Review Cheng, et al., A full Stokes subgrid scheme for simulation of grounding line migration in ice sheets using Elmer/ICE (v8.3)

Review based on the file named: *gmd-2019-244-manuscript-version4.pdf*

The manuscript aims to present a numerical scheme to deal with the friction inside elements partly floating in a (full-)Stokes formulation for the marine ice sheet simulation. The formulation and results are carried out in a 2D vertical domain, and possible extension to 3D domain is discussed. The reviewed version presents the corrections asked in the first review, mainly in the technical part (methods). The Introduction was changed, but additional "polishing" is needed before publishing. No additional simulations were carried out, and the presentation of the results was not modified.

General comments:

- The numerical scheme is better presented in this new version, although some minor corrections should be done. See specific comments.
- With the results presented along the manuscript it is hard to analyze the convergence (and consistency) of the subgrid scheme proposed. There is no convergence rate analysis or comparison with the cited reference work (Gagliardini et al., 2016). I strongly recommend additional simulations (mesh resolutions equal to 500 m and 250 m) and a comparison with the results from Gagliardini et al. (2016), mainly in terms of GL position against mesh resolution.
- The overall explanation of the subgrid scheme was improved, which helps the reproducibility of the results.
- The Introduction section must be improved yet. The reading is not smooth yet, and additional polishing is needed to make the reading "pleasant" enough for a scientific/technical paper.
- The citation style along the manuscript was corrected, but there are still corrections in some parts. See specific comments.

Specific comments:

- line 77: "for modeling of the flow" => "for modeling the flow"
- line 78: "These nonlinear" => "The nonlinear"
- line 102: "β" => "β (≥ 0)"
- line 117: "$z_b < 0$" => "$z_b > b(x)$"
- line 119: "The solution close to the grounding line" => "A first order solution close to the grounding line" or "A solution close to the grounding line from the boundary layer theory" or "A boundary layer' solution close to the grounding line". Note that this solution is based on a linear Stokes problem (i.e., $n = 1$ in Glen's flow law).

- line 121: “(Schoof, 2011)” => “Schoof (2011)”

- line 123: “u” => “the ice velocity u”

- line 124: “ice surface slope is continuous”: are you referring to slope or just the ice surface? Does this proposition come from Schoof (2011)? Also, why this is important/relevant for the subgrid scheme used here?

- line 128: “(Durand et al., 2009a)” = > “Durand et al. (2009a)”

- line 129: “(Schoof, 2011, Ch. 4.3)” => “Schoof (2011, Sect. 4.3)”

- line 129: “parameters” => “parameters,”

- line 133: “variables satisfy” => “variables satisfy (Schoof, 2011)” (if the citation is right)

- line 142: “(Norwicki and Wingham, 2008)” => “Norwicki and Wingham (2008)”

- line 143: “original variables”: what does it mean?

- line 149: The definition of “k” and “k^{**}” is weird. Why does the approximation space depend on the Glen’s flow law? Are these not referred to the polynomial order of the space? Please, check the definition and notation of these spaces.

- line 152: Please, change the citation style here

- line 156: the form “$b(\boldsymbol{v}, p)$” is not defined here (although it follows $b(\boldsymbol{u}, q)$)

- line 156: where is σ_{nt} in the expressions? Please, check the forms B_Γ and B_N

- line 156: How the forcing term ($F(\boldsymbol{v})$) is numerically considered in the element crossed by the grounding line? There is no mention of this along the text.

- line 171: Do you also split the in integral of the forcing term ($F(\boldsymbol{v})$))?

- line 173: Eq. (15): the forms B_Γ and B_N are already integrated. Please, fix the notation here.

- line 173: Eq. (15): where is the σ_{nt}? Please, check the forms here.

- line 173: Eq. (15): the forcing ($p_w n \cdot \boldsymbol{v}$) is considered here, but is it included in the stiff matrix? Please, could you make it clearer?

- lines 175-177: “With a strong formulation … into account”. This is phrase is not clear. I don’t understand why strong formulation is mentioned here.

- line 177: “no basis functions satisfies …”. I am not sure if this is true. There are lots of FEM schemes where the discontinuity is well accommodated (e.g., xFEM, CutFEM, etc). The phrase is only true if the standard FEM is used, and no specific refinement is made in the element crossed by the grounding line (as is the case of this paper).

