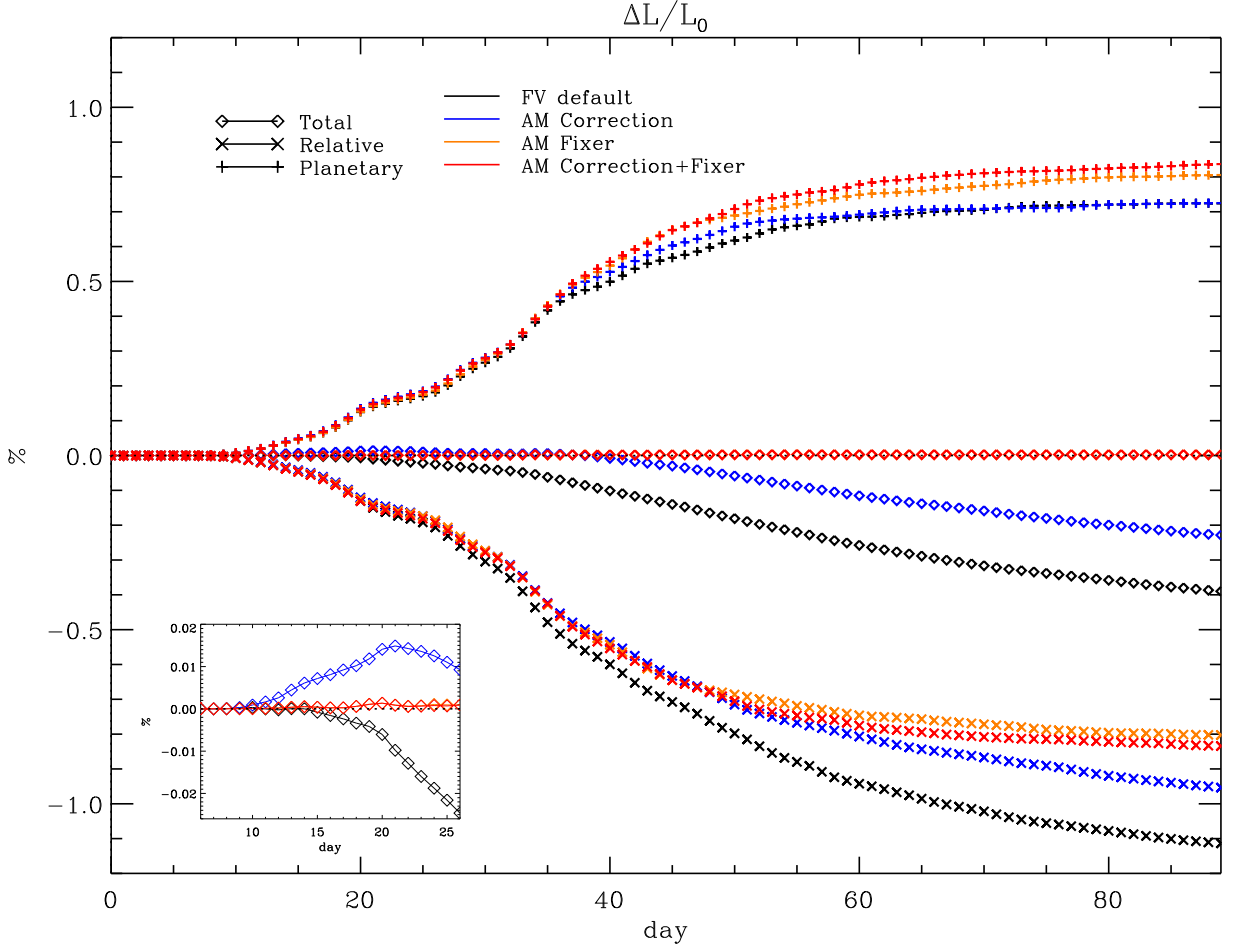


Figure 3: AM correction and fixer in adiabatic, frictionless baroclinic wave tests. Three sets of curves are shown for each of four different simulations with CAM FV, indicating the time evolution of global AM (diamond shapes) and its two components of planetary AM (vertical crosses) and relative AM (x-crosses) in each simulation. Total AM and each AM component are normalised to the initial total AM of the initial state, and differences with respect to initial values are shown, expressed in percentage. Standard CAM-FV is shown in black, CAM-FV with the AM correction only in blue, CAM-FV with the AM fixer only in yellow, and CAM-FV with both AM correction and fixer in red. The inset panel on the lower right of the Figure shows an enlargement for the initial evolution of total AM. Note that the four simulations are nearly indistinguishable before day 8, i.e. during the linear phase of the baroclinic wave. All simulations are run on the two-degree grid.

318 reference solution with original FV dynamical core. The latter is obtained at f19 resolution
 319 ($0.9^\circ \times 1.25^\circ$), which is sufficiently close to JW06's reference solution (cf JW06, Section 5(e),
 320 points (i) and (ii)) for our purposes. It may be seen that on this measure the solutions with and
 321 without the AM corrections are virtually indistinguishable during the stages of both linear and
 322 nonlinear baroclinic growth. A similar result holds for the phase (not shown).

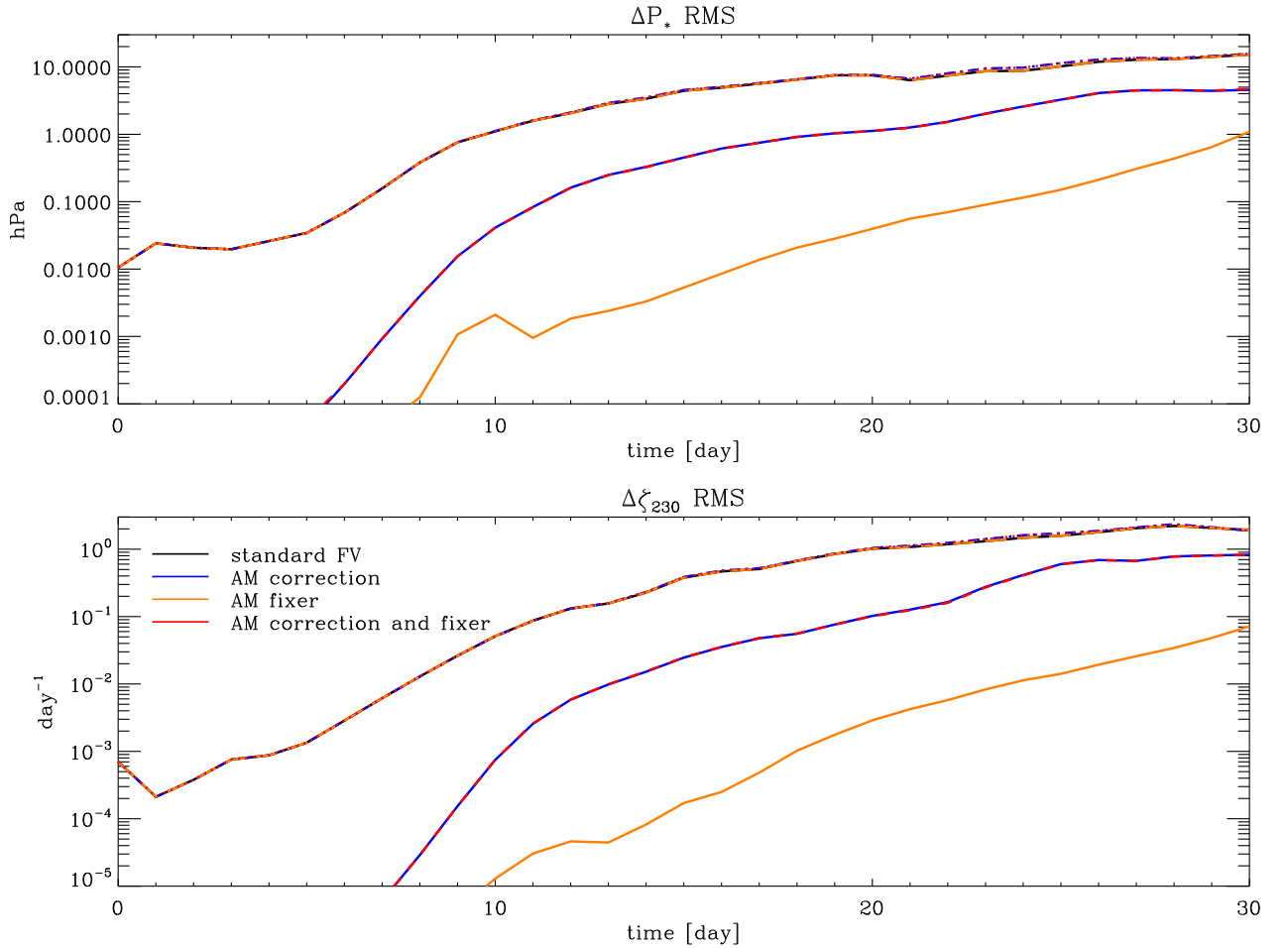


Figure 4: AM correction and fixer in adiabatic, frictionless baroclinic wave test. The simulations shown in Figure 3 are compared with a standard CAM-FV simulation at one degree resolution, and against each other. Each panel shows seven curves, four of which nearly overlap and form the top-most set of lines (including the reference simulation with standard FV). These represent the time evolution of the RMS difference of surface pressure (top panel) and relative vorticity at 230hPa (bottom panel) of each of the two-degree integrations and the control one-degree integration. Below that set of curves are two nearly overlapping curves, which show the RMS differences of the two-degree experiments with AM correction only and the control two-degree integration (blue lines), and of the experiment with both AM correction and fixer and the control integration (red lines). Finally, the single yellow lines at the bottom in each panel show the RMS differences of the two-degree integration with AM fixer only with the two-degree control integration.

