



P-model provides a simple but powerful method for predicting – rather than prescribing – light use efficiency and simulating terrestrial photosynthesis across a wide range of conditions. The model is available as an R package (*rpmodel*).

20 *Copyright statement.*

## 1 Introduction

Realistic, reliable and robust estimates of terrestrial photosynthesis are required to understand variations in the carbon cycle, monitor forest and cropland productivity, and predict impacts of global environmental change on ecosystem function (Prentice et al., 2015). Understanding how photosynthetic rates depend on temperature, humidity, solar radiation, CO<sub>2</sub> and soil moisture is at the core of this challenge. Process-based Dynamic Vegetation Models (DVMs) and Earth System Models (ESMs) in use today almost always use some form of the Farquhar-von Caemmerer-Berry (FvCB) model for C<sub>3</sub> photosynthesis (Farquhar et al., 1980; von Caemmerer and Farquhar, 1981



daily anomalies) (Secs. 4.6 - 4.2). We further address uncertainties associated with the fAPAR forcing (Sect. 4.4) and the uncertainties in the evaluation data by using GPP data derived from different flux decomposition methods (Sect. 4.5). The use of continuous GPP measurements, rather than experimentally disturbed measurements, makes it challenging to assess modelled GPP under extreme environmental conditions. We therefore make a further evaluation of simulated GPP during the course of soil moisture drought events (*fLUE droughts*, Sect. 4.3).

## 2 Theory

The theory underlying the P-model has been described by Wang et al. (2017a) and the derivation of equations is given therein. It is presented here again for completeness.

### 2.1 Balancing carbon and water costs

The P-model centres around a prediction for the optimal ratio of leaf-internal to ambient CO<sub>2</sub> concentration  $c_i : c_a$  (termed  $\chi$ ) that balances the costs associated with maintaining the transpiration stream and the cost of

with

$$110 \quad \xi = \sqrt{\frac{\beta(K + \Gamma^*)}{1.6\eta^*}} . \quad (9)$$

(See Appendix F1 for intermediate steps.) Because both terms in Eq. 3 are divided by  $A$ , the solution is independent of whether the Rubisco-limited rate  $A_C$  or the light-limited rate  $A_J$  (see below) are followed. With this prediction for  $\chi$ , we can use the *Coordination Hypothesis* (Chen et al., 1993; Haxeltine and Prentice, 1996; Maire et al., 2012) and the light-limited assimilation rate from the FvCB model to write

$$115 \quad A_J = \varphi_0 I_{\text{abs}} m , \quad (10)$$

with

135 In this equation  $A_J$  is no longer linear with respect to  $I_{\text{abs}}$  and thus does not have the form of a LUE model. However,  $J_{\text{max}}$  is assumed here to acclimate on longer time scales to  $I_{\text{abs}}$ , so that the marginal gain in assimilation  $A$  per unit change in  $J_{\text{max}}$  is equal to the unit cost ( $c$ ) of maintaining  $J_{\text{max}}$ .

$$\frac{\partial A}{\partial J_{\text{max}}} = c \quad (14)$$

The unit cost  $c$  is assumed to include the maintenance of light-harvesting complexes and various proteins involved in the electron transport chain. The cost of maintaining a given  $J_{\text{max}}$  is thus assumed to scale linearly with  $J_{\text{max}}$  and that this proportionality is constant ( $c$  is constant). By taking the derivative of Eq. 13 with respect to  $J_{\text{max}}$  and re-arranging terms (see Appendix F2 for intermediate steps), we obtain the  $J_{\text{max}}$  limitation factor  $L$  in

Here,  $M_C$  is the molar mass of carbon (12.0107 g mol<sup>-1</sup>) to convert from molar units to mass units, and PPFD is the photosynthetic photon flux density per square metre, integrated over a day (mol m<sup>-2</sup> d<sup>-1</sup>). fAPAR is unitless and integrates across the canopy, i.e., from fluxes per unit leaf area to fluxes per unit ground area. LUE is in units of g C mol<sup>-1</sup>. The intrinsic quantum yield parameter  $\varphi_0$  is modelled as temperature-dependent, and an additional (unitless) empirical soil moisture stress factor  $\beta(\theta)$  is included for modelling LUE.

