Response to Reviewer 1

We thank the reviewer for providing a supportive and constructive review of this work. Your insightful suggestions and comments have substantially improved the clarity of the revised manuscript. Below are responses addressing each comment. A revised manuscript is included following our responses to both reviewers.

Sincerely,

Aleah Sommers

RC1 comments (in blue):

General comments

1. The introduction should be more clearly focused (and quite possibly expanded) on the topic of subglacial hydrology. There is a fairly extensive body of literature about subglacial model development, including extensive work on alpine glaciers. The focus on outlet glaciers and sea level rise in the introduction is somewhat of an aside.

We have revised the introduction (Section 1) to be more general, but we feel that the implication of increased mass loss from glaciers and ice sheets as contributors to sea level rise provides broader context and is the primary big-picture motivation for this work and the increasing body of research on the response of ice sheets to climate change. For this reason, we retain some emphasis on outlet glaciers and sea level rise.

2. The motivation of the manuscript is somewhat unclear if the reader is un- indoctrinated into the world of subglacial hydrology. It would be useful to include a through description of viscous dissipation and why it hasn't been included in previous subglacial models in section 1.2 and clearly describe - before the model description - the goals of this modeling effort.

We have revised Sections 1.1 and 1.2 to include a more clear explanation of mechanical energy dissipation and the problems that have arisen with including this term in other formulations, as well as a clear statement of the goal to see if we could use a single set of governing equations to produce systematic self-organized channelization where it should occur.

3. The basal flux parameterization (Line 25) needs to be more carefully documented. There are several line notes to this effect, but essentially, the addition of the Reynolds number requires the selection of characteristic length scales and dimensionless parameters - reasoning behind how these values are assigned should be included in order to enhance the usefulness of this manuscript.

Section 2.1 in the revised manuscript now includes an elaboration of the flux formulation

(Eq. 5 in the revised manuscript), the different terms involved, and a description of its basis in fracture flow and derivation.

Specific comments

Page 1 Line 1. I am not sure "poorly understood" is the best phrase to use here. There is an extensive body of literature exploring the state and evolution of the subglacial hydrologic system and its representation in current models, while not perfect, are able to replicate many features of ice velocity fields. We know that the link between melt and ice motion is the subglacial system; however, there are parameters and parameterizations which are not well constrained.

This wording has been changed to "not fully understood".

1-2. The wording of this sentence is awkward.

This wording has been revised.

9. Much of the manuscript switches between 'channel' and 'efficient' drainage. Con- sider using something like '. . .over a wide range of drainage efficiencies. . .' or inefficient and efficient drainage to eliminate the "channel".

After careful consideration, we feel that using "channelized" and "sheetlike" drainage is more descriptive than "efficient" and "inefficient" drainage, and helps the reader better visualize the drainage configuration. In the revised manuscript, we have made an effort to be consistent in our use of the terms, while clarifying early in the paper that channelized drainage refers to efficient drainage, and sheetlike drainage refers to inefficient or distributed drainage.

15-22. While understanding ice sheet dynamics is important for the characterization of future sea level rise, it might be more correct to acknowledge that basal lubrication alone may not be a major uncertainty in sea level rise predictions (e.g., IPCC, sea level change chapter, pages 1168-1169; Shannon et al., 2013).

This text has been updated (lines 17-18 of revised manuscript).

21-22. Consider citing the chapter instead of the whole 'Physical Basis' document.

Citation changed (line 18): Church, J.A., P.U. Clark, A. Cazenave, J.M. Gregory, S. Jevrejeva, A. Levermann, M.A. Merrifield, G.A. Milne, R.S. Nerem, P.D. Nunn, A.J. Payne, W.T. Pfeffer, D. Stammer and A.S. Unnikrishnan, 2013: Sea Level Change. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

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7. There are a number of other publications that could be cited along with Cowton et al. (2013), including Bartholomew et al. (2010), Chandler et al. (2013), and Andrews et al. (2014).

Citations added (Page 2, lines 9-10).

8-9. This sentence seems out of place and doesn't provide much information.

This sentence was removed.

12-13. It seems that a description of the unknowns would logically follow this sentence instead of a description of a description of how the subglacial system works. It might be useful to remove references to unknowns - while this is certainly true - the main purpose of the manuscript is to rectify a persistent known problem - that models of subglacial hydrology tend to only represent 2 endmembers of the continuum of possible configurations.

This sentence was removed.

23-26. Reference the section numbers

Changes have been made in the revised manuscript to reference the section numbers (page 2, line 33 to page 3, line 2).

27. This section would benefit from explaining the motivation for subglacial hydrology model development as well. See Flowers (2015) for a great review of the topic.

It was a major oversight to neglect citing Flowers (2015) in this section. We are familiar with the review and have greatly benefited from it; we thank the reviewer for pointing out the omission. In the revised manuscript, we clearly point toward that paper as a resource (Section 1.1).

Page 3

3-4. This relates to the previous comment - it would be nice to detangle why the community ended up focusing on these two endmembers.

We have attempted to clarify this in the revised manuscript (Sections 1.1-1.2).

13. See comments regarding line p2L12-13 and p1L1.

Text changed: "Although the effects of surface melt on ice sheet dynamics are not yet entirely understood…" (page 3, line 21).

34 - p4L5. These sentences start to feel rushed. Also consider including more recent work by Rada and Schoof (2018) and Downs et al. (2018).

The text has been revised (page 4, lines 8-16).

Page 4 7-8. This sentence is a direct repeat of a sentence in the abstract. Consider revising.

This text has been revised (page 4, lines 18-19).

13-15. The instability that arises with the viscous dissipation has been discussed by a number of studies (Hewitt et al., 2012; Hoffman and Price, 2014; Kamb, 1987; Schoof, 2010; Schoof et al., 2012; Walder, 1986; Werder et al., 2013). In addition, Flowers (2015) has a nice summary of the reasoning behind and numerical approaches to switching between drainage elements. Because the primary contribution of this work is the inclusion of the viscous dissipation term and the representation of both turbulent and laminar flow, it is important to thoroughly discuss the reasoning and justification and numerics used in previous modeling work. This summary could readily follow lines 13-15.

We have attempted to address this by including a clearer description and explanation in the revised manuscript (Section 1.2).

18. If the isolated/weakly connected system is the primary scientific motivation behind the inclusion of viscous dissipation, then it would be beneficial to expand upon this topic (and include the body of work from alpine glaciers) (e.g., Andrews et al., 2014; Gordon et al., 1998; Hodge, 1979; Murray and Clarke, 1995), perhaps in a separate section or paragraph. However, the manuscript should also note that a through modeling effort to explore this future work.

The isolated/weakly connected system is not the main motivation for including the dissipation term, but it is a challenge that faces many existing subglacial hydrology models. We do not specifically attempt to produce unconnected regions, but the flexible configuration of the geometry in our model may be conducive for allowing these regions to exist with appropriate topography.

Page 5 4-5. It would be useful to expand on the representation of channels in this model compared to other models because they are very different - previous models represent channels along element edges (e.g., Hewitt, 2013; Schoof, 2010; Werder et al., 2013).

The text has been revised (Section 1.2).

5-6. Is two-way coupling implemented between ShaKTI and ISSM?

Not yet, although that is planned for upcoming work. We have clarified this in the revised text (page 5, lines 24-26; also see Section 4.1: page 15, lines 24-28).

21-22. Clearly define that β is a function of bedrock bump height and spacing and that it goes to zero when the gap height exceeds the bedrock bump height. This is essentially the delineation between 'cavity type' opening and 'channel type' opening and shouldn't be relegated to the Tables alone.

The text has been updated and β clearly defined (page 6, lines 23-25, Eqs. 3 and 4).

26. What does ω represent, more than simply the 'Parameter controlling nonlinear transition between laminar and turbulent flow'? In order to be useful to readers, some information about how it is chosen needs to be provided.

$1/\omega$ represents a Reynolds number at which a departure from laminar flow behavior becomes significant, and the square-root turbulent dependence becomes dominant. We have elaborated in the revised text to include a more detailed description and explanation of the flux formulation in the revised manuscript and the role of ω (page 7).

26. What is the characteristic length scale used in the calculation of the Reynolds number? This length scale should be associated with bedrock bump spacing and the gap height though some sort of hydraulic radius. How this is characterized and justification should be discussed. In this vein, q in Table 1 should probably have an equation associated with it.

The Reynolds number depends on the flux q, using the gap height as a characteristic length. This has been explicitly added to the revised manuscript (see Eq. 7). Additional explanation is included in Section 2.1 following Eq. (5) on page 7 to clarify the basis and derivation of our flux formulation. The equation for Re has also been added to Table 1 for clarity.