323 It may be noted that the largest impact on the RMS of surface pressure arises from the
 324 correction. Within the first 30 days this impact is formally always well below significance (as
 325 defined in JW06, cf their Figure 10), but it increases in time and eventually becomes appreciable
 326 as a full global meridional circulation is established. Similar results hold for the vorticity field,

327 as seen in Figure 4*b*).

328 Other aspects of the solution besides RMS differences also show limited sensitivity to the
329 application of the correction and the fixer. Figure 5 shows the evolution of the minimum pressure
330 in the developing baroclinic wave. By this measure, the solutions only start to diverge with the
331 filling of the primary cyclone and the deepening of the secondary wave after day 17. The solution
332 with the fixer deepens the secondary cyclone more quickly so that the minimum pressure is seen
333 to jump from first to the second wave minimum between days 18 and 19; this occurs one day
334 later with the unmodified dynamical core. A third transition after day 25 has higher central
335 pressure in the solutions with the fixer; by this time, however, rapid cyclogenesis is occurring
336 in the jet stream of the southern hemisphere, attaining a similar minimum pressure, which is
337 slightly deeper in the solutions with the fixer. In any case the pressure differences of the minima
338 remain of the order of a few hPa, and there is no systematic difference in their position.

339 **3.2 Other idealised tests**

340 Even if the impacts of the modifications of the FV dynamical core are relatively small on local
341 circulations over subseasonal time-scales, as shown above, the rationale for introducing them is
342 the hope of achieving a better simulation of the state of the atmosphere in integrations under
343 specified forcings. As explained in the introduction, one particular expectation is that the
344 subtropical easterlies should weaken, without affecting the circulation elsewhere too heavily. In
345 particular the role of the correction, which alone does not ensure AM conservation, must be
346 clarified, and its eventual use justified. Here we document the results of two sets of idealised
347 simulations that still have a simplified, equipotential lower boundary, but include non-vanishing
348 physical torques and heating tendencies.

349 The first set of such simulations adhere to the benchmark test of Held and Suarez (1994;
350 “HS” henceforth), where the forcing has the form of a relaxation towards a specified three-
351 dimensional atmospheric temperature field. Likewise, surface friction is represented by a damp-
352 ing of the winds within a set of levels near the bottom boundary. Apart from the small numerical
353 diffusion, these stresses are communicated to the rest of the atmosphere by means of momen-
354 tum advection in the mean circulation, and of pressure fluctuation in resolved transient motions
355 (including travelling waves). The second set of simulations follows the Aquaplanet (“AP”) test
356 first proposed by Neale and Hoskins (2000), where only a persistent field of bottom-boundary
357 temperatures is prescribed (the “QOBS” profile of Neale and Hoskins 2000), and the full set of

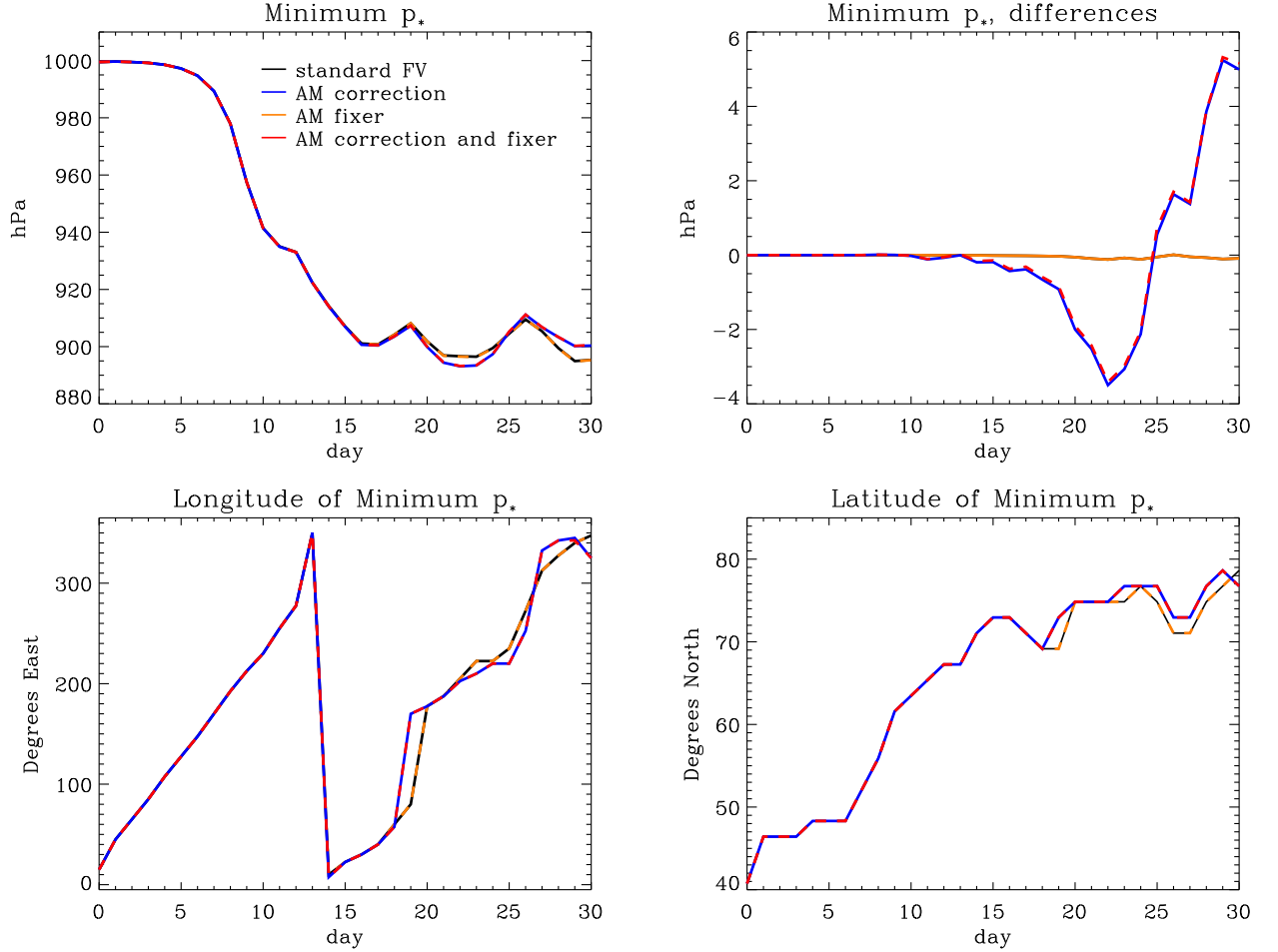


Figure 5: AM correction and fixer in adiabatic, frictionless baroclinic wave test. Evolution of minimum pressure (panel on the top-left) and its position (panels at the bottom) in the baroclinic-wave evolution from the integrations shown in Figure 3. Colour-coding of the lines is the same as in Figure 3. The panel on the top-right shows the differences in minimum pressure between the AM experiments and FV control, with the same colour coding as in the lower curves in Figure 4.

358 moist atmospheric physical parametrisations of CAM6 are used to force the circulation (except
359 for those specific to orographic processes). The bottom boundary is a notional static ocean with
360 unlimited heat and water capacity. Surface stresses are computed by the coupler, and passed
361 to the moist atmospheric boundary-layer parametrisation which then distributes those stresses
362 vertically. Momentum is also transported in moist convection, where active, and further adjust-
363 ments are made when the moist mass of the atmospheric column changes due to precipitation
364 and surface evaporation processes. To simplify the analysis, the gravity-wave parametrisation
365 of CAM6 was turned off in our AP tests. In both sets of tests, FV's advection scheme is used

366 at PPM’s standard fourth-order at all levels, i.e. the numerical diffusion obtained in standard
 367 CAM-FV integrations by employing low-order calculations near the model top is avoided. For
 368 initial conditions, HS simulations are cold-started with uniform surface pressure and geopo-
 369 tential, and vanishing wind fields except for a westerly perturbation identical to that used in
 370 the dry baroclinic wave tests (necessary in order to break zonal symmetry and to allow a non-
 371 vanishing correction). The AP simulations all take the same instantaneous atmospheric state
 372 from a previous spun-up run, even though this requires more adjustment for the corrected/fixed
 373 simulations than for the control.