- lines 185-186: what does “along the slope” mean?

- line 195: “The nonlinear equations …” => “The nonlinear equations, Eq. (14), …”
- line 197: “timestep” => “time step” (and elsewhere)
- line 198: “nonlinear iterations” => “nonlinear iterations (Picard)”
- Algorithm 1: All grounded nodes are marked as “GL nodes”? Please, could you make it clearer along the text? Also, check the text punctuation in Algorithm 1
- Algorithm 2: please, check the text punctuation in Algorithm 2
- line 208: “A stability problem” = > “A numerical stability problem”
- line 208: “(Durand et al., 2009a)” => “Durand et al. (2009a)” Same in lines 209, 238.
- line 215: “is updated implicitly”: is p_w also considered in the forcing term of Eq. (14)? Could you make it clearer?
- line 221: note that “n” was used before with another meaning
- line 242: “(Seroussi et al., 2014)” => “Seroussi et al., (2014)”. The same for Schoof citation
- line 249: “(11)” => “Eq. (11)”
- line 251: please, change the citation style here
- line 251: “analytical solution”: which one? From Schoof 2011’s paper? Same in line 254. Note that if it is from Schoof (2011), it is based on a linear Stokes problem $n = 1$ (Glen’s flow law).
- line 258: “between” => “between any”
- line 261: “basal surface of the ice” => “ice base”
- line 266: “external forces” => “external forces and boundary conditions” (maybe?)
- line 267: “geometrically grounded”: how is the element identified as geometrically grounded or geometrically floating, in the numerical framework? Could you make it clearer along the text?
- line 269: “(Gagliardini et al., 2016)” => “Gagliardini et al. (2016)”
- line 270: please, delete the extra “the”
- line 271: “fine mesh” => “fine mesh resolution (<100 m)” (maybe?)
- Fig. 2 and Fig. 3: “net forces” => “net forces in the vertical direction” (please, check also the text)
- Eq. (27): $\chi(x_i) = 0$, right? Or this is not zero in the numerical solution? Please, could you make it clearer along the text?
- line 280: “best numerical approximation”. I don’t know if “best” is the word here. Maybe mentioning that it is in the same order of the framework/scheme/approximation space

- line 284-285: “Considering … always stays”: maybe this phrase is unnecessary; even the numerical GL position stays on bedrock
- line 293: “bottom surface” => “ice base”
- line 296: please, delete “Then”
- line 297: “(Seroussi et al., 2014)” => “Seroussi et al., (2014)”. Same in line 299
- line 299: “condition” => “condition, respectively”
- line 299: “reasonable resolution” => “reasonable numerical accuracy”
- line 300: “required” => “used”. The integration points are defined over the GL element, right? And the step function makes the work of selecting the area to be integrated, right? Then, note that, depending on the situation, even a tenth order could not be enough to carry out the integration with enough numerical accuracy (as is the SEP2 method of Seroussi et al., 2014, where the distribution of the integration points follows the grounding line position inside the GL element). Besides that, the approach used here seems reasonable, and it is easier to be implemented in comparison to SEP2-type scheme.
- line 302: “fully on the ground”: geometrically, right?
- line 304: “basal surface” => “ice base”
- line 307: “fully grounded” => “fully geometrically grounded” (maybe?)
- line 308: “boundary elements” => “basal elements” (maybe?)
- line 311: “floating elements” => “fully geometrically floating elements” (maybe)
- line 313: “grounded” => “geometrically grounded” (maybe)
- line 313: “analytical solution”: maybe “numerical solution”? It is not clear what you meant here
- line 315: “3” => “Fig. 3”
- Eq. (29): check the notation of the forms B_{Γ} and B_{N}. Also, there is no σ_{tn} here
- lines 319-320: How are the phases (advance or retreat) defined? Comparing with previous (last time step) GL position? Please, could you make it clearer?
- Algorithm 3: please, check the text punctuation in Algorithm 3
- line 326: in the calculation of $\tilde{\chi}$, p_w is kept constant, right? Could you please make it clearer?
- line 330: “(Gagliardini et al., 2016)” => Gagliardini et al., (2016)”. The same in lines 331, 367, 369, 370, 372, 397, 398
- line 342: “both for” => “for both”
- line 344: please, correct the citation style here