Page 6 10-29. It would be useful to mention that the internal dissipation term in Equation 10 is not included in the Werder et al. (2013) formulation and perhaps nod to previous discussion of the inclusion (or lack thereof) of this term in previous modeling

The text has been revised (page 8, lines 15-17).

Page 7 3. Awkward phrasing.

This text has been revised (page 9, lines 4-5).

13-15. The over/under pressure problem is complex (Hewitt et al. (2012) and Schoof et al. (2012) only solve it in one dimension). It may be best to temper this statement and simply explain why subglacial pressures are constrained and how the forces are balanced.

The wording has been changed in the revised manuscript (page 9, lines 19-21).

15. Extra ';'

Removed in updated manuscript.

Page 8 19-20. The grid scale and the duration of the model run should be mentioned.

These details have been included in the revised manuscript (Section 3.1).

30-31. Rather than stating that the head and gap heights show a clear channelization structure, why not plot the 'degree of channelization'? This will remove any ambiguity.

Degree of channelization is included in the revised figures.

Page 9, 1. consider using the term ’arborescent’.

This is a word with nice imagery and historical context in subglacial hydrology; it has been included in the revised manuscript (page 11, line 12).

5-8. This should be moved and expanded into a model limitations section and the supplementary figure should move to the main text.

The supplementary figure has been revised and is now included in the main text as Fig. 4. We keep the discussion of mesh dependence for this example here where it first appears in the simulation results (Section 3.2) and revisit the topic in the new “Model Limitations” section 4.1.

7-8. Quite similar might be an overstatement, particularly because differences in the vicinity of channels is +30 meters - which is ~10% of the total ice thickness and ~50% of the total diurnal head variation measured by Andrews et al. (2014).

This wording has been changed and the figures revised. In the revised manuscript, the meshes were adjusted to ensure that each moulin input was truly located at the same location, and the results more clearly illustrate mesh convergence and the local variations that arise in regions of channelization (Figs. 3 and 4).

23-24. What low distributed input value? Does the choice of initial subglacial gap height affect the spin up time?

1 m a^{-1} distributed input (updated in text; page 12, lines 5-13). We acknowledge this is unrealistically high, as are the magnitudes of distributed melt input in our transient example; these extreme values are used to demonstrate the stable transition to self-organized channelization with very high forcing.

No, the initial gap height does not significantly affect the spin-up time for initial gap height of 0.001 to 0.1 m; this is now stated in Section 4.1 (Model Limitations).

28. Though the meltwater input during the winter is low, it really isn’t realistic. Is there a model stability reason for having winter meltwater input?

Yes. With zero meltwater input everywhere, the system tries to shut itself down and the nonlinear iteration has trouble converging. The model does perform well with zero distributed meltwater input if there are other point inputs somewhere in the domain (like in the single moulin and 10 moulin examples in this paper, Sections 3.1-3.2). In reality, englacial discharge may be lagged with some low input to the subglacial system occurring through winter.

Page 10 6-8. These sentences imply that the model is fully coupled with ISSM. Unless this is the case, consider adding citations to delineate that the described ice velocity behavior is what would be expected in the coupled model, or rephrase the sentences.

We hold velocity constant, but describe the pressure changes and how that would relate to sliding velocity in a coupled model. The text has been updated in the revised manuscript to avoid misleading wording (page 12, lines 23-27).

16-18. The last sentence in this paragraph is a bit out of place.

This sentence has been moved to Section 1.2.

Page 11 7-9. This sentence should have a citation to minimize confusion between the model results here and the link between ice velocity and the subglacial system.

The wording has been changed in the revised manuscript.

9-11. It makes sense to try and relate this work to observational work on outlet glaciers since those are the glaciers most likely to impact sea level rise, but the boundary conditions and the model domain presented here are more realistic for land-terminating regions of the ice sheet.

The text has been updated (page 14, lines 11-13). Our model can be applied to either land-terminating or marine-terminating glaciers, with different boundary conditions. We have tested the model on a marine-terminating glacier with successful results (forthcoming, not included in this paper).

15-20. This paragraph should be expanded into a ’model limitations’ section. It would also be nice to see some discussion of an ideal length scale. I imagine that when coupled with an ice dynamical model, there will be some grid size after which, a finer mesh won’t improve modeling results due to modeled ice characteristics.

A Model Limitations subsection has been added in the revised manuscript (Section 4.1) that includes discussion of an ideal length scale. Also note the additional exploration of mesh dependence included in the revised manuscript for the examples in Sections 3.2 and 3.3.

Figures

Figure 2. It would useful to see the ’degree of channelization’. Also consider using a non-linear color scale for gap height and flux.

This figure has been revised. (Fig. 2)

Figure 3. Can the gap height panels be plotted on the same scale? It would also be useful to see the ’degree of channelization’

This figure has been revised. (Fig. 3)

Figure 4. ’Box on’ for panels b and c. Panel labels are also needed. Consider adding ’degree of channelization’. Instead of using the log of gap height, consider just using a nonlinear color bar (for this and all other figures).

We have revised the figures to include degree of channelization and now use log-scale color bars for gap height, flux, and degree of channelization to show detailed structure rather than plotting the log of quantities.

SI figure. This figure should move to the main text and include difference plots of channelization and possibly gap height.

This figure has been revised and moved to the main text as Fig. 4. Due to the slightly offset locations of specific channels in unstructured meshes, difference plots may in some cases be misleading by indicating a large error, which is in reality a quite similar value but slightly offset due to mesh variations. To more clearly illustrate mesh dependence/convergence and areas of local variation, we have revised Figs. 3 and 4 to include a broader range of mesh resolution and compare y-averaged pressure distributions.

Response to Reviewer 2:

Thank you for your detailed and helpful review of this manuscript. We have attempted to address the concerns and incorporate your recommendations into a revised version of the manuscript. Below we respond to individual comments. A revised manuscript is included following the responses.

Sincerely,

Aleah Sommers

RC2 comments (in purple):

As elaborated below, I think the description of this new model has the potential to make a strong contribution to GMD if the authors consider the following revisions (roughly in order of importance):

(1) Adding technical model detail commensurate with (assumed) expectations for a journal focused on model development, including a more thorough elaboration of model boundary conditions, implementation and numerics; (2) Amplifying the description of the conceptual model and more thoroughly justifying the choices made in model formulation; (3) Reporting on the results of basic model testing: model convergence, consistency, efficiency, grid refinement (done to an extent already), etc. and presenting quantitative evidence of model performance (e.g. runtimes); (4) Addressing issues that plague many models of subglacial hydrology and being up-front about the shortcomings of the current model (or better showcasing the successes). Examples of these issues are: (a) low winter water pressures in contrast to observations, (b) englacial storage motivated by numerical need, (c) extreme sensitivity to initial and boundary conditions, (d) maintaining saturated conditions, (e) water-mass conservation when pressures are capped at overburden (f) convergence in the presence of substantial bed topography (g) fundamental continuum assumptions and omission of the unconnected bed (h) prescription of constant sliding speed and omission of two-way coupling (5) Streamlining the introductory material and omitting or condensing content that anyone reading this paper with the intention of using the model should already know very well; (6) Dialing back some of the stated advances of SHaKTI over existing models;

Detailed comments (page.line)

SHaKTI: Not sure exactly what “kinetic transient interactions” are and why this phrase forms an essential part of the model name. “kinetic transient interactions” sounds more like a biochemistry term. It would help the readers if the authors could use the full model name in a sentence to make it clear why this acronym was chosen, aside from its perhaps appealing phonetic similarity to “chakra”.

The name SHAKTI is intended to highlight the complex interactions through movement of water and ice (the term “kinetic” refers to motion in this context), and the fact that the subglacial system evolves with time (“transient”). A description has been added for clarity in Section 1 when the model name is first presented (page 2, lines 26-27), as well as a comma in the name after the word “kinetic” to help avoid confusion with chemical kinetics.

Note: “Shakti” is a Sanskrit term for energy, which gives form to everything in the universe. This could be seemingly unrelated to subglacial hydrology in particular, but we contend that it is highly relevant to all physical phenomena. In fact, the energy dissipated by flowing water plays an important role in generating the subglacial hydrologic system, as we see by including the dissipation term in our model formulation.

1.5 “changes the governing physics under different flow regimes” If this were a clunky IF-THEN sort of statement in standard models, then I see the point. Models like those of Hewitt (2013) and Werder et al (2013) have all the governing physics but simply apply the appropriate governing equations to different parts of the model mesh (edges versus cells). Perhaps the point to emphasize here is that SHaKTI, in principal, may capture intermediate flow regimes with the laminar-turbulent transition. One could argue, however, that the other models also do this by having channels and cavity systems operating simultaneously and in spatial proximity, thus together forming intermediate flow regimes.