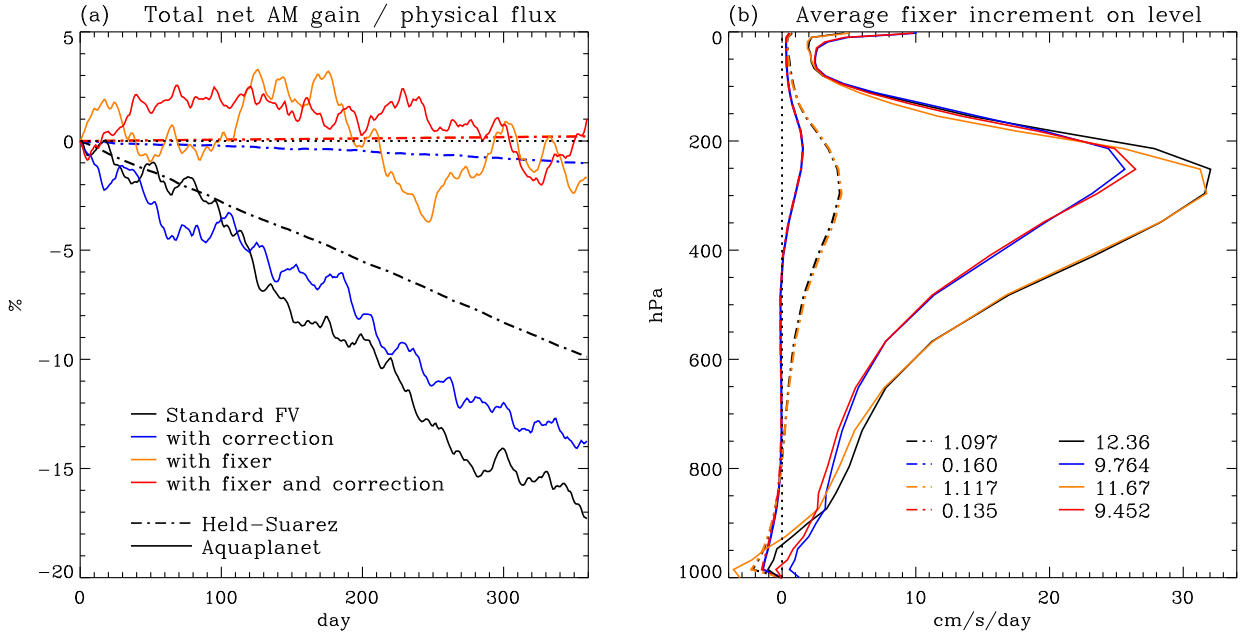


Figure 6: AM correction and fixer in Held-Suarez (HS) and Aquaplanet (AP) integrations. Panel (a) shows the time evolution of total AM for each of the integrations, similar to Figure 3 (diamond shapes) but normalised, separately for each integration, to the time-integrated physical (i.e. surface stress) torque at day 360. AP integrations are shown in solid, HS integrations in stippled lines. The colour coding is as in Figure 3. Panel (b) shows the time-mean numerical torque, averaged over days 120-360, arising at each model level from advective increments, as diagnosed by the fixer, and expressed as equatorial acceleration in a solid-body rotation required to compensate for the numerical sink. Line types and colours correspond to those shown in panel (a). The lists at the bottom of panel (b) indicate the time-mean equatorial accelerations of a *global* solid-body rotation, i.e. the increments shown by the lines but integrated vertically level by level, weighted with the appropriate moments of inertia.

374 Figure 6a indicates that the global AM conservation properties of the simulations in these
 375 tests are broadly in line with the expectations from the previous discussion. Standard FV tests
 376 (black lines) show a steady loss of AM in the atmospheric circulation, of a magnitude of the

377 order of 10-20% of the physical flux of AM through the atmosphere. (We count eastward stress
378 as positive, by which the atmosphere gains westerly momentum in the tropical surface easterlies,
379 and loses westerly momentum in the subtropical surface westerlies). Use of the correction leads
380 to an order-of-magnitude reduction of the numerical sink of AM in HS integrations, but it is
381 of limited effectiveness in full-physics AP integrations (blue lines). Integrations with the fixer,
382 with or without the correction (orange and red lines, respectively), maintain atmospheric AM
383 in the time mean. In HS simulations, there appears to be a very small residual drift of AM
384 notwithstanding the fixer. This is due to a small inconsistency in the application of the stress
385 terms, which are calculated and diagnosed in the “physics” part of the model time-stepping,
386 but applied later as velocity tendencies in the physics-dynamics interface on updated layer
387 masses. This is an intrinsic feature of the time-stepping of CAM-FV that we have not modified.
388 More notably, AP simulations differ from HS simulations in that they show obvious fluctuations
389 of total AM around the time mean or around the long-term drift, when there is one. Such
390 fluctuations are similar in all AP integrations, with a magnitude of a few percent of the physical
391 sources, and depend on non-conservation in CAM’s physics parametrisations. Fortunately, they
392 are not systematic and do not produce a noticeable long-term drift.

393 The effectiveness of the fixer in removing most of the AM drift confirms that the systematic
394 sink of AM in CAM-FV integrations arises predominantly from the shallow-water advection
395 calculations. The accuracy of the correction, by contrast, depends on the features of the cir-
396 culation, with good accuracy for numerically well-resolved features, as in the HS tests, but a
397 poorer one when grid-scale forcing associated with the water cycle occurs. Figure 6b gives more
398 details on the effect of the correction. Here, the time-average AM sink due to the dynamical
399 core is diagnosed using the fixer increments for the zonal velocity at the equator at each model
400 level. This diagnostic is produced irrespective of whether such increments are applied during
401 the integration. Apart from the smaller increments in HS integrations than in AP integrations,
402 which partly depend on the slower circulation (“surface” stresses are one order of magnitude
403 larger in the HS set-up than in the AP set-up), the advective AM sink has a distinctive shape in
404 pressure-level space, with a maximum in the upper troposphere and small values in the atmo-
405 spheric boundary layer. This shape partly reflects the underlying global-mean zonal wind field,
406 but the maximum sink lies below the maximum wind (at around 250 hPa rather than around
407 150 hPa). The profile of the impact of the correction, i.e. the reduction in fixer increments
408 when the correction is applied, has again a similar shape but with an even lower position of the
409 maximum, which better corresponds with the maximum in the vertical profile of level-integral

410 zonal momentum of the underlying flow. Combined with the off-line diagnostic information for
 411 the apparent AM sink from Figure 1, it can be deduced that the main loci of the time-mean
 412 AM sink in these simulations are found near the subtropical jet streams, where large zonal
 413 asymmetries occur in both the mass fields and the wind fields.

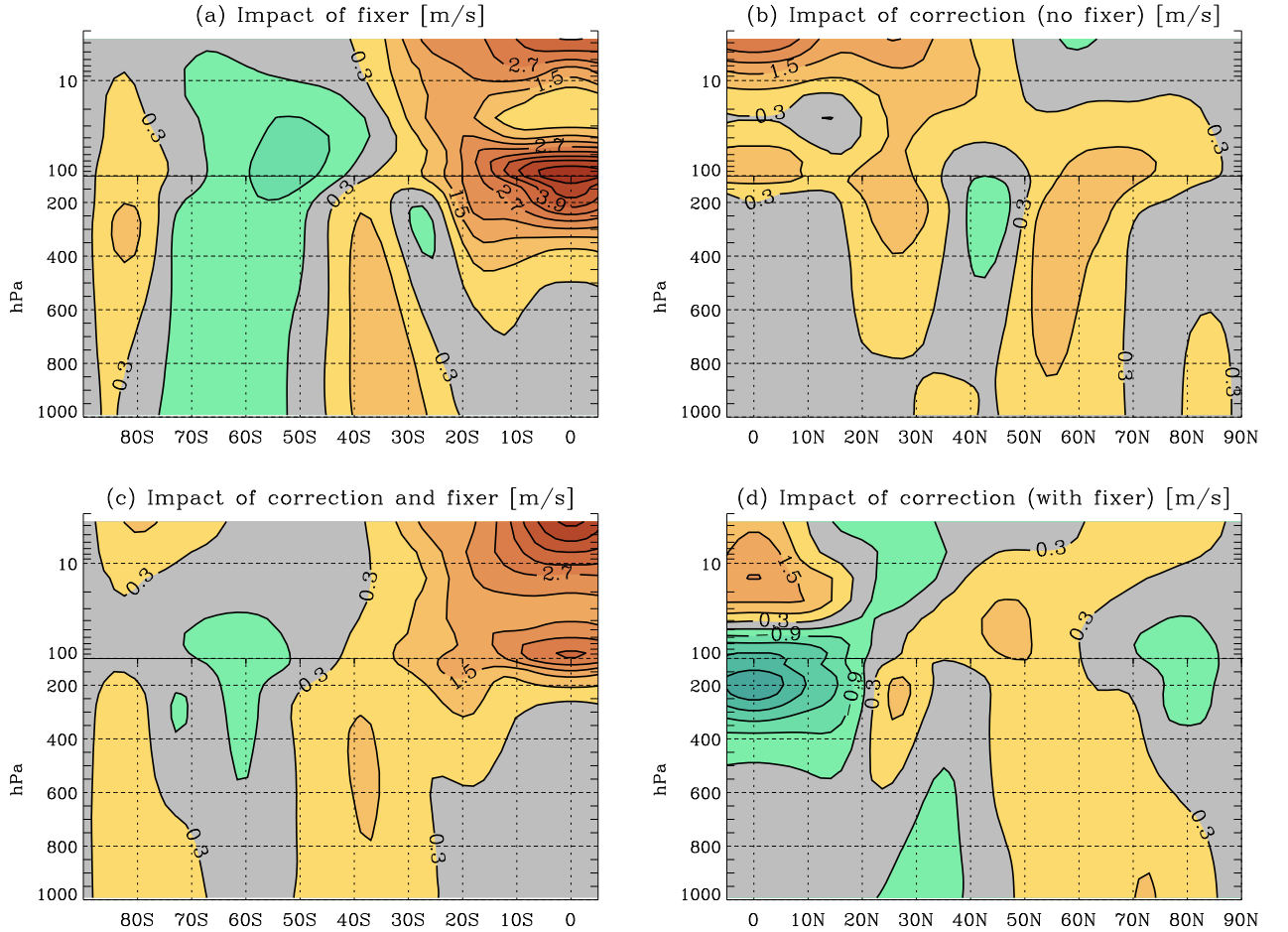


Figure 7: Impact of AM correction and fixer in Held-Suarez simulations. Time-mean latitude-pressure profiles of wind differences between HS simulations shown in the stippled lines in Figure 6. Panel (a) shows the zonal-mean zonal-wind time-average (days 120-360) difference field of the integration with AM fixer only and the control integration. Panel (b) shows the same field, but for the difference between the integration with AM correction and control. Panel (c) shows the difference between the integration with both AM correction and AM fixer and control, and panel (d) that between the integration with both AM correction and AM fixer and the integration with AM fixer only. The contour interval is 0.6 m/s, with blue hues indicating negative values, and red hues positive values. Values in the interval $[-0.3, +0.3]$ m/s are left in grey. The fields displayed have been symmetrised about the equator, since departures from symmetry are very small in the time mean for these hemispherically symmetric simulations. Accordingly, only one hemisphere, and the equatorial region, are shown in each panel.