- line 352: “(van Dongen et al., 2018)” => “van Dongen et al. (2018)”
- line 357: “(Schoof, 2011)” => “Schoof (2011)”
- line 357: “represented” => “captured”
- line 359: “Seroussi et al (Seroussi et al., 2014)” => Seroussi et al. (2014) (maybe?)
- Fig. 6: Note that the GL is close to a node. I suspect the same is observed for the other resolutions (2 and 4 km). So, the GL position also depends on the distribution of the nodes in 1D.
- line 363: “floatation criterion” => “hydrostatic floatation criterion”
- line 372: “asymptote” => “convergence asymptote” (maybe?)
- lines 374-375: “but the numerical solution of the velocity field, pressure as well as the two free surfaces are still determined by the coarse mesh …”: note that small bedrock features impact the GL dynamics, and they are important in short time scale simulations (decades). In general, mesh resolution equal to 500 m is required to capture these bedrock features near the GL. Also, from figures 6 and 7, there are expressive changes in the fields near the GL (thickness, surface, horizontal and vertical velocities). These changes are only “well” captured with enough mesh resolution (<1 km or less). Besides that, no error estimator was used here; therefore, the term “determined” doesn’t fit here. The subgrid scheme tends to accelerate the rate of convergence in comparison to NSEP-type schemes (by decreasing the numerical error of one source, the boundary condition at the base), but relatively fine mesh resolution (I would say 500 m) is yet required around the GL to numerical error control (from other sources, e.g., bedrock geometry, intrinsic solutions variations around GL, effect of ocean-induced basal melting, etc).
- line 377: “following way” => “following way (considering linear Lagrange functions)” (maybe?)
- line 382: “An alternative to a subgrid scheme is to introduce dynamic adaptation of the mesh”: I don’t think mesh adaptation is an alternative, strictly speaking. They are complementary to each other. The subgrid scheme tends to decrease the error on the boundary condition, accelerating the rate of convergence (ideally); the mesh adaptation helps save computation effort, since enough mesh resolution (~500 m) is needed around the GL. They can (should) be used together, indeed.
- line 383: please, correct the citation style here
- line 386: “shorter timesteps are necessary for stability when the mesh size is smaller in a mesh adaptive method” => “shorter time steps are necessary for numerical stability in dynamic mesh adaptation schemes”. Note that it depends on the numerical implementation; some schemes are more stable than others.

- line 387: “Introducing a time dependent mesh adaptivity into an existing code requires a substantial coding effort and will increase the computational work considerably. Subgrid modeling is easier to implement and the increase in computing time is small.” I don’t totally agree here. Yes, mesh adaptivity is a substantia coding effort, and there are drawbacks in scalability. But at the end, the computational effort is (or should be) much less in comparison to a fine uniform mesh. The improvement of a subgrid scheme for the basal condition (friction) makes the 25 m-mesh resolution requirement to a 500 m-mesh resolution requirement. But yet, a 500 m-mesh resolution is expressively fine in comparison to a typical horizontal scale of ice sheets (order of 1,000 km).
A static mesh adaptation (performed during the domain discretization) could be used instead of dynamic mesh adaptation (considering the GL will not migrate beyond the adapted/refined region). For short-term simulations (decades) this is feasible, but this is not totally true for paleo-ice sheets simulations. Therefore, using subgrid scheme with dynamic mesh adaptation should work properly (in the sense of convergence of the GL dynamics with reduced computational effort).
- lines 390-392: “A subgrid scheme … (Feldmann et al., 2014)”: this phrase could be migrated to the discussion part. Also, correct the citation style here.
- line 395: “function $\chi(\boldsymbol{x})$” => “function $\chi(\boldsymbol{x})$ based on a first order approximation of the basal stress balance” (maybe?) Again, note that the solution from Schoof (2011) considers $n = 1$ (Glen’s flow law), as you have well pointed along the text.
- line 396: “is modified” => “is modified to accommodate the discontinuities in the boundary conditions”
- line 399: “Solving for … GL position”: I think this phrase could be deleted.