The text has been revised in the updated manuscript (page 1, lines 4-6): “Imposing a distinction that applies different equations to capture the dominant physics in different parts of the model domain, however, may not allow for the full array of drainage characteristics to arise”.

1.13-14 “supporting the notion that. . .” delete. Too obvious.

This text has been deleted.

1-2. Suggest condensing introduction and omitting textbook-level content, e.g., lines

16-18.

The Introduction (Section 1) has been revised while attempting to maintain sufficient background explanation for anyone not familiar with subglacial hydrology (pages 1-4). Reviewer 1 of this paper suggested that the introduction should be broader, so we have aimed to strike a balance between the recommendations of both reviewers. We hope that the revised introduction helps to better place this work in context of previous work in subglacial hydrology and the overall context of ice sheet response to climate change.

2.28-3.10. Suggest omitting or highly condensing this very basic background material. See Flowers (Proceedings of the Royal Society A, 2015) for a convenient citation to replace much of this content. Ditto for most of page 3.

We have revised Section 1.1 and included a clear reference to Flowers (2015) at the start. Although this paper is focused on the model formulation, we feel it is necessary to include an ample description of previous work to place our model in context of what has come before. This follows the same line of reasoning as our response to the previous comment.

4.16. I'm not sure most glaciologists would agree that using different governing equations for fundamentally distributed and channelized drainage systems is questionable. Perhaps emphasize the lack of intermediate flow regimes as in the next sentence. Here I think the drawbacks of existing models are overstated.

As pointed out by the reviewer, there is indeed a continuum of flow morphologies that develop in subglacial environments that include intermediate flow regimes. From a fundamental perspective, there is no need to use different governing equations for distributed versus channelized drainage systems – each system requires statements of water mass conservation, ice mass conservation, momentum balance and energy balance, including relevant terms. Parameterizations of various terms in the governing equations may vary – for example, the creep closure term can be parameterized for channels by invoking the approximately semi-circular geometry, whereas a different parameterization is used for distributed systems where creep closure occurs between supporting bedrock bumps. We do not disagree that different parameterizations of various terms in the governing equations may be suitable for distributed versus channelized drainage. However, the governing equations should be similar. More specifically, we meant to emphasize the inclusion or not of the dissipation term in the energy equation in the distributed drainage system here. To avoid misunderstanding, we have revised the text in Section 1.2.

4.26. Replace "it is satisfying"

This text has been revised.

4.28. Not clear how this model allows "high-resolution" exploration in particular.

Because the model would not capture effects of channelization on a very large grid, it is not necessarily appropriate for very coarse resolution studies (elements spanning several km or larger), which is what we intended to convey. We have added additional discussion relating to mesh dependence to the revised manuscript in Sections 3.2, 3.3, and 4.1. We agree with the reviewer that the term "high-resolution" does not contribute much meaning, and the text has been revised.

5.1-8. Please give an overview of the conceptual model here. How is the drainage element envisioned? How does this relate to the fracture-flow formulation of q?

The subglacial drainage system is represented as a sheet with variable gap height. This is now more clearly stated in the revised text (page 5, lines 18-19).

5.10-11. Conservation of water AND ice mass? "basal water flux" => "horizontal water flux"; define "internal melt generation". Not clear if that would be englacial melt that makes its way to the basal drainage system or something else.

The text has been revised for clarity (page 5, line 30 to page 6, line 1).

5.12-13. I struggle to see how SHaKTI "can be viewed as an approximation to a multi-dimensional generalization of the governing equations for glacial conduits described by Spring and Hutter (1981) and Clarke (2003)." These references describe only channel physics, not opening by sliding as in cavities. Clarke uses conduit distensibility in the governing equations and accounts for thermal advection, in contrast to SHaKTI. Easiest just to omit this text. I don't think trying to explain the statement would add much.

The most general form of the conservation equations for subglacial hydrology would be an extension of the Spring-Hutter (or Spring-Hutter-Clarke) equations to two dimensions, with augmentation to account for opening by sliding. In fact, the conduit deformation term referred to by the reviewer is analogous to the creep closure term in subglacial hydrology models. In general, a complete set of governing equations for subglacial hydrology models should include acceleration terms in the momentum equation, and advection and in-plane conduction terms should be included in the energy equation, as in the Spring-Hutter equations. This is what we were referring to. Subglacial hydrology models typically neglect the acceleration terms in the momentum equation and employ an approximate energy equation wherein all dissipated mechanical energy is locally used to produce melt (furthermore, as noted above, some models neglect internal melt generation in the sheet). We have rewritten the text for improved clarity to acknowledge the complete form of the equations and the approximations that are made here and in other models (page 6, lines 3-9).

5.15. It looks like the theory is developed for fully saturated flow, so this should be stated explicitly.

The text has been revised (page 6, lines 13-14).

5.18. "input rate" = "internal melt rate" above?

No, $i_{e \rightarrow b}$ is the external meltwater input rate (to represent surface water making its way to the bed). This has been clarified in the text (page 6, line 13). The internal melt rate is described in Eq. (11) of the revised manuscript and calculated using an energy balance at the bed.

5.20 (Eqn 2). Better described as "evolution of gap height" than "gap dynamics"? State what these terms are before the end of the paragraph, ideally before the equation. Eqn (2) would appear to allow for creep opening, not just creep closure. Is this intentional? If not, why write the creep term in this way? Is creep opening permitted in the numerical implementation of the model? If so, it should be justified.

The text has been revised to better explain this equation, and "gap dynamics" changed to "evolution of gap height" as suggested in Eq. 2 (page 6, lines 15-27).

Creep opening is not allowed, as we currently limit the water pressure to not exceed ice overburden pressure (Section 2.2).

5.26 (Eqn 3). State that this is the formulation for fracture flow, or how this formulation came to be adopted. Define Re here as in table, else the laminar-turbulent transition doesn't make sense. Here the reader really needs to know what the conceptual model is in order to make sense of the flux formulation.

We have revised the text to include substantial elaboration following Eq. (5) to more clearly describe the basis and derivation of our flux formulation, including the Reynolds number definition and role of the transition between laminar and turbulent regimes (page 7).

6. This reader is wondering how b and h are going to be related in the model, as the treatment of gap height and water pressure/hydraulic head forms a key difference in various models. Perhaps mention this early on when saturation conditions are noted.

We obtain h by solving a nonlinear elliptic equation (or parabolic if storage is used), in which b controls the conductivity (see Eq. 15).

6.6. fracture flow: this is a description of the conceptual model that should appear earlier.

Text has been added for clarity in the opening paragraph of Section 2 (page 5, lines 18-19).

6.7 "Most" => "Many"

This wording has been changed.

6.14-15. "heat consumed due to changes in water pressure" More physically based to explain that it is the heat consumed or released in maintaining the water at the pressure-melting temperature in the presence of changing water pressure.

This wording has been revised (page 8, lines 9-10).

6.17-18. Good place to cite Clarke (2003) for heat advection and Creyts and Clarke (2010) for supercooling.

Citations have been added to the revised manuscript (page 8, line 14).

6.21. Please state rationale for including englacial storage. Werder et al (2013) do this, but is it needed here for numerical stability?

No, englacial storage is not needed for stability but is included for completeness in being able to simulate situations that may involve substantial storage. The simulations presented in this paper use e_v=0 (no storage). This has been explained in revised text (page 8, lines 18-22).

6.28-30. Expressing K as a tensor here, given that it is assumed isotropic, seems needlessly complicated. An even more compact way to write the first term in Eqn (9) is \nable \dot q.

In writing the original manuscript, we considered whether to define K as a tensor or not, and decided that it was most complete to do so, since K could potentially be anisotropic and easily made so in the model.

In Eq. (15) of the revised manuscript (previously Eq. 9 in the original submitted manuscript), we write the first term in this form to make the dependence on h obvious to the reader.

7. Section 2.2. Boundary conditions are key for model implementation. It would seem to make sense to articulate them mathematically. I think Werder et al (2013) set a nice example of the balance between the mathematical and descriptive exposition of a model, including boundary conditions and method of solution.

The text has been updated to more clearly define boundary conditions (Section 2.2).

7.9. So, negative water pressures are permitted in the model? If so, how big are they? Do they have a significant influence on creep closure?

Negative water pressures can be calculated in the model, and we have seen them occur with steep slopes in bed topography. Examining the effects on creep closure is a wonderful suggestion that will be considered in upcoming work that focuses on real topography where negative pressures arise. In the present paper, our intention is to simply lay out the model formulation with simple simulations (in which negative pressures do not occur).