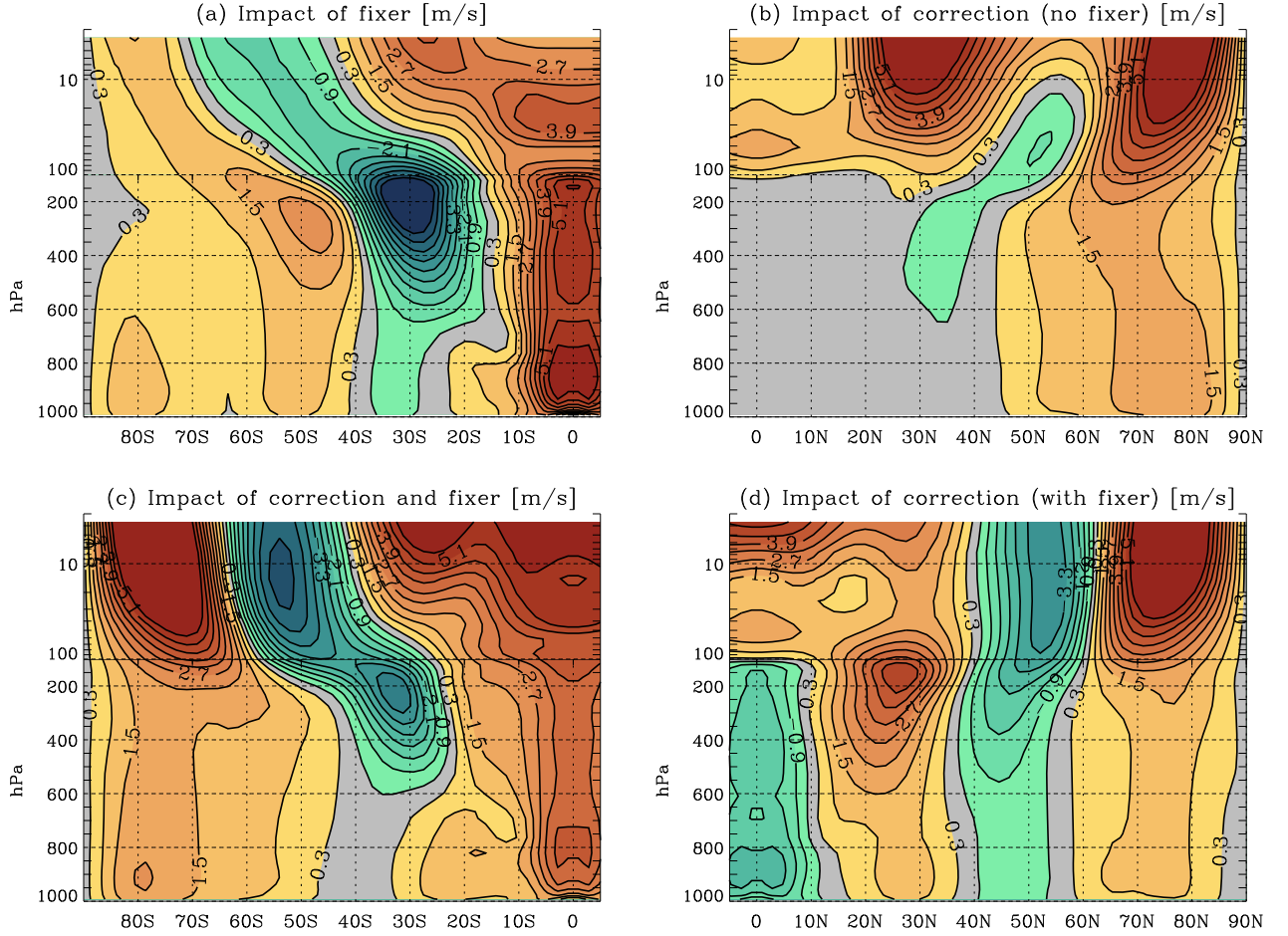


Figure 8: Impact of AM correction and fixer in Aquaplanet simulations. Same as Figure 7, but for the AP simulations shown in the solid lines in Figure 6.

414 The effect on the mean circulation of applying the correction and/or the fixer are shown in
 415 Figures 7 and 8 for HS and AP simulations, respectively. The zonal-mean zonal winds are shown,
 416 which is the quantity that both the correction and the fixer directly modify. Nevertheless, it
 417 should be remembered that the net effect is indirect, since the zonal winds remain in the time-
 418 average close to geostrophic balance with the (equivalent) temperature field. In HS simulations,
 419 the local temperature differences between simulations are simply proportional to the difference
 420 in temperature advection by the meridional and vertical circulation, which is modified primarily
 421 through a “tea leaves” mechanism. As already seen in the Introduction, the leading-order effect
 422 of the fixer is a weakening of this circulation, and thus of the associated advective temperature
 423 tendencies. These tend to cool the lower troposphere in the subtropical easterlies, cool the upper
 424 troposphere near the equator, and warm the troposphere poleward of the jet streams. The effect

425 of the fixer on the zonal-mean zonal wind shown in Figure 7a is generally consistent with this
426 expectation, with an equatorward retreat of the surface easterlies and weaker westerlies in the
427 higher latitudes. There is, however, an additional large westerly difference near the equatorial
428 tropopause, which is a direct consequence of the westerly forcing of the fixer, which is greatest
429 at the Equator. This is clearly an undesirable effect of the fixer on the simulations. A more
430 selective effect on the circulation is produced by the correction (Figure 7b). As seen above, its
431 main action is in where the greatest sink of AM is located, i.e. on the flanks of the subtropical
432 jet stream. By correcting part of the AM non-conservation, it also acts to limit the action of
433 the fixer (Figure 7d). As a result, the combination of correction and fixer together, as well
434 as ensuring good global AM conservation, is less severe in terms of its upper-level equatorial
435 westerly effect (Figure 7d). This suggests that the fixer is best employed in combination with
436 the correction.

437 In AP simulations, a slow-down of the meridional circulation is still expected and found,
438 but the interaction between dynamical forcing by the fixer or the correction and the physics
439 tendencies is much more complex and difficult to predict. The fixer now produces large westerly
440 differences near the equator at all levels, and a marked weakening of the subtropical jet stream
441 (Figure 8a). The equatorial winds above 300hPa become westerly. The correction is less effective
442 overall than in HS simulations, and its impacts are mostly confined to levels close to the model
443 lid or to the high latitudes (Figure 8b). Nonetheless, its use is still beneficial in terms of limiting
444 the action of the fixer, at least in the troposphere (Figure 8d). The result of the combined
445 correction and fixer can be seen in Figure 8c. In terms of tropospheric impacts, it appears
446 acceptable; equatorial winds remain easterly below 200hPa, and weak above. The weakening
447 of the equatorial and tropical easterlies compared with the control simulation implies greater
448 similarity with simulations with AM-conserving spectral models. Large changes however can
449 be seen near the model lid, especially in the four model layers with pressures less than 25
450 hPa. This is a consequence of momentum accumulation within these layers. In CAM's default
451 configuration, the order of FV's PPM advection scheme is reduced here, which results in large
452 numerical dissipation. Effectively, these levels are used as sponge layers and are thus not part of
453 the valid computational domain of the model. In full-model configurations it is therefore advised
454 to keep the reduced order of advection and turn off both the correction and the fixer in these
455 layers. The large mean-state changes seen near the top in Figure 8d then vanish. Considering
456 the troposphere only, the conclusion obtained from HS simulations can be seen to hold also
457 for full-physics AP model simulations, in that the combined application of the fixer and the

458 correction results in smaller overall mean-state changes of the solution compared to default FV
 459 without modifications, while ensuring good conservation of AM.