### 3.1.1 Temperature dependence of the intrinsic quantum yield of photosynthesis

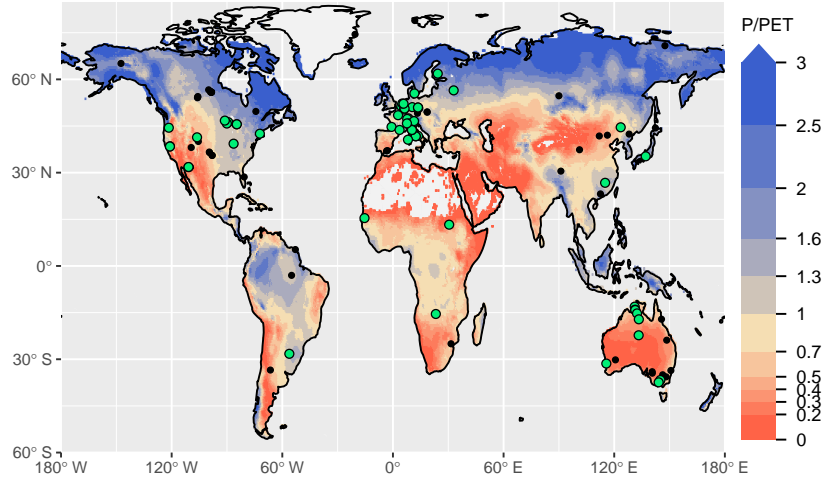
The temperature dependence of the intrinsic quantum yield ( $\varphi_0(T)$ , mol mol<sup>-1</sup>) is modelled following the temperature dependence of the maximum quantum yield of photosystem II in light-adapted leaves, determined by Bernacchi et al. (2003) as



**Table 1.** Model setups. The standard fAPAR data is MODIS FPAR MCD15A3H, where the original data, given at 4-day intervals, is splined to daily values (‘spl.’). Alternative greenness forcing data are based on MODIS EVI MOD13Q1, splined from 8-day intervals to daily, and MODIS FPAR MCD15A3H, linearly interpolated (‘itpl.’) from 4-day intervals to daily. Standard observational GPP data, used for model calibration and evaluation, are from FLUXNET 2015, based on the nighttime flux decomposition method (‘NT’ in the table, variable GPP\_NT\_VUT\_REF in FLUXNET 2015). Alternative GPP data used based on the daytime flux decomposition method (‘DT’ in the table, variable GPP\_DT\_VUT\_REF), and based on an alternative method (Wang et al., 2017a) (‘Ty’ in the table). For setups ORG, BRC, FULL, FULL\_FPARitp, and FULL\_EVI, data used for the model calibration is from all dates where NT data are available. For setups FULL\_DT, FULL\_Ty, and FULL\_NTsub, calibration data are from all dates where data is available for all three methods DT, NT, and Ty. Column  $\varphi_0(T)$  specifies whether the temperature dependence of intrinsic quantum yield is included. Column  $\beta(\theta)$  specifies whether soil moisture stress is included. Columns  $\widehat{\$







**Figure 1.** Overview of sites selected for model calibration (green dots) and evaluation (black dots). All sites and additional information are listed in Tab. A1. The color key of the map represents aridity, quantified as the ratio of precipitation over potential evapotranspiration from Greve et al. (2014).

295 meteorological variables, emerging from a previous analysis (Stocker et al., 2018). For the evaluation, we used all sites except those classified as croplands or wetlands, and seven sites where  $C_4$  vegetation is mentioned in the site description available through the FLUXNET2015 dataset (AU-How, DE-Kli, FR-Gri, IT-BCi, US-