7.12. How is the Pw=Pi restriction implemented without violating conservation of mass? If it's not, it would be good to report the amount of mass-conservation viola- tion this restriction imposes.

The pressure cap is imposed within the nonlinear Picard iteration loop, so the system will iterate further by solving for the head field again with this cap activated at some computational nodes, and mass balance is thus always satisfied.

Note: the simulations included in this model development paper do not encounter pressures that run into the overburden limit. In future tests on more complex terrain and with thicker ice, this should certainly be addressed in detail.

7.17. "Euler-Backward" => "backward Euler" seems more conventional, unless this means something else.

The text has been changed (page 9, line 23).

7.17. Picard iteration. This is a common methodology, but one not known for its speed. Though not mentioned in the manuscript, I surmise that a major advantage of this modeling approach (unified physics applied everywhere) over others could be its efficiency, but perhaps not, depending on the numerical implementation. This reader would be very interested to known if the model formulation had the potential to be fast, and whether the numerical implementation was designed with this in mind. Given the disparity in timescales between ice flow and water flow which typically necessitates comparatively small timesteps for hydrological models, many model users will be looking for hydrological models that do not add unnecessary computational burden to their ice-sheet models.

The Picard iteration (or some form of nonlinear iteration) is a necessary component of an implicit backward Euler formulation. Picard iteration is much simpler to implement than a Newton iteration for this problem largely because of the complex nonlinearities for which Jacobian matrix computations for the latter will become very involved. There are limitations with Newton's method as well – non-convergence is often encountered. For this reason, we implemented Picard iteration, but future computational enhancements of our model could consider alternative nonlinear solvers such as Jacobian-Free-Newton-Krylov methods.

We discuss the importance of time step in the new Section 4.1 of the revised manuscript ("Model Limitations"). Our model does require relatively small time steps compared to the years-scale of ice flow models. At the same time, it should be noted that the time scales associated with important physical processes and forcing terms in ice sheet dynamics versus subglacial hydrology are indeed vastly different. Attempting to run subglacial hydrology models on the same time-steps as used in ice sheet models will not be ideal, simply because the relevant physics are not properly resolved. We anticipate that the future development of coupled ice-sheet and subglacial hydrology models will use different time-steps in each sub-model, and two-way coupling will not implemented in every subglacial hydrology model time-step. Alternatively, seasonal time-scale coupled simulations could inform the representation of sliding in longer decadal and centurial simulations.

7.19-20. It sounds like gap height and hydraulic head are not solved simultaneously (or iteratively). Why not? Explicit time-stepping is simple but can lead to large errors. Can the authors reassure the readers that this has been investigated and propose corresponding limits on the time step?

We employ a semi-implicit solution strategy. We solve for head with a nonlinear iterative approach in each time step, with the gap height from the end of the previous time-step held fixed. Using the new head field, the gap height is updated before proceeding to the next time-step. Thus, the gap height is being lagged by one time-step. This is not an uncommon strategy in highly nonlinear problems, but it does place a restriction on the time-step in transient simulations (for steady input simulations, this is not an issue since the geometry and pressure converge to steady states). Additional discussion of time-step size has been added in the revised manuscript (Section 4.1).

8.1. Is the model convergence sensitive to prescribed initial gap height?

No (at least for initial gap heights within reason). For initial gap height ranging from at least 0.001 to 0.1 m, or randomly perturbed within that range, the model converges to the same state, and model run times vary only by a few seconds at most. This has been added to Section 4.1 (Model Limitations).

8.7. Curious why closure is not included in "degree of channelization". If closure balances opening at small gap heights for any opening mechanism, it seems channelization would be suppressed.

The degree of channelization is calculated in this way to be consistent with the way it was calculated for the Subglacial Hydrology Model Intercomparison Project (deFleurian et al., currently in review at *Journal of Glaciology*). It was intended to give an idea of where the model behaves more as "sheetlike" vs. "channelized" based on the opening mechanism (to facilitate comparison to other models that differentiate between the two).

8.11-12. The software-style description seems a little strange. I guess the key thing here is that the output is ascii, not binary or something else? It seems like output from any model could be visualized in contour plots, timeseries, etc, and in any software.

Model output is binary, and the output could indeed be visualized in any software. The text has been revised for clarity (page 10, lines 17-23).

8.15. Somewhere above this it should be noted that the mesh is irregular.

Text has been added earlier in Section 2.3 to state that the model can be run on a structured or unstructured mesh. Using an unstructured mesh typically reduces bias in channel configuration, although the model performs well on either type.

8.16. "Application". Here I was expecting to see some multi-faceted demonstration of the model performance (e.g. accuracy, consistency, convergence, efficiency) prior to the demonstration that the model produces qualitatively familiar results in some basic tests. Model performance metrics are not often reported in journal articles focused on model applications, but I expected this would be different in Geoscientific Model Development. The Editor can decide if this suggestion is misguided; it could be misinterpreting the purpose and expectations of the journal.

We include results demonstrating convergence in the face of grid refinement (see Figs. 3, 4, 7, and 8) and time-step refinement (Fig. 10). Due to the highly nonlinear nature of subglacial hydrology, it is not straightforward to compare the numerical simulations to any analytical solutions. However, the model is part of a Subglacial Hydrology Model Intercomparison Project (SHMIP; deFleurian et al., under review), where it is being compared to other models on a range of benchmark problems.

If the editor has specific suggestions on a more formal evaluation of model performance beyond what we have included in the revised manuscript, we will be happy to consider them.

8-9. Subglacial hydrology models frequently have trouble in the presence of bed topography. The tests presented here omit bed topography with the exception of a gentle slope. It would be useful to know if this model does better than others in the presence of realistic bed topography. It's ok if it doesn't.

The model does perform well with bed topography, although negative water pressures are calculated with very steep slopes (as mentioned in the text). The aim of this paper is to document the model formulation with simple illustrative example simulations. We have applied the model to simulate subglacial hydrology of the Store Glacier in west Greenland with real topography. That work that is beyond the scope of the current paper, but will be submitted soon.

9.3. “drainage configuration . . . affected by . . . bed topography” Except in this test the bed is flat. Is this just a general statement?

Yes, the text has been revised to reference how drainage configurations arise in general (page 11, lines 13-14).

9.4 “unstructured mesh” Please mention this when model implementation is described.

This has been added in Section 2.3 (see response to 8.15 above).

9.10. The test domains seem very small. It would be useful to report something on model runtimes. Is it practical to run this model coupled to an iceflow model for a large catchment?

The test domains are intentionally small to demonstrate features on a small scale. Model run times are dependent on the number of processors used and the type of machine, and will obviously increase with larger domains. Run time is mentioned in Section 3.1 of the revised manuscript, although we point out that run times will vary with model size, duration, complexity, time step size, number of processors, etc.

The SHAKTI model is best with relatively fine resolution to capture the effects of channelization, designed for simulation of individual glaciers, but it can be used on large domains. The new Section 4.1 includes some thoughts and guidance for selecting a mesh size.

10.3. “do not include storage term” Meaning englacial storage?

Correct. The simulations presented in this paper use zero englacial storage. This has been clarified in the text here (page 12, lines 18-19) and earlier (see response to 6.21).

10.6-7. Paper should make clear that ub is prescribed and constant, thus there is no two-way coupling with sliding, meaning the negative feedback associated with sliding is absent (Hoffman and Price, 2014?)

This has been clarified in the text (page 12, lines 23-27, also see Section 4.1).

10.9. This sounds like a problem that plagues most models (c.f. Downs et al, 2018: https://agupubs.onlinelibrary.wiley.com/doi/full/10.1002/2017JF004522), so should be noted as a common shortcoming with a citation or two.

We have mentioned the problem of low winter water pressures in subglacial hydrology models at a few places in the revised text (page 4, lines 13-16; page 14, lines 15-17). Addressing this particular problem is not the aim of this model formulation and the illustrative examples included in this paper do not attempt to capture this behavior.

10.21/ "impose potential channel locations" Indeed this is a limitation in some models, but here the "channel" locations are a function of the mesh, just as they are in the models of Hewitt, Schoof and Werder. In the latter case, the channels may lie anywhere along the mesh edge. In this model, they may lie anywhere in the mesh elements. In the models of Hewitt, Schoof, Werder, increases in grid size mean small channels cannot be represented; in the current model, increases in grid size mean channels become unrealistically wide. Both are limitations in different ways.

It is true that gap height is calculated across elements, but we are not calculating a specific cross-sectional area of a channel, and a channel is not restricted to be along interface elements (see the intricate details in gap height distribution in Fig. 6). It is true that with a coarse grid, the apparent effects of channelized pathways become diffuse as shown in Figs. 3 and 6. We have included additional discussion in the revised manuscript of mesh dependence in Sections 3.2, 3.3, and 4.1.

