Answer to Anonymous Referee nr.2

We thank Anonymous Referee nr.2 for their thoughtful comments and corrections which allowed us to fix some important typos in the manuscript and encouraged us to rethink about the way in which the model is implemented. We provide below a point by point answer to all of their comments. The referee’s comments are highlighted in blue, while changes to the manuscript are highlighted in red.

R2C1)

It is stated on line 10 of page 4 that any organic matter that does not reach the sediments is instantaneously remineralised. This decision seems quite reasonable, but it is contradicted by the equations, in which organic matter is not only remineralized but is also advected and mixed. I recommend to do either one thing or the other but not both. If suspended particles are going to be mixed around in the model then they should have their own separate ordinary differential equations and state variables. Alternatively, if particle flux and remineralisation are made instantaneous, as stated in the manuscript, then there should be no mixing or advection of particulate organic matter. Whichever way it is done, the descriptions in the text need to be made consistent with the equations.

We thank Referee nr.2 for their observation, and we agree with them on the fact that our choice of combining an implicit sinking scheme with lateral advection and mixing can be troublesome. For this reason, we have modified the code in order to switch off the lateral advective and vertical mixing fluxes of POC. In our new baseline run, POC is exclusively redistributed throughout the boxes by vertical sinking, in line with the Referee’s suggestion. In order to account for these changes in the model, we have modified the manuscript as described below.

The description of the $P_{org}$ physical fluxes in section 2.2.2 has been reformulated as:

The implicit representation of the organic matter in the water column implies that no organic matter is accumulated in the ocean. In our baseline version of the model, corresponding to the results presented in this manuscript, SPorg and LPorg are redistributed throughout the watercolumn exclusively by implicitly modelled gravitational sinking before being either buried, accumulated in the sediments or remineralized. Even though the vertical export by downwelling and mixing (Stukel and Ducklow, 2017), and the lateral organic matter redistribution (Lovecchio et al., 2017;Inthorn et al., 2006) may be important when working with suspended SPorg ($z_{remS} = 0$), these fluxes are not currently accounted for in the model.

We have modified Figure 1 and 2 in order to account for this change in the model.

We have run the revised model both with and without the lateral advective and vertical mixing fluxes in order to understand the impact of removing these fluxes onto our model results. We include below a few significant plots which show overall higher oxygen levels and production in the absence of mixing/advective fluxes of $P_{org}$. Our main conclusions remain unchanged.
Figure R2-1: Ocean P and O₂ budgets for 2 model configurations: without POC lateral advection and vertical mixing (subplots (a) and (b), new baseline); with POC lateral advection and vertical mixing (subplots (c) and (d)).
Figure R2-2: Oxygen concentrations in the deep shelf (ds) and in the deep open ocean (do) for 2 model configurations: without POC lateral advection and vertical mixing (subplots (a) and (b), new baseline); with POC lateral advection and vertical mixing (subplots (c) and (d)).

Figure R2-3: Total ocean production for 2 model configurations: without POC lateral advection and vertical mixing (subplots (a) and (b), new baseline); with POC lateral advection and vertical mixing (subplots (c) and (d)).
Tables 1 to 3 are very helpful. Another table needs to be added, listing the state variables in the model and stating their units. In addition, the units of all equations (the left-hand side) should also be stated, if they are not already given in the tables.

As suggested by Referee nr. 2, we have added a table (Table 1 of the revised manuscript) which indicates names and units of the state variables of the model.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{ss}$</td>
<td>Inorganic phosphorus in surface shelf sea box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$P_{ds}$</td>
<td>Inorganic phosphorus in deep shelf sea box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$P_{so}$</td>
<td>Inorganic phosphorus in surface open ocean box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$P_{do}$</td>
<td>Inorganic phosphorus in deep open ocean box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$O_{ss}$</td>
<td>Molecular oxygen in surface shelf box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$O_{ds}$</td>
<td>Molecular oxygen in deep shelf box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$O_{so}$</td>
<td>Molecular oxygen in surface open ocean box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$O_{do}$</td>
<td>Molecular oxygen in deep open ocean box</td>
<td>mmol m$^{-3}$</td>
</tr>
<tr>
<td>$O_{at}$</td>
<td>Oxygen mixing ratio in atmosphere (mol mol$^{-1}$)</td>
<td>-</td>
</tr>
<tr>
<td>$S_{ss}$</td>
<td>Organic phosphorus in the sediments of the shelf sea</td>
<td>mmol m$^{-2}$</td>
</tr>
<tr>
<td>$S_{so}$</td>
<td>Organic phosphorus in the sediments of the open ocean</td>
<td>mmol m$^{-2}$</td>
</tr>
<tr>
<td>$P_{TOT}$</td>
<td>Diagnostic variable: total P budget from sources and sinks only</td>
<td>Tmol P</td>
</tr>
<tr>
<td>$O_{TOT}$</td>
<td>Diagnostic variable: total O budget from sources and sinks only</td>
<td>Pmol O$_2$</td>
</tr>
</tbody>
</table>

Table 1: List of the model’s state variables and of their units

We mention the new table in subsection 2.1 as follows:

The entire set of the model’s state and diagnostic variables and their units are listed in Table 1.