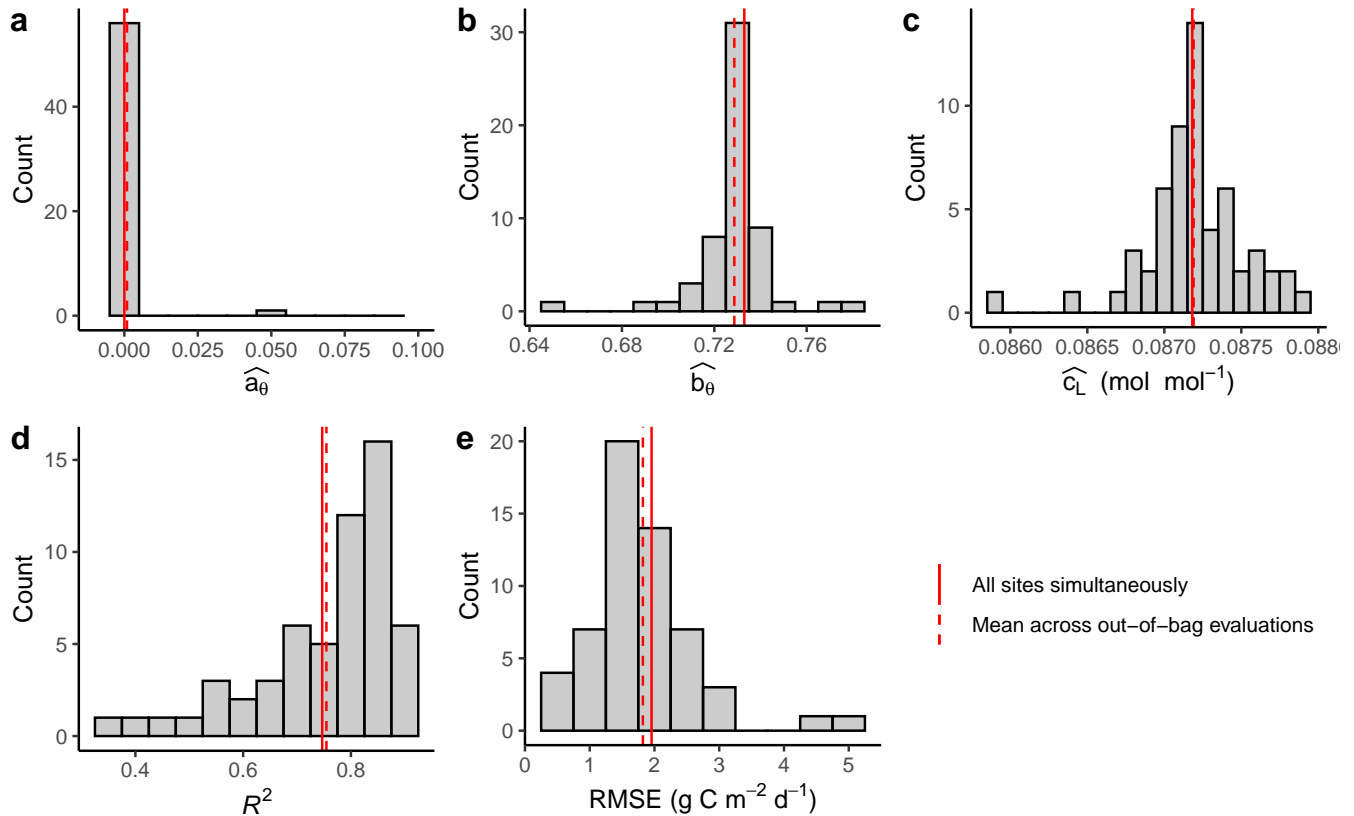
(Jiang and Ryu, 2016), BEPS (He et al., 2018; Chen et al., 2016), and VPM (Zhang et al., 2017b). A more detailed description of these models and aggregation to a common grid of  $0.5^\circ$  and monthly resolution can be found in Luo et al. (2018). For sun-induced fluorescence (SiF), we use data from GOME-2A and GOME-2B, based on v.2 (V27) 740 nm terrestrial chlorophyll  
 315 fluorescence data from the MetOp-A and MetOp-B satellites (Joiner et al., 2013, 2016). Data were aggregated to monthly and  $0.5^\circ$  resolution by mean, as further described in Luo et al. (2018).

### 3.6 Evaluation methods

We evaluated both simulated LUE and GPP. The P-model (Sect. 2) predicts variations in LUE across sites (space) and months (monthly  $\text{LUE} = m'$ ), while simulated GPP is affected by the PPFD and f