10.25-26. This seems like a big deal, and a true potential advantage over the other models out there.

Yes, this is why we hope the model will be a useful contribution to the field. We thank the reviewer for recognizing the value in the inclusion of the dissipation term.

11.10-11. "Supports the notion" Too obvious. Suggest deleting.

Deleted.

11.13-14. Arguably the unconnected bed requires additional model physics, but this regime has been parameterized by Hoffman et al (2016) and Downs et al (2018).

We do not include specific physics to represent unconnected bed. However, in principle, the model formulation is capable of incorporating disconnected regions bounded by very small gap heights. A related issue is whether the numerical solution of the nonlinear Eq. (15) will be hampered by the occurrence of disconnected regions. We plan to investigate this issue further in future work based on strategies that we have employed previously in the context of rock fractures.

Additional citations have been added (page 14, line 17).

11.25 suggest "channels" => "pathways". Reword "sorts itself out".

The text has been revised.

2.22. "the model..., a model formulation" => "we describe the model formulation of SHaKTI, which allows for. . ."

Text revised.

5.19 and 5.25. These lines and the text that follows them do not form sentences.

Text revised.

6.9. ditto above

Text revised.

References: more than 15 of the references are incomplete. Authors should check the list thoroughly. “Truffer” is missing an “r”. “et al” is used where it probably shouldn’t be. Sometimes journal titles are written out, sometimes they are not.

Thank you for alerting us to the fact that the reference list was not entirely accurate. We have updated the references.

Tables: check superscripts. I have great respect for SI, but please give ub in m/a also.

The tables have been updated.

SHAKTI: Subglacial Hydrology And Kinetic, Transient Interactions v1.0

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Abstract. Subglacial hydrology has a significant influence on glacier and ice sheet dynamics, yet remains not fully understood. The drainage geometry and flow mechanics are constantly changing, with complex feedbacks that play out between the water and ice. A clear tradition has been established in the subglacial hydrology modeling literature of distinguishing between channelized (efficient) and sheetlike (inefficient or distributed) drainage systems or components. Imposing a distinction that applies different equations to capture the appropriate physics in different parts of the model domain, however, may not allow for the full array of drainage characteristics to arise. Here, we present a new subglacial hydrology model: SHAKTI (Subglacial Hydrology And Kinetic, Transient Interactions). In this model formulation, a single set of governing equations is applied over the entire domain including melt from dissipation, with a spatially and temporally varying transmissivity that allows for representation of the wide transition between turbulent and laminar flow, and the geometry of each element is allowed to evolve accordingly to form sheetlike and channelized configurations. The model is implemented as part of the Ice Sheet System Model (ISSM). We include steady and transient examples to demonstrate features and capabilities of the model, and we are able to reproduce seasonal behavior of the subglacial water pressure that is qualitatively consistent with observed seasonal velocity behavior in many Greenland outlet glaciers.

1 Introduction

One of the significant consequences of contemporary climate change is rising sea level. A large component of sea level rise is due to the transfer of ice from glaciers and ice sheets into the ocean via melt, runoff, and iceberg calving (Church et al., 2013). Future ice dynamics remains a major uncertainty in sea level rise predictions involving many uncertain factors, including basal lubrication and effects on sliding velocities from subglacial drainage (e.g., Church et al., 2013; Shannon et al., 2013).

Although massive outlet glaciers of West Antarctica may be on the verge of irreversible collapse in the next 200 to 1,000 years (Joughin et al., 2014; DeConto and Pollard, 2016), the Greenland ice sheet is currently the single largest contributor to sea level rise (Shepherd et al., 2012). Considering the substantial amount of water held in this frozen reservoir, it is important to improve understanding of its behavior, including the subtleties of its drainage, which affects ice velocity through sliding. Since 1990, many Greenland outlet glaciers have displayed dramatic accelerations and frontal retreats, yielding substantial changes on the rapid timescale of decades or years (Joughin et al. 2010). Other glaciers, however, have accelerated less rapidly or even decelerated over the same period (McFadden et al., 2011), and the mechanisms driving these contrasting responses are still not entirely understood. The recent accelerations observed in marine terminating outlet glaciers, which exhibit some of the greatest accelerations and are highly sensitive to changes in terminus conditions, may be in response to changing ocean temperatures (Nick et al., 2009, Rignot et al., 2010, Andresen et al., 2012), but their diverse behaviors have been found to depend on more factors than ocean temperature alone, such as bed topography and subglacial discharge distribution (Slater et al., 2015; Rignot et al., 2016). In land terminating glaciers, the observed accelerations are likely driven largely by water inputs to the ice sheet from the surface via crevasses and moulins, similar to alpine glaciers (e.g., Anderson et al., 2004; Bartholomaus et al., 2008). Meltwater inputs have been shown to drive variation in ice velocities on the Greenland ice sheet (e.g., Zwally et al., 2002; Bartholomew et al., 2012), as well as seasonal changes in the efficiency of the subglacial drainage system (e.g., Bartholomew et al., 2010; Chandler et al., 2013; Cowton et al., 2013; Andrews et al., 2014).

The hydrology of meltwater on the surface, within, and beneath glaciers and ice sheets should ideally be viewed and modeled as a complex system of processes, considering the interconnectedness of surface mass balance, meltwater retention, discharge at the ice margin, and feedbacks between hydrology and ice dynamics (e.g., Rennermalm et al., 2013; Nienow et al., 2017). Water delivered to the bed through englacial conduits drives basal sliding, which has important effects on flow in some regions (Vaughan et al., 2013), and year-round sliding can occur with temperate bed conditions (Colgan et al., 2011). Increased meltwater input to the bed, however, does not necessarily imply increased basal sliding, contrary to what might seem intuitive. For example, as meltwater input increases, water pressure under the ice increases, leading to enhanced basal lubrication and higher sliding velocity (Zwally et al., 2002). But with sustained meltwater input over a melt season, more efficient drainage channels can develop, decreasing the water pressure (Schoof, 2010). Characteristics of individual outlet glaciers such as bed topography, ice geometry, surface temperature, and other factors all play into the intricate choreography of the seasonal evolution of the subglacial drainage system and its influence on ice velocity. Subglacial hydrology models have had success in simulating realistic drainage behavior, but challenges still remain.

The goal of this modeling effort was to see if we could use a single set of governing equations to produce systematic, self-organized channelization where it should occur. In this paper, we describe the model formulation of SHAKTI (Subglacial Hydrology And Kinetic, Transient Interactions), which allows for flexible evolution of the subglacial drainage system configuration and flow regimes using a single set of governing equations over the entire domain. The model aims to represent the complex interactions due to (kinetic) movement of ice and water and (transient) changes of the subglacial system through time. We hope this unified formulation may be used to facilitate exploration of the conditions under which different drainage system types form and persist, and the flow regimes experienced in different areas of a domain. With upcoming application to actual glaciers, this type of model could provide useful insights into the seasonal evolution of real subglacial drainage systems and their influence on mass loss from the Greenland ice sheet, with the potential for broader application to Antarctica and alpine glaciers.

The paper is structured as follows: in Sections 1.1-1.2, we provide a brief summary and review of historical and recent subglacial hydrology modeling progress to put our model in context. We then present the model's governing equations and the numerical framework in Section 2, with illustrative simulations to demonstrate key model features and capabilities in Section 3, and a discussion of implications and model limitations in Section 4.

1.1 Subglacial hydrology modeling context

Subglacial hydrology has long been an area of interest, initially in the context of geomorphology, groundwater, and surface hydrology from alpine glaciers, and more recently in the context of its influence on ice sheet dynamics. Below is a brief and selective summary of previous subglacial hydrology modeling work motivated by glacier sliding. We direct readers to Flowers (2015) for a comprehensive review of the full subject history, recent advancements, and current challenges.

The first major efforts to quantitatively model subglacial hydrology began in the 1970s. Shreve (1972) described a system of arborescent subglacial channels, and Röthlisberger (1972) formulated equations for semi-circular channels melted into the base of the ice sheet, in a state of equilibrium between melt opening and creep closure. Nye (1973) expanded the work of Röthlisberger to consider channels incised into bedrock or subglacial sediments, and more fully developed the equations into models for explaining outburst floods (Nye, 1976). In a different approach, Weertman (1972) considered subglacial drainage through a water sheet of approximately uniform thickness. In the following decade, different plausible drainage configurations were also proposed, such as a system of “linked cavities”, spaces that open behind bedrock bumps as a result of glacier sliding (Walder, 1986; Kamb, 1987). By the mid-1980s, it was recognized that the major components of subglacial hydrology could be classified as either efficient (channels or canals) or inefficient (thin sheets, flow through porous till, or distributed systems of linked cavities, often represented in continuum models as a sheet). While channels themselves emerge as a result of self-organized selective growth from a linked cavity system, a clear distinction between these two subsystems was established.