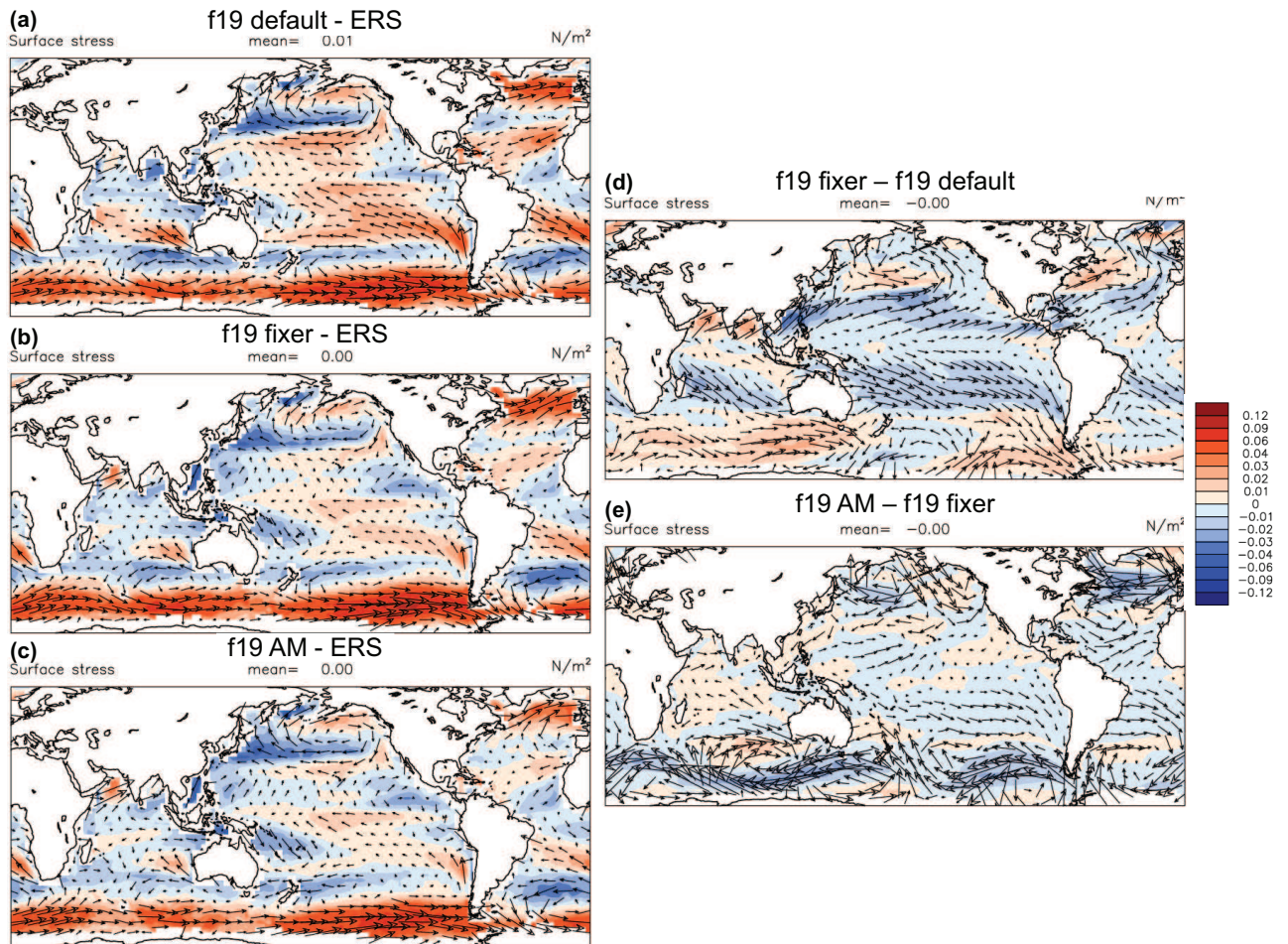


Figure 9: Impact of AM correction and fixer in F2000 simulations. Panels (a), (b) and (c) show maps of surface wind-stress vector differences (arrows) and wind-stress magnitude differences (colours) between "F2000" simulations with CAM-FV at $1.9^\circ \times 2.5^\circ$ degree resolution ("f19") and a climatology obtained from satellite scatterometer observations (ERS; Quilfen et al. 1999). Panel (a) shows the annual-mean climatological bias in the f19 control integration; panel (b) for a f19 simulation with AM fixer only; and panel (c) for an f19 simulations with both AM correction and AM fixer. Panels (d) and (e) show the same fields, but for the differences between the simulation with fixer only and control, and between the simulation with both fixer and correction and that with fixer only. The colour scale for all plots is on the right of panels (d) and (e). These plots were produced with the AMWG diagnostics package developed by the Atmospheric Model Working Group of the University Corporation for Atmospheric Research and the National Center for Atmospheric Research.

4 Simulations of the observed climatology

The relevance of the AM modifications to the FV dynamical core for CAM simulations in realistic configuration is investigated here using “F2000” cases, which are AMIP-type simulations (Gates 1992) where SSTs and all compositional forcings are prescribed as a repeating annual cycle obtained from an observed climatology of the decade spanning the turn of the century. We test at two grid resolutions, one of $1.9^\circ \times 2.5^\circ$ (“f19”) as in all integrations already discussed above, and one of $0.9^\circ \times 1.25^\circ$ (“f09”), to test the impacts of AM modifications in a case that is scientifically supported by NCAR at this time. The CESM model version used (here as above) is release 2.1.1¹

Figure 9 illustrates the effects of the fixer and the correction on f19 simulations. The control simulation shows a characteristic easterly surface wind-stress bias throughout the Tropics (Figure 9a). In addition, there are excessive westerlies at southern high latitudes. The effect of the fixer is to reduce the tropical biases (Figure 9b), with an evident westerly effect on the simulations nearly symmetrically about the equator (Figure 9d). By that same token, however, the high-latitude westerly errors are enhanced (Figure 9b). The application of the correction in addition to the fixer not only brings further improvements in the tropics, but also corrects the westerly effect of the fixer in high latitudes (Figure 9e). The result is a significant improvement in the simulation of the surface wind-stress field over the entire ocean domain.

In general, we obtain a similar conclusions as for the AP simulations. The impact of the correction on the global conservation of AM is modest, removing only about 15% of the sink at f19 resolution. However, its action is stronger on upper-level winds (cf. Figure 6b), which leads to proportionally reduced fixer increments at those levels, and thus to smaller impacts by the fixer on areas affected by baroclinic instability.

Figure 10 and Figure S3 in the supplementary information shows the seasonally resolved impacts on the zonal-mean zonal winds from applying the combination of fixer and correction in F2000 simulations at both f19 and f09 resolutions (cf also Figure S3 in the supplementary information, for JJA). In all cases, the reduction of biases in both easterly and westerly wind regimes is noticeable, the latter especially at the sub-polar latitudes of the winter hemisphere.

More in detail, it may be noted that the benefits of the AM modifications appear more

¹More precisely, we used a pre-release of CESM2.1.1 (#20, 22 March 2019). In terms of the simulations presented in this paper, the differences with the full 2.1.1 release only affect the F2000 cases at f19 resolution, where slightly different emission datasets are used to force the simulations. The impacts of this are of negligible consequence for the results discussed in this Section.