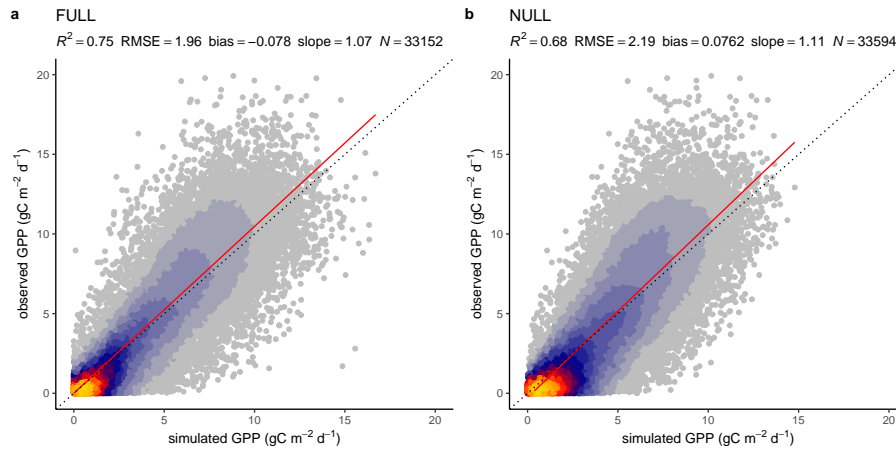
**Table 2.** Description of Koeppen-Geiger climate zones and number of sites for which data is available per climate zone and hemisphere. Sites are classified based on Falge et al. (2017), and Beck et al. (2018). Only zones with data from more than three sites are shown.

Code	<i>N</i> north	<i>N</i> south	Description
Aw	–	5	Tropical savannah
BSk	5	–	Arid steppe cold
Cfa	11	–	Warm temperate fully humid with hot summer
Cfb	20	5	Warm temperate fully humid with warm summer
Csa	12	–	Warm temperate with dry and hot summer
Csb	4	–	Warm temperate with dry and warm summer
Dfb	17	–	Cold fully humid with warm summer</



**Table 3.**  $R^2$  of simulated and observed GPP based on different model setups and for different components of variability.

Setup	8-daily	Spatial	Annual	Seasonal	var(daily)	var(annual)
FULL	0.75	0.69	0.69	0.73	0.27	0.09
BRC	0.72	0.65	0.63	0.72	0.25	0.06
ORG	0.70	0.63	0.60	0.69	0.24	0.05
NULL	0.68	0.65	0.58	0.71	0.21	0.03
FULL_FPARItp	0.73	0.71	0.69			