Since 2000, a renewed surge of interest in subglacial hydrology has been sparked as mass loss increases from glaciers and ice sheets and sea level rise is increasingly perceived as an imminent reality, generating a flurry of new observations and modeling advances. Although the effects of surface melt on ice sheet dynamics are not yet entirely understood (e.g., Clarke, 2005; Joughin et al., 2008), observations have reinforced the fact that surface meltwater significantly influences flow behavior in alpine glaciers and ice sheets (e.g., Mair et al., 2002; Zwally et al., 2002; Bartholomaus et al., 2008; Howat et al., 2008; Shepherd et al., 2009; Bartholomew et al., 2010; Hoffman et al., 2011; Sundal et al., 2011; Bartholomew et al., 2012; Meierbachtol et al., 2013; Andrews et al., 2014). Along with more detailed observations, several efforts were made in the early 2000s to accurately simulate subglacial hydrology. Some of these studies treated the subglacial system as a water sheet of uniform thickness (e.g., Flowers and Clarke, 2002; Johnson and Fastook, 2002; Creyts and Schoof, 2009; LeBrocq et al., 2009). Arnold and Sharp (2002) presented a model with both distributed and channel flow, but only one configuration could operate at a time. Kessler and Anderson (2004) introduced a model using discrete drainage pathways that could transition between distributed and channelized modes, and Flowers et al. (2004) used a combination of a distributed sheet in parallel with a network of efficient channels. Schoof (2010) developed a 2D network of discrete conduits that could behave like either channels or cavities, and found that with sufficiently large discharge an arborescent network of channel-like conduits would form, although the resulting geometry was highly dependent on the rectangular grid used. Hewitt (2011) developed a model that used a water sheet to represent evolving linked cavities averaged over a patch of bed (an effective porous medium), coupled to a single channel.

More recent studies tied together key elements of subglacial drainage to form increasingly realistic 2D models. Hewitt (2013) introduced a linked-cavity continuum sheet integrated with a structured channel network. In that model, channels open by melt, while the distributed sheet opens only by sliding over bedrock bumps (neglecting opening by melt from dissipative heat). Melt from dissipative heat contributes only to opening in channels. Werder et al. (2013) presented a model that involves water flow through a sheet (representative of averaged linked cavities) along with channels that are free to form anywhere along edges of the unstructured numerical mesh, exchanging water with the surrounding distributed sheet. Approaching the problem in a different way, Bougamont et al. (2014) reproduced seasonal ice flow variability through the hydro-mechanical response of soft basal sediment in lieu of simulating the evolution of a subglacial drainage system. To capture broad characteristics of subglacial drainage without resolving individual elements, DeFleurian et al. (2014) employed a 2D dual-layer porous medium model, and Bueler and Pelt (2015) formulated equations for a 2D model that combines water stored in subglacial till with linked cavities. To help explain observations of high water pressure in late summer and fall, recent observations and modeling efforts have highlighted the importance of representing hydraulically isolated or "weakly connected" regions of the bed (Hoffman et al., 2016; Rada and Schoof, 2018), and addressed the problem by facilitating seasonal changes in the hydraulic conductivity (Downs et al., 2018).

1.2 Distinction between channelized and sheetlike drainage, and the problem of dissipation

A common theme in the subglacial hydrology modeling literature is a distinction between channelized (efficient) and sheetlike (inefficient or distributed) drainage systems or components. In most existing 2D models, either only one of these forms is considered, or else slightly different equations are applied to coupled channel and sheet components. For the sheetlike system, these models only consider opening (i.e. growth of the sheet thickness) due to sliding over bedrock bumps, disregarding opening by melting of the upper ice surface. Melt is generated by the thermal energy obtained from dissipated mechanical energy (commonly referred to as energy loss or head loss). However, these models redirect the generated thermal energy into adjacent channel components that are allowed to melt and grow. Channel components are allowed to form in pre-specified locations or to evolve along the edges of sheetlike elements, as in Werder at el. (2013). The main reason that most of these models disregard melt opening in the sheetlike system is to avoid the unstable behavior that has been found to occur when it is included, leading to unstable growth where the melt opening rate exceeds the closure rate, sparking channelization (Hewitt, 2011) or driving initiation of glacial floods (Schoof, 2010). The transition to a channelized state has been described elegantly in previous work (e.g., Walder, 1986; Kamb, 1987; Schoof, 2010; Hewitt, 2011; Schoof et al., 2012; Werder et al., 2013; Hoffman and Price, 2014).

In reality, the subglacial hydrologic system is comprised of a wide array of drainage features, of which the sheet and channel are two end-members. Imposing a sharp distinction between the treatment of the melt opening term, and dividing the governing equations between different model components may not allow for the full array of drainage features to arise. It is also a bit artificial to redirect the opening by melt in sheetlike elements to nearby channels. In the model formulation described in this paper, a single set of governing equations is applied over the entire domain, including the melt opening term everywhere. In our formulation, the hydraulic transmissivity of the subglacial domain is allowed to vary spatially and temporally, allowing for a continuum of drainage features. We also account for laminar, turbulent and intermediate flow regimes, based on an experimentally verified flow law for rough-walled rock fractures (Zimmerman et al., 2004). The gap thickness of each computational element in a discretization of the governing equations is allowed to evolve flexibly, and sequential elements with high gap growth rates typically link up to produce channelized features. The ability to represent co-existing turbulent, laminar and intermediate regimes regimes appears to be a promising approach to overcoming the previously mentioned instability that occurs when the melt generated by mechanical energy dissipation is retained in the sheet system equations. Even with the melt opening term included everywhere in the domain, we are able to generate steady and transient drainage configurations that include channel-like efficient drainage pathways. Our model does not aim to simulate every individual cavity or specific channel cross-section, but rather captures the homogenized effects of these elements on a discrete mesh. As we demonstrate in Section 3, although the resolution of subglacial geometry in our approach is mesh/grid-sensitive, the simulated basal water pressure and effective pressure (which are most relevant for calculating sliding velocities in ice dynamics models) are relatively insensitive to mesh dimensions.

2 SHAKTI model description

This flexible subglacial hydrology model can handle transient meltwater inputs, both spatially distributed and localized, and allows the basal water flux and geometry to evolve according to these inputs to produce flow and drainage regimes across the spectrum from sheetlike to channelized. The subglacial drainage system is represented as a sheet with variable gap height, and we employ a flux formulation based on fracture flow equations. Channelized locations are not prescribed a priori, but can arise and decay naturally as reflected in self-organized formation of connected paths of large gap height (calculated across elements) and lower water pressure (calculated at vertices) than their surroundings. In contrast, previous models allow for efficient channels to arise along element or grid edges and calculate a specific cross-sectional channel area (e.g., Schoof, 2010; Hewitt et al., 2013; Werder et al., 2013).

The parallelized, finite element SHAKTI model is currently implemented as part of the Ice Sheet System Model (ISSM; Larour et al., 2012; http://issm.jpl.nasa.gov), with full two-way coupling with the ice dynamics model planned for upcoming work. Below, we present the equations involved in the SHAKTI formulation. The governing equations are similar to those used in Werder et al. (2013), with some key differences that enable application of the same set of equations everywhere in the domain.

2.1 Summary of model equations

The SHAKTI model is based upon governing equations that describe conservation of water and ice mass, evolution of the gap height, water flux (approximate momentum equation for water velocity integrated over the gap height), and internal melt generation (approximate energy equation for heat produced at the bed). All variables used in the equations are summarized in Table 1, with constants and parameters summarized in Table 2.

In general, a complete set of governing equations for subglacial hydrology models should include acceleration terms in the momentum equation, and advection and in-plane conduction terms should be included in the energy equation. The most general form of the conservation equations for subglacial hydrology would be a multi-dimensional extension of the equations described by Spring and Hutter (1981) and Clarke (2003), with augmentation to account for opening by sliding. Our model formulation and most existing subglacial hydrology models typically neglect the acceleration terms in the momentum equation and employ an approximate energy equation in which all dissipated mechanical energy is locally used to produce melt, and the equations presented here should be viewed as an approximation to the more general equations.