Too many of the equations in the model are dimensionally inconsistent. That is to say, the units on the left-hand side of the equation do not match the units on the right-hand side of the equation when the different terms are combined together. As an example, equation 4 on page 4 is an equation for the rate of organic matter production in units of Moles y$^{-1}$ or mmol m$^{-3}$ y$^{-1}$ or similar. Because this is an ongoing flux rather than a one-off transfer, it must be expressed as a rate of transfer per unit time. However, none of the terms on the right-hand side of the equation have time anywhere in their units. The equation is formulated in such a way that it appears to be aimed at converting a fraction of the surface phosphorus concentration into production at each timestep, but the way it is actually formulated means that the rate of
conversion of surface phosphate into production (organic matter) will depend on the timestep used. Shorter time steps will convert phosphate to organic matter more rapidly than longer timesteps whereas ODE equations should be timestep-independent. Equation 5 is another example, where, according to the equation, ‘Coag’ must have units of organic matter concentration squared per year, which makes no sense. Before resubmission, I recommend that every equation in the model is checked for dimensional (units) consistency: multiplying through the terms of the terms on the right-hand side should produce the units of the term on the left-hand side.

We thank Referee nr.2 for having spotted a few mistakes in the manuscript’s equations. We have double checked the entire set of equations, the parameters’ units and we have corrected a few typos.

As highlighted by the Referee, the uptake rate $P_{eff}$ (Equation 4) as well as the coagulation factor $cg_r$ (Equation 5) were missing part of their units (see Table 4 of the revised manuscript). We have now corrected their units accordingly (see also Gruber et al., 2006). We have explained better the values and magnitudes of the parameters in the table’s caption. We have also spotted another few inconsistencies in the units and fixed them both in the tables and in the equations.

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R2C4

I did not notice a statement anywhere that conservation of mass (or, more properly, conservation of total inventories of elements) has been checked and found to be stable. This is easy to do for a box model, whether it is closed or open. Obviously for a closed model, if there are no errors in the equations, the total sum of atoms of a given element should be constant over time. For an open model, the changes in the total inventory over time should exactly match the sum of external inputs over time minus the sum of the outputs from the system over time (i.e. $\Delta Inv = -$). This can easily be checked by adding two extra differential equations to the model: one to track the sum of the external inputs to the system and another to keep track of the sum of the losses from the system as a whole. From the model equations, I suspect that the model does not properly conserve phosphorus and oxygen but rather there is some (unintended) cumulative creation/destruction over time. This will interfere with the ability of the model to be run over long timescales to address the geological timescale questions of interest. It is a little bit unclear, but it appears that the amount of phosphate removed per unit time from the surface box as particle export is not identical to the amounts of phosphate added per unit time to the deep box and sediments combined. The euphotic zone depth appears in the equation for the former but not the latter, for instance, whereas if it appears in one then it should also appear in the other. Again, checks can be made by adding extra (book-keeping) ODEs to the model. For this example, one extra ODE could tally up the cumulative export from the surface box and another extra ODE could tally up the sum of the cumulative inputs to the deep and sediment boxes. At the end of each model run, a quick numerical check can be made to ensure that the tallies are identical within the precision of numerical rounding errors.

We agree with Referee nr.2 that a check on conservation of mass is essential for the model’s evaluation. Even though we did not discuss it explicitly, our model does go to equilibrium on the long term, as now visible from Figure 3 of the revised manuscript.

As a further check we have now added 2 extra variables to the model, $P^{TOT}$ and $O^{TOT}$, which represent the total P and O budgets. These two variables (in units of TmolP and PmolO$_2$, respectively) are initialized to the total sum of the two tracers across all boxes at time zero.
At each time step, $P_{TOT}$ and $O_{TOT}$ evolve according to the sum of all sources and sinks in the model. Their derivatives are as follows:

\[
\frac{dP_{TOT}}{dt} = \left[ P_{in} - \left( CaP_{form} \cdot A_{ocean} \cdot P_{shelf} + CaP_{form}^0 \cdot A_{ocean} \cdot (1 - P_{shelf}) \right) \right] \cdot 10^{-15}
\]

\[
\frac{dO_{TOT}}{dt} = \left[ OProd^{ss} \cdot V^{ss} + OProd^{so} \cdot V^{so} - \left( (SedRemO^{ds} + AerRem^{ds}) \cdot V^{ds} + (SedRemO^{do} + AerRem^{do}) \cdot V^{do} + OxyWheat \cdot (Mol_{atmo} \cdot 10^{3}) \right) \right] \cdot 10^{-18}
\]

At the end of each model run we plot the sum of the total $P$ and $O$ content in the model at each time step by multiplying the concentrations in each box by the box volume or area (the latter for the sediments), and check that this corresponds to the time evolution of $P_{TOT}$ and $O_{TOT}$ calculated at run time regardless of the partition of the tracers among the boxes.

In order to include these new diagnostic variables in the model’s description, we have modified the first paragraph of subsection 2.1 as follows:

The box model resolves explicitly for each relevant box the local concentrations of three types of tracers: molecular oxygen $O_2$ (O), inorganic dissolved phosphorus (P) and sediment organic phosphorus (SedP$_{org}$). The total budgets of P and O, respectively $P_{TOT}$ and $O_{TOT}$, are also independently integrated from the net sources and sinks of the two tracers over the entire model domain, for the purpose of checking mass conservation.

We have also added the following statement to subsection 3.1 of the model evaluation:

Starting from the initial values listed in Table 3, the modelled state variables evolve towards equilibrium for any couple of values of $z_{rem}^{S}$ and $z_{rem}^{L}$ in the explored interval. Simple mass conservation checks show no hidden source or sink of tracers in the model’s boxes.

We include below the result for the choice of $z_{rem}^{S}$ and $z_{rem}^{L}$ used for Figure 3 of the revised paper.
Figure R2-4: Comparison between the evolution of the total tracer budgets \((P_{TOT} \text{ and } O_{TOT})\) calculated at run time from the tracer’s net input fluxes (total sources + total sinks), and the tracer budgets calculated integrating the local concentrations in the boxes at each time step from the model’s output.