**Figure 3.** Correlation of observed and modelled GPP values of all sites pooled, mean









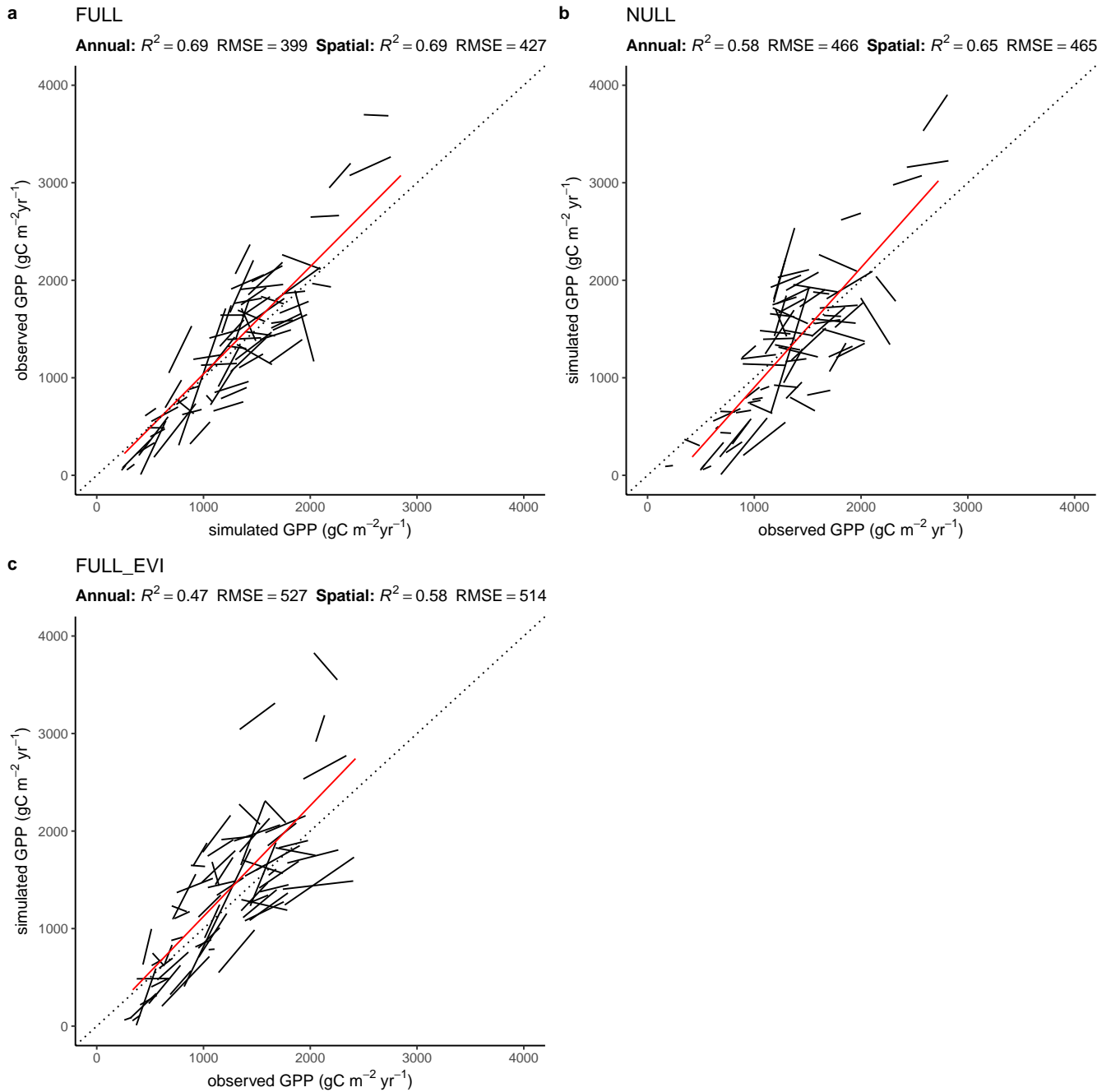


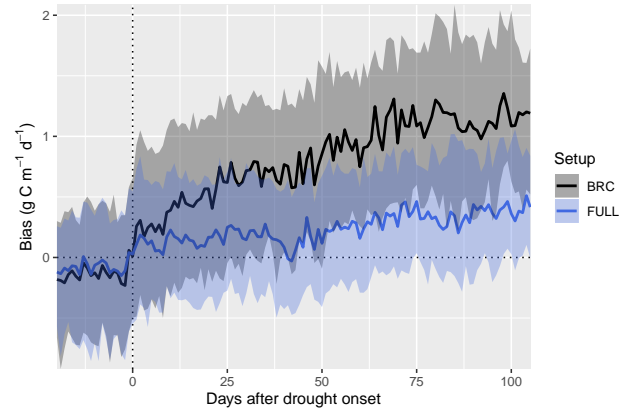




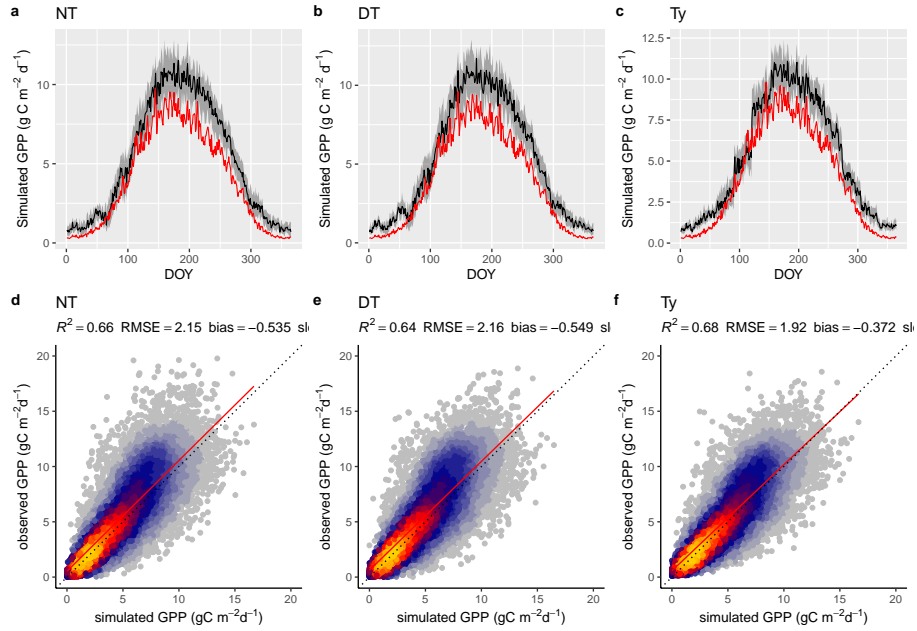
585 model driven by remotely sensed vegetation greenness. Using optimality principles for the formulation of the P-model reduces its dependence on uncertain or vegetation type-specific parameters and enables robust predictions of GPP and its variations through the seasons, between years, and across space. Further work is required to develop a distinct treatment of  $C_4$  vegetation for global applications and additional evaluations are needed to examine the P-model's sensitivity to increasing  $CO_2$ . We have shown that accounting for the effects of low soil moisture and the reduction in the quantum yield efficiency under low tem-  
590 peratures improves model performance. There is potential to include below-ground water limitation effects in the mechanistic optimality framework of the P-model.