The continuity equation is written as a water and ice mass balance:

$$\frac{\partial b}{\partial t} + \frac{\partial b_e}{\partial t} + \nabla \cdot \boldsymbol{q} = \frac{\dot{m}}{\rho_w} + i_{e \to b} \tag{1}$$

where b is subglacial gap height, b_e is the volume of water stored englacially per unit area of bed, $\boldsymbol{q}$ is basal water flux, $\dot{m}$ is basal melt rate, and $i_{e \to b}$ represents the input rate of surface meltwater from the englacial to subglacial system. This water balance assumes that the subglacial gap is always filled with water.

Evolution of the gap height (subglacial geometry) involves opening due to melt and sliding over bumps on the bed, and closing due to ice creep:

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} + \beta u_b - A|p_i - p_w|^{n-1}(p_i - p_w)b \tag{2}$$

where A is the ice flow law parameter, n is the flow law exponent, p_i is the overburden pressure of ice, p_w is water pressure, β is a dimensionless parameter governing opening by sliding, and u_b is the magnitude of the sliding velocity. In most existing 2D models that include both channel and distributed sheetlike drainage components, melt opening is typically considered “channel opening” and opening by sliding over bumps on the bed is considered “cavity opening”, with the different terms applied to the appropriate components within the model. Our model diverges from others in that we include both opening terms everywhere in the domain. The opening by sliding parameter β is a function of typical bed bump height (b_r) and bump spacing (l_r), as well as local gap height (so that opening by sliding only occurs where the gap height is less than the typical bump height). In defining β, we follow Werder et al. (2013):

$$\beta|_{b<b_r} = \frac{(b_r - b)}{l_r} \tag{3}$$

$$\beta|_{b \geq b_r} = 0 \tag{4}$$

The horizontal basal water flux (approximate momentum equation) is based on fracture flow equations:

$$\boldsymbol{q} = \frac{-b^3 g}{12\nu(1+\omega Re)}\nabla h \qquad (5)$$

where g is gravitational acceleration, ν is kinematic viscosity of water, ω is a dimensionless parameter controlling the nonlinear transition from laminar to turbulent flow, Re is the Reynolds number, and h is hydraulic head. The momentum equation is approximate in the sense that acceleration terms are neglected. Equation (5) is a key piece of our model formulation, in that it allows for a spatially and temporally variable hydraulic transmissivity in the system, and facilitates representation of both laminar and turbulent flow regimes, coexistence of laminar and turbulent flow in subregions, as well as flow that pertains to the wide transition between laminar and turbulent, where the linearity of laminar flow is not valid, but the square root dependence doesn't fully apply. Many existing subglacial hydrology models prescribe a hydraulic conductivity parameter and assume the flow to be turbulent everywhere. Equation (5) is based on flow equations for rock fractures and has been employed in that context previously (Zimmerman et al., 2004; Rajaram et al., 2009; Chaudhuri et al., 2013).

This form of the momentum equation can be derived based on laminar flow theory for flow between parallel plates. Integrating the Navier-Stokes equations twice to obtain the laminar flow for plane Poiseuille flow, we obtain:

$$q_{lam} = \frac{-b^3 g}{12\nu}\nabla h \qquad (6)$$

where ν is the kinematic viscosity of water. The definition of Reynolds number follows the precedent in fracture literature, using the gap height b as a characteristic length scale:

$$Re = \frac{|vb|}{\nu} = \frac{|\boldsymbol{q}|}{\nu} \qquad (7)$$

where v is the bulk velocity of the flow. The laminar flux (Eq. 6) is modified to allow for transition to a turbulent regime, and Eq. (5) can be written in the form:

$$\frac{\boldsymbol{q}}{\nabla h} = \frac{-b^3 g}{12\nu(1+\omega\frac{|\boldsymbol{q}|}{\nu})} = \frac{-b^3 g}{12(\nu+\omega|\boldsymbol{q}|)} \qquad (8)$$

Using a value of $\omega = 0.001$, the transition to turbulent flow occurs around $Re = 1,000$. For turbulent flow with high Reynolds number ($Re >> 1,000$), $\omega Re >> 1$, and the flux $\boldsymbol{q}$ is proportional to the square root of the head gradient magnitude:

$$\boldsymbol{q}^2_{turb} = \frac{-b^3 g}{12\omega}\nabla h \qquad (9)$$

For laminar flow with low Reynolds number ($Re << 1,000$), $\omega Re << 1$, and $\boldsymbol{q}$ is proportional to the head gradient magnitude (Eq. 6). For intermediate Reynolds numbers in the wide transition between laminar and turbulent, $\boldsymbol{q}$ exhibits a combined proportionality to the head gradient or its square root.

Hydraulic head is calculated as:

$$h = \frac{p_w}{\rho_w g} + z_b \tag{10}$$

where ρ_w is density of liquid water and z_b is bed elevation. Internal melt generation is calculated through an energy balance at the bed:

$$\dot{m} = \frac{1}{L}(G + |\boldsymbol{u_b} \cdot \boldsymbol{\tau_b}| - \rho_w g \boldsymbol{q} \cdot \nabla h - c_t c_w \rho_w \boldsymbol{q} \cdot \nabla p_w) \tag{11}$$

where L is latent heat of fusion of water, G is geothermal flux, $\boldsymbol{u_b}$ is the ice basal velocity vector, $\boldsymbol{\tau_b}$ is the stress exerted by the bed onto the ice, c_t is the change of pressure melting point with temperature, and c_w is the heat capacity of water. Melt is therefore produced through a combination of geothermal flux, frictional heat due to sliding, and heat generated through internal dissipation (where mechanical kinetic energy is converted to thermal energy), minus the heat consumed or released in maintaining the water at the pressure-melting temperature in the presence of changing water pressure. We note that this form of the energy equation assumes that all heat produced is converted locally to melt and neglects transport of dissipative heat. We assume that the ice and liquid water are isothermal, consistently at the pressure melting point temperature. These assumptions may not be strictly valid under certain real conditions that may have interesting heat transfer implications, such as heat advection (Clarke, 2003) or supercooling (Creyts and Clarke, 2010), or where meltwater enters a system of cold ice (below the pressure melting point), but we leave these potential model extensions for future work. As mentioned previously in Section 1.2, Werder et al. (2013) and similar models do not include the internal dissipation term in their sheetlike drainage components, but assign any melt from dissipation to contribute to opening in the nearest channel component.

For the sake of versatility, we also include an option to parameterize storage in the englacial system (note that this is not necessary for numerical stability; we use zero englacial storage in the example simulations presented in Section 3 of this paper). Following Werder et al. (2013), the englacial storage volume is defined as a function of water pressure:

$$b_e = e_v \frac{\rho_w g h - \rho_w g z_b}{\rho_w g} = e_v (h - z_b) \tag{12}$$

where e_v is the englacial void ratio ($e_v = 0$ for no englacial storage).

Equations (1), (2), (5), and (11) are combined to form a parabolic, nonlinear partial differential equation (PDE) in terms of hydraulic head, h:

$$\nabla \cdot \left[\frac{-b^3 g}{12\nu(1+\omega Re)} \nabla h \right] + \frac{\partial e_v (h - z_b)}{\partial t} = \dot{m} \left[\frac{1}{\rho_w} - \frac{1}{\rho_i} \right] + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e \to b} \tag{13}$$

With no englacial storage ($e_v = 0$), Eq. (13) takes the form of an elliptic PDE.

Defining a hydraulic transmissivity tensor:

$$\boldsymbol{K} = \frac{b^3 g}{12\nu(1+\omega Re)} \mathbf{I} \tag{14}$$

Equation (13) can be written more compactly as:

$$\nabla \cdot (-\boldsymbol{K} \cdot \nabla h) + \frac{\partial e_v(h - z_b)}{\partial t} = \dot{m}\left(\frac{1}{\rho_w} - \frac{1}{\rho_i}\right) + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e\to b} \tag{15}$$

Although we employ an isotropic representation of the hydraulic transmissivity tensor in Eq. (14), our model formulation can be readily generalized to incorporate anisotropy. The source terms on the right side of the equation depend on h, so solving for the head distribution is a nonlinear process.

2.2 Boundary conditions

Boundary conditions can be applied as either prescribed head (Dirichlet) conditions or as flux (Neumann) conditions. To represent land-terminating glaciers, we typically apply a Dirichlet boundary condition of atmospheric pressure at the edge of the ice sheet:

$$h_{front} = z_b \tag{16}$$

To represent marine terminating glaciers, the outlet boundary condition can be set to the overlying fjord water pressure. Neumann boundary conditions are imposed on the other boundaries of the subglacial drainage domain:

$$\nabla h_{bound} = f \tag{17}$$

where f can be set to represent no flux ($f = 0$) or a prescribed flux, which can be constant or time-varying.