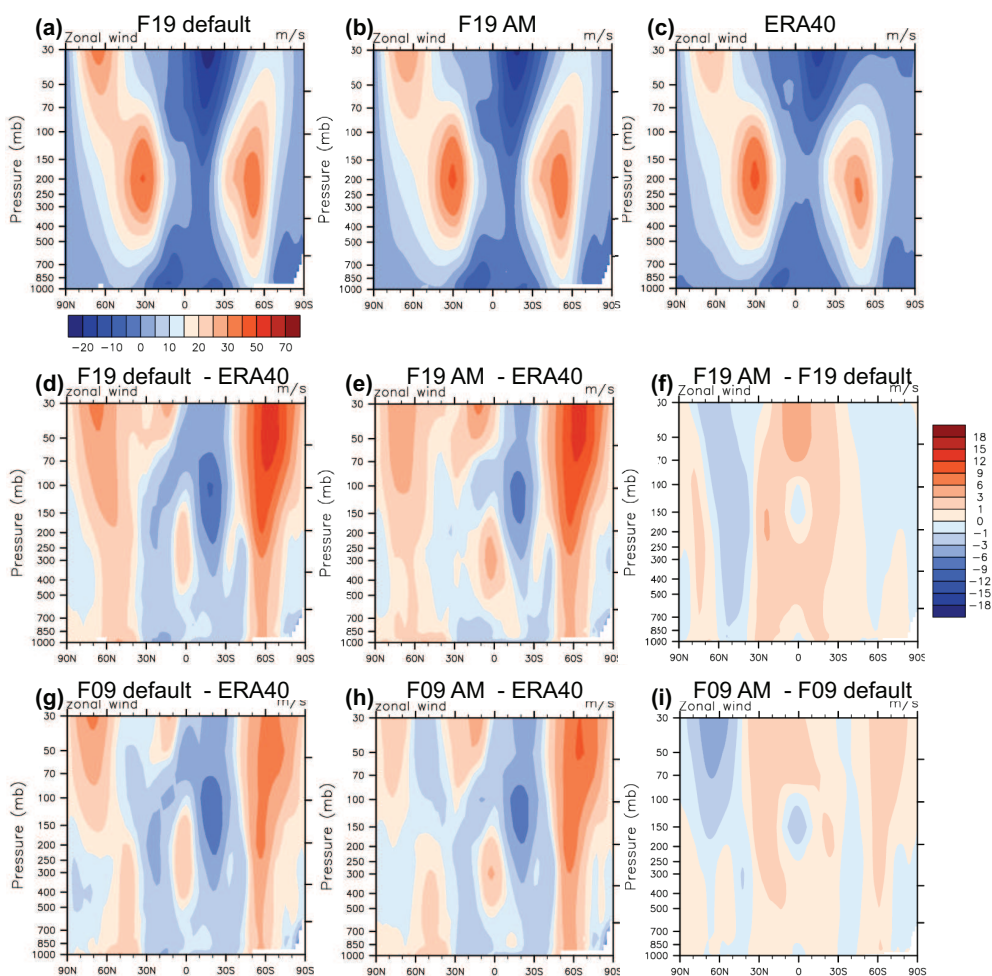


Figure 10: Impact of AM correction and fixer in F2000 simulations. Latitude-pressure maps of zonal-mean zonal wind climatologies for boreal winter (DJF). Panels (a), (b) and (c) show total fields for the CAM-FV f19 control simulation, (panel (a)) for the f19 simulation with both AM fixer and AM correction (panel (b)), and for the ERA40 reanalysis (Uppala et al., 2005). The colour scale is at the bottom of panel (a). Panels (d) and (e) show the differences of each of the two f19 integrations and ERA40, and panel (f) shows the differences between the two f19 simulations. The colour scale is on the right of Panel (f). Panels (g), (h), and (i) are analogous to panels (d), (e), and (f), respectively, but for CAM-FV simulations at $0.9^\circ \times 1.25^\circ$ resolution. These plots were produced with the AMWG diagnostics package developed by the Atmospheric Model Working Group of the University Corporation for Atmospheric Research and the National Center for Atmospheric Research.

489 clearly for the winds in the simulation at the lower resolution, where the numerical sink of AM
 490 is indeed larger. These benefits however are not limited to the zonal-mean zonal winds, and
 491 they are also appreciable at the f09 resolution. Most notable is the reduction in the strength
 492 of the Hadley circulations (cf Figure S4 in the Supplementary Information), which is expected

493 from the arguments set out in the Introduction. This has consequences for many aspects of the
494 global circulation. Figure 11 shows a summary of the impacts on the quality of the simulations
495 in relation to the observed climatology. The improvements at f09 seems particularly remarkable
496 considering that the unmodified simulation is a scientifically supported case that has been fully
497 tuned for a best match to observations. It may be noted that no additionally tuning whatsoever
498 is involved in the simulation with AM modifications shown here, and that the AM modifications
499 themselves have no free parameters as they follow directly from an effort to reduce the numerical
500 sink stemming from the FV dynamical core. The better quality of this simulation thus follows
501 entirely from better adherence of the solution to a fundamental property of the equations of
502 motion. Indeed, it should be kept in mind that the AM modification of the FV dycore

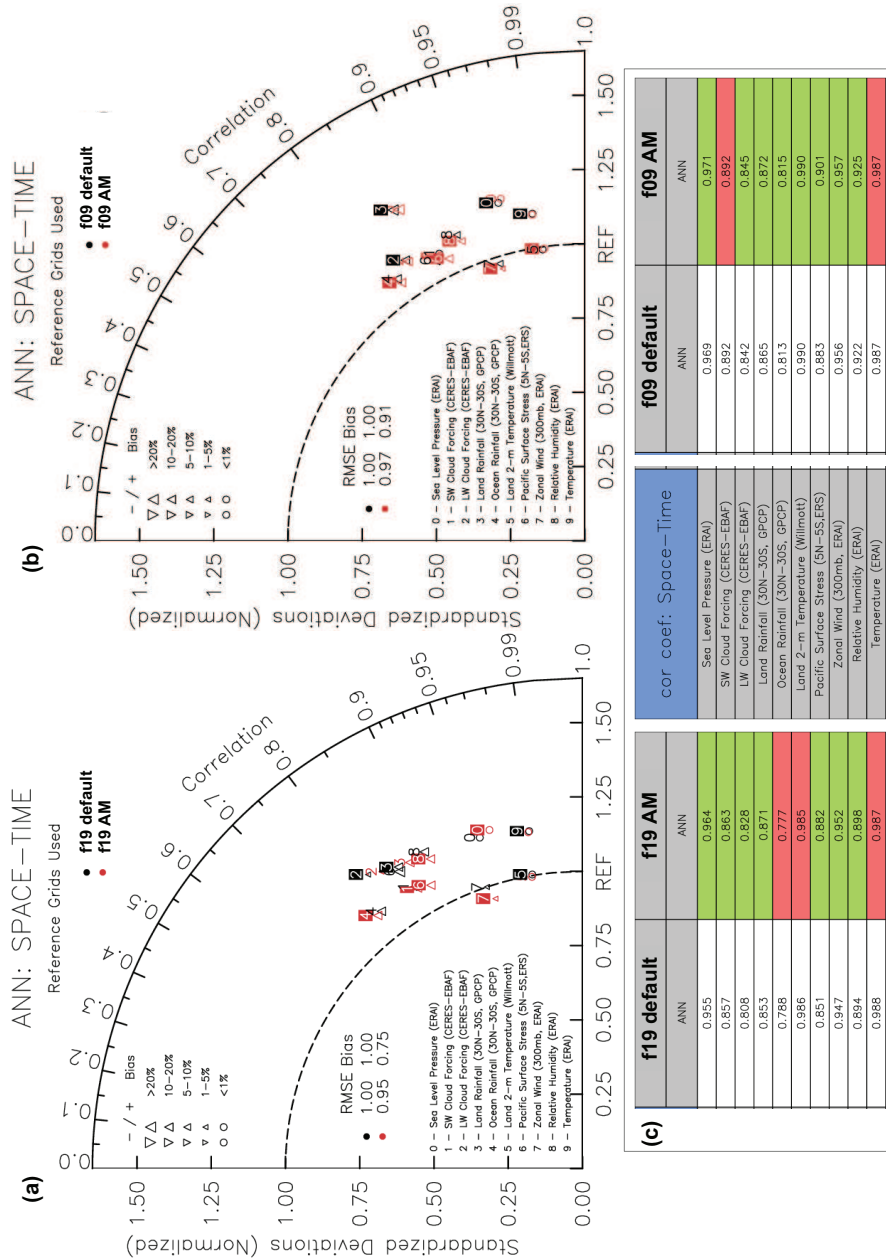


Figure 11: Impact of AM correction and fixer in F2000 simulations. Panels (a) and (b) show Taylor (2001) diagrams for the validation of the CAM-FV “F2000” simulations at f19 (panel (a)) and at f09 (panel (b)) resolution against observations for a standard set of diagnostic fields, listed in the panels. Black symbols represent RMS differences to observations for the control simulations without modifications, and red symbols for the simulations using both the AM correction and the AM fixer. For the overall RMSE and bias scores, those from the control simulations are used as normalisation. Panel (c) summarises the correlation values from simulated and observed diagnostic fields as listed in the central table. Green fields mark all instances where one of the AM-modified simulation represents an improvement over the respective control simulation. These plots were produced with the AMWG diagnostics package developed by the Atmospheric Model Working Group of the University Corporation for Atmospheric Research and the National Center for Atmospheric Research.

Table 1: Simulation set-ups and the effect of AM modifications. The percentage figures represent the numerical source (negative for sink) of global total atmospheric AM relative to the global total physical eastward torque acting on the atmosphere (terms T_x and C_λ in Eq.(A1), when only the positive part of the integrands are summed). The column “Experiments” indicate which modification to CAM-FV are used (the relevant sections of this paper are indicated in the footnotes). The three columns under “Simulations” are for results obtained with model integrations in Held-Suarez mode (Held and Suarez, 1994), in aquaplanet mode (Neale and Hoskins, 2000), and in “F2000” mode, i.e. an AMIP-type (Gates 1992) simulation with annually repeating present-day climatological SSTs.

Experiments	Simulations (f19, $1.9^\circ \times 2.5^\circ$)		
	HS	AP	F2000
geometry and pressure only ¹	-7.1%	-23.8%	-26.5%
AM correction ²	0.3%	-19.8%	-24.7%
AM correction and fixer ³	0.7%	1.9%	0.8%

¹Sections 2.2 and 2.1

²Section 2.3

³Section 2.3 and Section 2.4

5 Summary and Conclusions

AM conservation in CAM-FV has been substantially improved by means of a correction that reduces the zonal-mean numerical sink of Lin and Rood’s (1997) shallow-water scheme, and a fixer that ensures conservation of global angular momentum under advection. The effectiveness of these modification in terms of AM conservation in the simulations presented here is summarised in Table 1. We show that aside from global AM conservation, they have other significant impacts on the simulations, consistent with the “tea-leaves” mechanism (Einstein 1926) that rapidly redistributes pressure forces in a rotating fluid in response to zonal accelerations. The most notable effect is a reduction of the excessive easterlies of the model, with a concomitant slow-down of the Hadley circulation. As a result of such changes, the simulations of the observed climatology shows marked improvements.

The zonal-mean correction of the shallow-water scheme is not necessary for enforcing global conservation, as this can be achieved by the fixer alone. Indeed, the correction is quite ineffective

516 in realistic simulations of the atmosphere in terms of global conservation. Nevertheless, we
517 find that its concomitant application with the fixer has positive impacts on the simulations.
518 In particular, it reduces the effects of the fixer in the mid-latitudes. This can be explained
519 with the greater effectiveness of the correction in the baroclinically unstable regions around
520 the subtropical jet streams, where the zonal-mean numerical sink appears to be largest. Even
521 so, because of its potentially large local effects, the utilisation of the correction under different
522 set-ups should be tested on a case-by-case basis according to its impacts on the results.