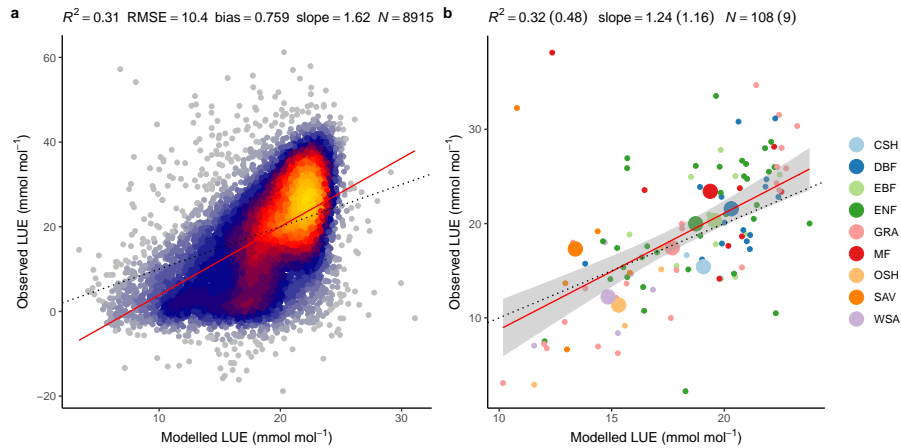
*Code and data availability.* The P-model is implemented as an R package (*rpmodel*) and available through CRAN and Zenodo (Stocker, 2019a). A documentation is on <https://stineb.github.io/rpmodel/>. Code for all evaluations presented here is available through [https://github.com/stineb/eval\\_pmodel](https://github.com/stineb/eval_pmodel). The current *rpmodel* code version, producing outputs consistent with results shown here, is v1.0.3. Model outputs  
595 are available on *Zenodo* (Stocker, 2019b).



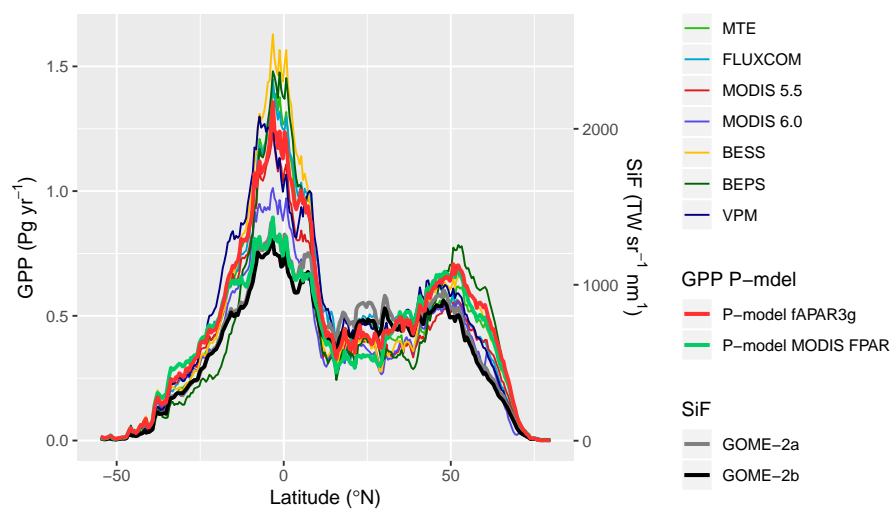


**Figure 6.** Bias in simulated GPP during the course of drought events. Simulated GPP is from a simulation with (FULL) and without (BRC) accounting for soil moisture stress. The timing of drought events is taken from Stocker et al. (2018) and is identified by an apparent soil moisture-related reduction of observed light use efficiency at 36 FLUXNET sites. The bias is calculated as simulated minus observed GPP. Data from multiple drought events and sites are aligned by the date of drought onset and aggregated across all sites and events (lines for medians, shaded ranges from the 33% and 66% quantiles).









