Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2019-122-RC1, 2019

Geoscientific Model Development Discussions

Interactive comment on "Extending Square Conservation to Arbitrarily Structured C-grids with Shallow Water Equations" *by* Lilong Zhou et al.

Anonymous Referee #1

Received and published: 19 August 2019

General comments

The manuscript considers the numerical solution of shallow water equations on quasi-uniform spherical meshes. It explains how an energy-conserving spatial discretization can be combined, using a suitable change of prognostic variables, with a square-conserving temporal discretrization to obtain a scheme that is exactly energy conserving.

There are some problems with the presentation in the manuscript (details below), which it should be possible to fix. However, the key ideas needed to obtain exact energy conservation on arbitrary meshes have been around for a while; this paper merely brings them togther. Also, temporal truncation errors tend to be much smaller than spatial truncation errors in atmospheric models, so only a small improvement (if

any) is obtained by replacing an energy-conserving spatial discretization by an energy-conserving space-time discretization, (as the results in this paper confirm). Thus, I think the manuscript lacks the originality and significance needed to justify publication in GMD.

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Specific comments

Sections 3.1, 3.2. The notation $\mathcal{L}F$, $\mathcal{M}u$, $\mathcal{N}\phi$ suggests that $\mathcal{L}$, $\mathcal{M}$, $\mathcal{N}$ are linear functions of F, u, and ϕ respectively. In fact they are all nonlinear functions. Moreover, $\mathcal{M}$ and $\mathcal{N}$ are actually functions of both u and ϕ. These two sections are over-elaborate and presented in a very confusing way. In several places it is not obvious what is assumed and what is claimed to be proved. All that is really needed is the fact that the energy at a point can be written as a squared quantity by making a certain change of variables.

P1 line 27, also P2 lines 3-5. The opening sentence is too categorical. For a quantity like energy or potential enstrophy, in a numerical model the total is made up of resolved and unresolved contributions. Therefore it is not obvious that conserving the resolved contribution is necessary for a good solution; indeed it may be necessary to dissipate the resolved contribution (e.g. to prevent 'spectral blocking'). One can argue for a conservative numerical method by saying that we want to parameterize any dissipation, not leave it to numerical errors, but the opposite argument can also be made. Such ideas are extensively discussed in the literature.

P1 line 29, Figs. 3b, 5b, 7b, P19 line 17. On a spherical domain the vanishing of the global integral of vorticity is a purely kinematic identity. Provided the vorticity and its integral are calculated in a self-consistent way, the same result must hold in the discrete case (e.g. P10 line 20). Thus conservation of the global integral of vorticity is a test only of the fact that the vorticity is calculated consistently; it says nothing about the properties of the numerical methods used to solve the equations.

P1 line 29. '...five basic physical conserved properties'. Actually potential enstrophy is just one member of an infinite family (so-called Casimirs).

P3 line 23. This form of the equations is usually called 'vector-invariant' (as on P19 line 16).

P4. Equation (3) is inconsistent with the definition of the 2-norm on line 5. This paragraph seems to be mixing up point values and global integrals.

Figure 1. Note that the grid used need not be uniform and regular (as suggested by the figure).

P5. Note that the sign convection for u_e is related to the direction of the unit normal - this is crucial to get everything to work out. Also crucial for energy conservation is that $Q_e^{\perp}$ is constructed to satisfy the equation on P20 line 9.

P5 line 22. '...new type of Runge-Kutta'. Not so new (1996).

P6 line 11. It would be helpful to give a reference for 'IAP transformation'.

P11 lines 22-23. The text does not make sense - it seems to be mixing up point values and global integrals.

Section 5.2. The fact that the solid body rotation flow eventually breaks down, despite the conservation properties of the scheme, is intriguing. Could this be a manifestation of the 'Hollingsworth instability' (as discussed, for example, in Skamarock et al 2012)?

P19 line 26. Sign error? Line 30. What is h_e ?

References are not in alphabetical order.

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