523 Improving the quality of the simulation of the global distribution of surface wind-stress should
524 be expected to bring particular benefits to coupled atmosphere-ocean simulations. An adequate
525 discussion of such coupled simulation would exceed the scope of the present manuscript, which is
526 aimed primarily at presenting the method. In particular, due to their computational expense, at
527 the present time it is not possible to produce well spun-up coupled simulations that can provide
528 an assessment of the impact of the AM modifications.

529 The modification to the FV dynamical core that we describe and utilise are relatively crude,
530 and cause local loss of accuracy due to violation of vorticity conservation under advection.
531 Nevertheless, the associated detrimental impacts appear to be fairly limited, with insignificant
532 differences under standard tests such as the Jablonowsky and Williamson (2006) baroclinic wave
533 test, which should be sensitive to local conservation. Even so, it is clear from the very same tests
534 that simulations over weather time-scales are not sensitive to AM conservation, so that for such
535 application it is not advisable to trade enforcing such conservation for a loss of accuracy. On the
536 longer time-scales of climate simulations, by contrast, our results demonstrate the importance
537 of global conservation of atmospheric AM in order to obtain a realistic global circulation.

538 **Code and data availability.**

539 The code used in the numerical simulations of this paper is available under

540 <https://zenodo.org/badge/latestdoi/214872045>

541 CAM6 is published in the open-access CESM ESCOMP git repository, freely available under
542 <https://github.com/ESCOMP>. The AM options can be switched on by setting standard CAM
543 namelist parameters to non-default values (i.e. T instead of F; there are no free numerical
544 parameters). Apart from these switches, all atmosphere model configurations presented in this
545 paper are standard CESM cases that can be set up and run using the scripts provided in the
546 repository. Users can obtain technical support if requested.

547 **Author Contributions:** Thomas Toniazzo conceived the idea, proposed the work, made the
548 calculations, implemented the code, ran the simulations, evaluated them, produced all figures,
549 and wrote the manuscript. Mats Bentsen supported this activity through national infrastructure
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A Off-line diagnostics of numerical torque in model simulations

The diagnosis of the residual torque that violates AM conservation in CAM simulations follows from the hydrostatic Primitive Equations (cf. White et al. 2005). In our zonally and vertically integrated diagnostics such as in Figure 1 the AM source is calculated as

$$S_M = \partial_t L_r + D_L - T_x - C_\lambda \quad (\text{A1})$$

where the first term on the r.h.s. represent the tendency of relative atmospheric AM, the second term represent the divergence of the flux of relative AM, the third the external torque (which in all simulations presented in Sections 1, 2, and 3, when non-vanishing, is exclusively due to surface stresses or linear friction in the PBL), and the last term is the tendency of planetary atmospheric AM due to the vertically integrated divergence of atmospheric mass. In formulas:

$$\begin{aligned} L_r &= \int_0^{2\pi} \int_{p_*}^{p_{top}} (ua \cos \varphi) \frac{dp}{g} a \cos \varphi d\lambda \\ D_L &= \frac{1}{a} \frac{\partial}{\partial \varphi} \int_0^{2\pi} \int_{p_*}^{p_{top}} (uva \cos \varphi) \frac{dp}{g} a \cos \varphi d\lambda \\ T_x &= \int_0^{2\pi} (\tau_x a \cos \varphi) a \cos \varphi d\lambda \\ C_\lambda &= -\frac{a\Omega \sin 2\varphi}{g} \partial_t \int_0^{2\pi} \int_0^\varphi p_* a^2 \cos \varphi' d\varphi' d\lambda \quad , \end{aligned}$$

where a is the Earth's radius, φ the latitude, λ the longitude, g the gravitational acceleration in Earth's surface, Ω the angular speed of Earth's rotation, and u , v , p_* and τ_x are the zonal wind component, the meridional wind component, the surface pressure, and the zonal component of the surface or frictional stress acting on the air in the model simulations. Note that to obtain C_λ the continuity equation was used. Note that for the time-average values of S_M , the time differentials become increments between the initial and the final state; terms T_x and C_λ are linear in the wind-stress and the surface pressure, respectively. Terms L_r and D_L are bi- and trilinear in the model prognostic quantities u , v , p_* , so an on-line computation of the time averages of the integrands are required for these terms. CAM provides time-mean diagnostic of the zonal wind u and of the product of the wind components uv conservatively interpolated onto standard pressure levels, and the integrals in Eq.(A1) are computed with their help.

659 **B Formulation and approximations for the AM correc-** 660 **tion in CAM-FV**

661 The local conservation equation for the shallow-water equations is

$$\begin{aligned}
 \partial_t [\Delta p (ua \cos \varphi + \Omega a^2 \cos^2 \varphi)] = & \\
 & - \frac{1}{a \cos \varphi} \partial_\varphi [\Delta p (ua \cos \varphi + \Omega a^2 \cos^2 \varphi) v \cos \varphi] \quad (\text{A2}) \\
 & - \frac{1}{a \cos \varphi} \partial_\lambda [\Delta p (ua \cos \varphi + \Omega a^2 \cos^2 \varphi) u] ,
 \end{aligned}$$

662 where (φ, λ) are latitude and longitude, respectively, Δp is the layer thickness in terms of
663 hydrostatic pressure, (u, v) are the zonal and meridional wind components, a is the Earth's
664 radius, and Ω the Earth's angular velocity. Note that we are ignoring pressure and geopotential
665 terms here, as we focus exclusively on the process of advection. Accordingly, Δp , i.e. the
666 layer under consideration, may be arbitrary, except that it satisfies the shallow-water mass
667 conservation equation, i.e. we follow Lin's (2004) "vertically Lagrangian" approach by following
668 the vertical motion of the layer. Integrating Eq.(A2) over longitude, we obtain:

$$\int d\lambda \partial_t (\Delta p ua \cos^2 \varphi) = - \int d\lambda \partial_\varphi (\Delta p uv \cos^2 \varphi) + \int d\lambda \Delta p fva \cos^2 \varphi, \quad (\text{A3})$$

669 where f is the Coriolis parameter. To address the FV scheme's violation of this conservation,
670 we apply an additional, zonally uniform increment of the zonal wind, $\overline{\delta u}$, such that, over each
671 shallow-water sub-step δt (we shall refer to this simply as the "time-step" in this section) of the
672 dynamical core:

$$\begin{aligned}
 \frac{1}{\delta t} \int d\lambda \cos \varphi [\Delta p_n (u_n + \overline{\delta u}) - \Delta p_o u_o] \cos \varphi = & \\
 & - \int d\lambda \cos \varphi \frac{1}{a \cos \varphi} \partial_\varphi (\Delta p uv \cos^2 \varphi) \quad (\text{A4}) \\
 & + \int d\lambda \cos^2 \varphi \Delta p f v .
 \end{aligned}$$

673 Here, "old" prognostic quantities (i.e. valid at the beginning of the time-step) and "new"
674 prognostic quantities (i.e. valid at the end of the time-step, before any correction) are indicated
675 by the sub-scripts "o" and "n", respectively; quantities without subscripts are intended as time-
676 centred representing advective fluxes over the time-step. To obtain the correction, we solve this
677 equation for the required increment $\overline{\delta u}$ and substitute for u_n the actual FV zonal wind increment
678 over the time-step:

$$u_n = u_o + \left(\xi_o v - \frac{1}{a \cos \varphi} \partial_\lambda K \right) \delta t, \quad (\text{A5})$$

679 where ξ is the absolute vorticity, and K is the kinetic energy term as discretised in LR97's
 680 scheme. The result is:

$$\begin{aligned}
 \left(\int d\lambda \Delta p_n \right) \overline{\delta u} &= - \int d\lambda \Delta p_n \left(\zeta_o v - \frac{1}{a \cos \varphi} \partial_\lambda K \right) \delta t \\
 &- \int d\lambda (\Delta p_n - \Delta p_o) [u_o + (\xi_o v - \zeta_o v) \delta t] \\
 &- \int d\lambda \frac{1}{a \cos^2 \varphi} \partial_\varphi (\Delta p uv \cos^2 \varphi) \delta t.
 \end{aligned} \tag{A6}$$

681 The term in the second line on the right-hand side representing advection of planetary vorticity
 682 is written in a roundabout way for later convenience.

683 We note two aspects of this expression. First, there is a significant numerical cancellation
 684 between the second and the third lines on the right-hand side. Second, all advective terms in
 685 the first two lines on the right-hand side can be easily discretised according to standard LR97's
 686 prescription, and are thus automatically defined on D-grid u-points, i.e. where required for $\overline{\delta u}$.
 687 However, all mass factors are defined on scalar points, i.e. on the A-grid. Furthermore, the
 688 integrand in the third line on the rhs has no natural expression in LR97's discretisation, and
 689 both zonal and meridional winds in that expression need to be interpolated onto the A-grid.
 690 Hence, additional interpolation is required for these terms. Notwithstanding these issues, we
 691 found that this correction, when implemented, gave accurate conservation of AM. However, it
 692 also proved to cause numerical instability, such that the integration crashed within seven or
 693 eight time-steps. Analysis suggested that the last term on the rhs had to be recast in a different
 694 form.