## Appendix A: Site information

Table A1 provides meta information and references for each site from the FLUXNET2015 Tier 1 dataset, used for model calibration and evaluation in the present study.

## Appendix B: Temperature and pressure dependence of photosynthesis parameters

### 600 B1 Photorespiratory Compensation Point $\Gamma^*$

The temperature and pressure-dependent photorespiratory compensation point in absence of dark respiration  $\Gamma^*(T, p)$  is calculated from its value at standard temperature ( $T_0 = 25^\circ\text{C}$ ) and atmospheric pressure ( $p_0 = 101325 \text{ Pa}$ ), referred to as  $\Gamma_{25, p_0}^*$ . It is modified by temperature following an Arrhenius-type temperature response function  $f_{\text{Arrh}}(T_K, \Delta H_{\Gamma^$

as :

$$K_c(T_K) = \exp(38.05 - 79.43/(T_K R)) \quad (\text{B4a})$$

$$625 \quad K_o(T_K) = 1000 \cdot \exp(20.30 - 36.38/(T_K R)) \quad (\text{B4b})$$

$$V_{o,\max}(T_K) = \exp(22.98 - 60.11/(T_K R)) \quad (\text{B4c})$$

$$V_{c,\max}(T_K) = \exp(26.35 - 65.33/(T_K R)) \quad (\text{B4d})$$

By substituting the temperature-dependency equations for each term in Eq.



## C2 Intrinsic water use efficiency

680 The intrinsic water use efficiency (iWUE, in Pa) has been defined as the ratio of assimilation over stomatal conductance (to water) (Beer et al., 2009) as  $iWUE = A/(1.6g_s)$ . The factor 1.6 accounts for the difference in diffusivity between CO<sub>2</sub> and H<sub>2</sub>O. Using Fick's Law (Eq. 5), this is simply

$$iWUE = \frac{c_a(1 - \chi)}{1.6}, \quad (C2)$$

or, using the prediction of optimal  $\chi$  given by Eq. 8, this can be expressed as

$$685 \quad iWUE = \frac{1}{1.6 \left(1 + \frac{\xi}{\sqrt{D}}\right)} ($$

where  $b_0 = 0.015$  (Atkin et al., 2015). Dark respiration follows a slightly different instantaneous temperature sensitivity than  $V_{\text{cmax}}$  following Heskell et al. (2016):

$$R_d = R_{d25} f_R \quad (\text{C9})$$

$$f_R = \exp(0.1012(T_{K,0} - T_K) - 0.0005(T_{K,0}^2 - T_K^2)) \quad (\text{C10})$$

By combining Eqs. C6, C8, and C9,  $R_d$  at growth temperature  $T$  can directly be calculated from  $V_{\text{cmax}}$  as

$$R_d = b_0 \frac{f_R}{f_V} V_{\text{c$$

where

$$k_{\text{PWP}} = -0.024 \cdot f_{\text{sand}} + 0.487 \cdot f_{\text{clay}} + 0.006 \cdot f_{\text{OM}} \quad (\text{D9})$$

$$+ 0.005 \cdot (f_{\text{sand}} f_{\text{OM}}) \quad (\text{D10})$$

$$- 0.013 \cdot (f_{\text{clay}} f_{\text{OM}}) \quad (\text{D11})$$

$$+ 0.068 \cdot (f_{\text{sand}} f_{\text{clay}}) \quad (\text{D12})$$

$$+ 0.$$

755 Eq. 3 can thus be written as

$$a \frac{1.6 D}{c_a (1 - \chi)^2} = b \frac{K}{c_a \chi^2} \quad (\text{F3})$$

and solved for  $\chi$ :

$$\chi = \frac{\xi}{\xi + \sqrt{D}} \quad (\text{F4})$$

$$\xi = \sqrt{\frac{\beta K}{1.6 \eta^*}} \quad (\text{F5})$$

760 Where  $b/a = \beta/\eta^*$ . The exact solution, without the simplification  $\Gamma^* = 0$ , and following analogous steps, is

This can be re-arranged to

$$(1 - u)^{1/2} = \frac{1}{\sqrt{1 + k^2}} \quad (\text{F14})$$

The right-hand term now corresponds to the  $J_{\max}$  limitation factor  $L$  in Eq. 13, and we get Eq. 15.

To sum up, the P-model calculates GPP as

$$780 \quad \text{GPP} = I_{\text{abs}} \varphi_0(T) \beta(\theta) m' M_C, \quad (\text{F15})$$

where

with

$$\omega = -(1 - 2\theta) + \sqrt{(1 - \theta) \left( \frac{1}{\frac{4c}{m} (1 - \theta \frac{4c}{m})} - 4\theta \right)}. \quad (\text{F22})$$

805 Using this,  $A_J$  can be written analogously to Eq. 16, but with

$$m' = m \frac{\omega^*}{8\theta}, \quad (\text{F23})$$

and

$$\omega^* = 1 + \omega - \sqrt{(1 + \omega)^2 - 4\theta\omega}. \quad (\text{F24})$$

The cost

**Table A1.** Sites used for evaluation. Lon. is longitude, negative values indicate west longitude; Lat. is latitude, positive values indicate north latitude; Veg. is vegetation type: deciduous broadleaf forest (DBF); evergreen broadleaf forest (EBF); evergreen needleleaf forest (ENF); grassland (GRA); mixed deciduous and evergreen needleleaf forest (MF); savanna ecosystem (SAV); shrub ecosystem (SHR); wetland (WET).

Site	Lon.	Lat.	Period	Veg.	Clim.	N	Calib.	Reference
AR-SLu	-66.46	-33.46	2009-2011	MF	Bwk	448		Ulke et al. (2015)
AR-Vir	-56.19	-28.24	2009-2012	ENF	Csb	747	Y	Posse et al. (2016)

**Table A2.** Continued from Table A1

Site	Lon.	Lat.	Period	Veg.	Clim.	N	Calib.	Reference
CA-NS1	-98.48	55.88	2001-2005	ENF	Dfc	1067		NA
CA-NS2	-98.52	55.91	2001-2005	ENF	Dfc	1123		NA
CA-NS3	-98.38	55.91	2001-2005	ENF	Dfc	1395		NA
CA-NS4	-98.38	55.91						





**Table A5.** Continued from Table A1

Site	Lon.	Lat.	Period	Veg.	Clim.	N	Calib.	Reference
US-SRG	-110.83	31.79	2008-2014	GRA	BSk	2117	Y	Scott et al. (2015a)
US-SRM	-110.87	31.82	2004-2014	WSA	BSk	3354	Y	Scott et al. (2009)
US-Syv	-89.35	46.24	2001-2014	MF	Dfb	2365	Y	Desai et al. (2005)
US-Ton	-12							

**Table A6.** Fixed parameters. 'SC' stands for 'at standard conditions' (25 °C, 101325 Pa). 'MM coef.' refers to 'Michaelis Menten coefficient'.

Symbol	Value	Units	Description	Reference
$\beta$	146.0	1	Unit cost ratio, Eq. 3	This study
$\Gamma_{25,p_0}^*$	4.332	Pa	Photorespiratory compensation point, SC	Bernacchi et al. (2001)
$K_{c25}$	39.97	Pa	MM coef. for CO <sub>2</sub> , SC	Bernacchi et al. (2001)
$K_{o25}$	27480	Pa	MM coef. for O <sub>2</sub> , SC	Bernacchi et al. (2001)
$\Delta H_{\Gamma^*}$				

**Table A8.** Variables returned by the function `rpmodel()`. Variable names correspond to the named elements of the list returned by the `rpmodel()` function call. Symbols correspond to their use in this paper.

Variable name	Symbol	Description	Units	Reference
<code>ca</code>	$c_a$	Ambient CO <sub>2</sub> partial pressure	Pa	Sect. 2.1
<code>gammastar</code>	$\Gamma^*$	Photorespiratory compensation point	Pa	Sect. B1
<code>kmm</code>	$K$	Michaelis-Menten coefficient for photosynthesis	Pa	Sect. B3
<code>ns_star</code>	$\eta^*$	Change in the viscosity of water, relative to its value at 25 °C	unitless	Huber et al. (2009)
<code>chi</code>	$\chi$	Ratio of leaf internal-to-ambient CO <sub>2</sub> </		

*Author contributions.* B.D.S. designed the study, wrote the model code, conducted the analysis, and wrote the paper. H. W. developed the  
820 model and wrote the an initial version of the model description. N.G.S. developed the model and implemented model code. S.P.H. contributed  
to designing the study and writing the manuscript. T.K. contributed to study design, model implementation and manuscript writing. D.S.  
implemented the water holding capacity model. T.D. wrote an initial version of the model code and model documentation. I.C.P. developed  
the model and contributed to designing the study.

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