695 We therefore chose to approximate the last term, as follows:

$$\frac{1}{a \cos^2 \varphi} \partial_\varphi (\Delta p uv \cos^2 \varphi) \approx \left[\frac{1}{a \cos \varphi} \partial_\varphi (\Delta p v \cos \varphi) \right] u + \left[\frac{v}{a \cos \varphi} \partial_\varphi (u \cos \varphi) \right] \Delta p. \tag{A7}$$

696 The approximation here consists in using C-grid (advective) fluxes in the partial differentials
 697 on the right-hand side. Considering this as a calculation for the advective fluxes of zonal
 698 momentum, which is its physical meaning, this appears to be a valid interpretation for v . For
 699 the values of Δp and u outside the operators, we adopt the substitutions

$$\begin{aligned}
 u &=: u_o + \delta_h u + \delta'' u \\
 \Delta p &=: \Delta p_n - \delta_h \Delta p + \delta'' \Delta p,
 \end{aligned}$$

700 where

$$\delta_h \Delta p := \frac{\Delta p_n - \Delta p_o}{2}, \quad \delta_h u := \frac{u_n - u_o}{2}, \tag{A8}$$

701 and $\delta''u$ and $\delta''\Delta p$ are formally $o(\delta t)$. The increments are still understood as advective only, i.e.
 702 they exclude pressure force terms. By further using the identities

$$-\frac{\delta t}{a \cos \varphi} \partial_\varphi (\Delta p v \cos \varphi) = \Delta p_n - \Delta p_o + \frac{\delta t}{a \cos \varphi} \partial_\lambda (\Delta p u) \quad (\text{A9})$$

$$-\left[\frac{1}{a \cos \varphi} \partial_\varphi (u_o \cos \varphi) \right] v \delta t = \left(\zeta_o - \frac{1}{a \cos \varphi} \partial_\lambda v_o \right) v \delta t, \quad (\text{A10})$$

703 we finally arrive at the expression for our approximate angular-momentum conserving zonal-
 704 mean zonal wind correction:

$$\begin{aligned} \left(\int d\lambda \Delta p_n \right) \overline{\delta u} &= \int d\lambda (\Delta p_n - \delta_h \Delta p) \left[\frac{1}{a \cos \varphi} \partial_\lambda K - \zeta_{\lambda o} v \right] \delta t \\ &+ \int d\lambda \left[\frac{1}{a \cos \varphi} \partial_\lambda (\Delta p u) \delta t \right] (u_o + \delta_h u) \\ &+ \int d\lambda \left[2\delta_h \Delta p + \frac{1}{a \cos \varphi} \partial_\lambda (\Delta p u) \delta t \right] \delta''u \\ &+ \int d\lambda \delta''\Delta p [\xi_o v - \zeta_{\lambda o} v] \delta t, \end{aligned} \quad (\text{A11})$$

705 where we have used the shorthand $\zeta_{\lambda o} := \frac{1}{a \cos \varphi} \partial_\lambda v_o$.

706 We note that setting the higher-order terms to zero implies that the correction has no effect
 707 on a zonally symmetric flow. If, in addition, the flow is in an exact steady-state, then the
 708 correction always vanishes identically, regardless of these terms. It can further be shown that,
 709 if the term in K is the true gradient of the kinetic energy in the original scheme, for any values
 710 of $\delta''u$ and $\delta''\Delta p$ that are first order in δt or higher, the correction (A11) is formally third-order
 711 in δt or higher. In other words, the correction will not affect solutions that are already locally
 712 angular-momentum conserving.

713 In Equation (A11), all mass terms must be averaged over φ ; by contrast, all advective terms
 714 (in square brackets) represent fluxes as discretised according to the standard LR97 algorithm.
 715 The discretised expression of Equation (A11) thus corresponds with Equation (7). The only
 716 additional PPM calculation required to calculate this correction is the meridional advection of
 717 the partial relative vorticity, ζ_λ , with a minimal additional computational cost that is hardly
 718 detectable in CAM simulations.

C Formulation and implementation of the AM fixer in CAM-FV

As we explain in section 2.4, the fixer is based on diagnosing the global change of atmospheric AM due to advective increments only, which should vanish identically according to the continuous equations. When applied, the fixer counteracts that change at every advective sub-step; irrespectively, its time-mean increments can always be used to diagnose AM non-conservation in the simulations, in a manner that is completely independent of the physics parametrisations or boundary conditions used, and hence independent of the particular configuration of the simulations itself. All the calculations related to the fixer and the quantification of the numerical (advective) AM source are internal to the dynamical core only, indeed of its shallow-water part.

So, for each time-step and at each level k , we require the advective shallow-water equation increments to satisfy:

$$\delta \left\{ \sum_{i,j} [u_{i,j} \cos e_j + u_{i,j+1} \cos e_{j+1} + a\Omega (\cos^2 e_j + \cos^2 e_{j+1})] \cos c_j \Delta p_{i,j} \right\}_k = 0, \quad (\text{A12})$$

where the indices (i, j) refer to longitude and latitude, respectively; e_j are the latitudes of the u-velocity points of the D-grid; and c_j the latitudes of the scalar points (A-grid). The other symbols have the same meaning as in the previous section, and δ represent the purely advective increment obtained in the dynamical core, which may include the correction discussed above. The action of the fixer in this context is represented by an additional increment $\delta\varpi_k$, so that the total increment of the zonal wind becomes $\delta u_{i,j,k} + a\delta\varpi_k \cos e_j$. We obtain:

$$\delta\varpi_k = -\frac{T_k}{I_k} \quad (\text{A13})$$

where the numerical torque is

$$T_k = a \sum_{i,j} \cos e_j (\cos c_j + \cos c_{j-1}) \{ \delta u_{i,j} \overline{\Delta p_{i,j}^\varphi}(t + \Delta t) + [u_{i,j}(t) + a\Omega \cos e_j] \delta \overline{\Delta p_{i,j}^\varphi} \}_k \quad (\text{A14})$$

and the moment of inertia is

$$I_k = a^2 \sum_{i,j} \cos^2 e_j (\cos c_j + \cos c_{j-1}) \overline{\Delta p_{i,j,k}^\varphi}(t + \Delta t). \quad (\text{A15})$$

In these expressions,

$$\overline{\Delta p_{i,j,k}^\varphi} := \frac{\Delta p_{i,j,k} \cos c_j + \Delta p_{i,j-1,k} \cos c_{j-1}}{\cos c_j + \cos c_{j-1}}. \quad (\text{A16})$$

740 Equation (A13) gives the required angular acceleration of the entire atmospheric shell at model
 741 level k . The action of the “level” fixer is therefore to add an increment to the zonal wind:

$$\delta^f u_{i,j,k} = a \delta \varpi_k \cos e_j. \quad (\text{A17})$$

742 In some regions of the model domain, it is not desirable to apply a fixer, since dissipation is
 743 explicitly built into in the dynamical core formulation. This is the case near the upper boundary
 744 of CAM’s domain (the lower boundary in pressure space), where the fixer is accordingly switched
 745 off. In general, a weight $w_k \leq 1$ can be applied at each level, so that Eq.(A13) becomes

$$\delta \varpi_k = -w_k \frac{T_k}{I_k}, \quad (\text{A18})$$

746 where only a fraction w_k of the numerical torque at level k is compensated by the fixer at that
 747 level.

748 The “global” fixer applies the same solid-body rotation increment to all levels within the
 749 domain where it is required. When all weights are unity, this is simply

$$\delta \varpi_g = -\frac{\sum_i T_i}{\sum_j I_j}; \quad (\text{A19})$$

750 when $\exists k : w_k < 1$, the vertical integrals must be weighted accordingly, and the weights applied
 751 to the correction at each level, so that

$$\delta \varpi_{g,k} = -w_k \frac{\sum_i w_i T_i}{\sum_j w_j I_j}. \quad (\text{A20})$$

752 It can be seen that $\sum_k I_k \delta \varpi_{g,k} = -\sum_k w_k T_k$ so that the numerical torque associated with the
 753 domain of interest is fully compensated also by this fixer. Experimentation has shown that
 754 tapering the global fixer so as to exclude its action from levels in the stratosphere was necessary,
 755 in order to avoid distortions of the dynamics in layers where it is sensitive to small amounts
 756 of zonal acceleration; and where, moreover, thanks to the predominance of solenoidal dynamics
 757 (before gravity-wave drag, which is applied in the physics parametrisations), the dynamical core
 758 performs well in terms of AM conservation. For the latter reason, no tapering (i.e. any weights
 759 other than 1 in the valid domain, and 0 in the filtered layers near the model lid) is in fact
 760 required for the level fixer.

761 For diagnostic purposes, fixer increments are always calculated as in Eq.(A13) and provided
 762 in output. Use of the increments in Eq.(A13) lead to conservation of total AM in idealised
 763 spin-up or spin-down experiments with no physical sources or sinks of momentum (cf. Figure

764 3), as well as an accurate balance of the surface torques in Held-Suarez or Aquaplanet simu-
765 lations where only surface stresses are present (and accurately diagnosed). Hence, we obtain
766 two important conclusions. First, all numerical sources of AM indeed reside in the advective
767 wind increments of the shallow-water part of the dynamical core; second, the fixer diagnostics
768 return an accurate estimate of the apparent numerical AM source for any CAM-FV integration,
769 irrespective of physics parametrisations or boundary fluxes (including orographic form drag).