In our current formulation, there is no lower limit imposed on the water pressure; this means that unphysical negative pressures can be calculated in the presence of steep bed slopes, as in Werder et al. (2013). While suction and cavitation may occur in these situations, the flow most likely transitions to free-surface flow with the subglacial gap partially filled by air or water vapor. At high water pressure, we restrict the value to not exceed the ice overburden pressure, which would in reality manifest as uplift of the ice or hydrofracturing at the bed. These extreme “underpressure” and “overpressure” regimes are important situations that have been considered in other studies (e.g., Tsai and Rice, 2010; Hewitt et al., 2012; Schoof et al., 2012), but are quite complex in 2D, and remain to be addressed carefully in future developments.

2.3 Computational strategy and implementation in the Ice Sheet System Model (ISSM)

Within each time step, the nonlinear Eq. (15) is solved using an implicit backward Euler discretization and Picard iteration to obtain the head (h) field. From h, we calculate p_w, $\boldsymbol{q}$, Re, and $\dot{m}$, to be used in the subsequent iteration (in each iteration, p_w, $\boldsymbol{q}$, Re, and $\dot{m}$ are lagged from the previous iteration). Once the Picard iteration has successfully converged to a solution for h, the gap height geometry (b) is then updated explicitly based on basal gap dynamics using Eq. (2) to advance to the next time step. A schematic of this numerical procedure is presented in Fig. 1.

SHAKTI is implemented within ISSM, an open source ice dynamics model for Greenland and Antarctica developed by NASA's Jet Propulsion Laboratory and University of California at Irvine (Larour et al., 2012; http://issm.jpl.nasa.gov). ISSM uses finite element methods and parallel computing technologies, and includes sophisticated data assimilation and sensitivity analysis tools, to support numerous capabilities for ice sheet modeling applications on a variety of scales. The SHAKTI hydrology model solves the equations presented above in a parallel architecture using linear finite elements (i.e. P1 triangular Lagrange finite elements), which can be based on a structured or unstructured mesh. The source code is written in C++ and we rely on data structures and solvers provided by the Portable, Extensible Toolkit for Scientific Computation (PETSc, http://www.mcs.anl.gov/petsc). The user interface in MATLAB is the same as for other solutions implemented in ISSM, designed to facilitate model set up and post processing (see Documentation, https://issm.jpl.nasa.gov/documentation/hydrologyshakti/). The nonlinear iteration is performed to solve Eq. (15) for hydraulic head using the direct linear solver MUMPS in PETSc, but other solvers provided by PETSc could be easily tested in future work.

Model inputs include spatial fields of bed elevation, ice surface elevation, initial hydraulic head, initial basal gap height, ice sliding velocity, basal friction coefficient, typical bed bump height and spacing, englacial input to the bed (which can be constant or time-varying, and can be spatially distributed or located at discrete points to represent moulin input), and appropriate boundary conditions. Parameters that can either be specified or rely on a default value are geothermal flux, the ice flow law parameter and exponent, and the englacial storage coefficient.

Model outputs include spatiotemporal fields of hydraulic head, effective pressure, subglacial gap height (the effective geometry representative of an entire element), depth-integrated water flux, and "degree of channelization" (the ratio of opening by melt in each element to the total rate of opening in that element by both melt and sliding). Head and effective pressure are calculated at each vertex on the mesh; gap height, water flux, and degree of channelization are calculated over each element (these quantities are based on the head gradient). Instructions for setting up, running a simulation, and plotting outputs can be found in the SHAKTI model documentation (https://issm.jpl.nasa.gov/documentation/hydrologyshakti/) and in an example tutorial (https://issm.jpl.nasa.gov/documentation/tutorials/shakti/).

3 Application

To demonstrate the capabilities of SHAKTI, here we present simple illustrative simulations that highlight some of its features.

3.1 Channel formation from discrete moulin input

In this first example, we consider a 1 km square, 500 m thick tilted ice slab with surface and bed slope of 0.02 along the x direction. Steady input of 4 m^3 s^{-1} is prescribed at a single moulin at the center of the square ($x = 500$ m, $y = 500$ m). Water pressure at the outflow (left edge of the domain, $x = 0$) is set to atmospheric pressure, with zero flux boundary conditions at the other three sides of the domain. All other constants and parameters are as described in Table 2. We use an unstructured triangular mesh with typical edge length of 20 m (with 4,004 elements). The model is run to a steady configuration (steady state is reached by 12 days) starting from an initial gap height of 0.01 m. A channelized drainage pathway emerges from the moulin to the outflow, with higher effective pressure (i.e. lower head and water pressure), larger gap height, and higher basal flux than its surroundings (Fig. 2). Scripts for running this example are included as a tutorial in ISSM (https://issm.jpl.nasa.gov/documentation/tutorials/shakti/), and can serve as a template for more sophisticated simulations. Run times will vary by machine and number of processors, but to run this simulation on 24 processors for 30 days with a time step of 1 hour, the core solution elapsed time is 38 seconds.

3.2 Channelization with multiple moulins

For the next example, we consider a rectangular domain 10 km long and 2 km wide, with a flat bed ($z_b = 0$ everywhere) and parabolic surface profile with a minimum thickness of 300 m and a maximum of 610 m. Ten moulins are located at arbitrarily chosen locations in the domain, each with a steady input of 10 $\mathrm{m}^3\ \mathrm{s}^{-1}$. The model is run to 365 days with a time step of 1 hour (steady state is reached before 50 days), starting from an initial gap height of 0.01 m. The resulting steady distributions shown in Fig. 3 on five different meshes show a clear channelized drainage structure. Rather than each moulin forming a unique channel to the outflow, the moulin inputs influence each other, warping the pressure field and forming arborescent efficient pathways that combine downstream. For this specific arrangement of moulin inputs, a single principal drainage channel emerges. The unique drainage configuration that evolves in a particular circumstance and setting is affected by many factors, including bed topography, ice thickness, sliding velocity, meltwater input location, and input intensity.

The exact configuration of self-organizing channels also depends to some extent on the mesh. The five unstructured meshes used in this example have typical edge lengths ranging from 50 m (12,714 elements) to 400 m (205 elements). Using an unstructured mesh reduces bias in channel direction compared to a structured mesh, but the orientation and size of the elements does still affect the resulting geometry. The different cases shown in Fig. 3 provide a qualitative view of dependence of channelization structure on mesh size. Figure 4 presents quantitative plots of the mean head and effective pressure (averaged in the y direction) for the five meshes. Across much of the domain, they converge remarkably well, but diverge slightly in the region of significant channelization.

3.3 Seasonal variation and distributed meltwater input

Next we consider a transient example involving a seasonal input cycle of meltwater, with input distributed uniformly across a rectangular domain 4 km long and 8 km wide. The bed is flat ($z_b = 0$ everywhere). The ice surface follows a parabolic profile, with ice thickness ranging from 550 m at $x = 0$ to 700 m at $x = 4$ km, and is uniform across the y direction. We begin with an initial subglacial gap height of 0.01 m, perturbed with random variations drawn from a normal distribution with standard deviation of 1%. The purpose of these random variations in the initial gap height is to serve as triggers for potential instability and channelization, which is an important phenomenon in subglacial hydrologic systems (Walder, 1986; Kamb, 1987; Schoof, 2010; Hewitt et al., 2011). Even in nature, the gap height is unlikely to be uniform and the ubiquitous irregular variations in the gap height and bedrock surface will act as natural perturbations to initiate instabilities and channelization. As the ice slides over bedrock, abrasion processes may also serve to generate irregularities. In the literature on the self-organized formation of dissolution channels in rock fractures (e.g. Cheung and Rajaram, 2002; Szymczak and Ladd, 2006; Rajaram et al., 2009), it has been established that under conditions that lead to self-organized channel formation, the specific nature of the initial random variations do not influence the structure and spacing of the channels; rather they serve as a trigger for the initiation of channels. In unstructured meshes, it is also possible for mesh-related asymmetries to introduce perturbations that can serve as triggers for this instability. In stable regimes, however, the same perturbations will not produce channelization.

The model is first run with steady distributed input of 1 m a^{-1} in a spin-up stage with a time step of 1 hour (steady state achieved in 4 days). After a steady configuration is achieved, a cycle of meltwater input variation is imposed and run for 1 year (365 days), also with a time step of 1 hour. Seasonal meltwater input in m a^{-1} is approximated by a cosine function between 0.4-0.7 a (days 146 and 255):

$$i_{e\to b} = -492.75 \times \cos(2\pi/0.3(t-0.4)) + 493.75 \tag{18}$$

This yields a maximum meltwater input at the peak of the summer of 986 m a^{-1}, with a winter minimum of 1 m a^{-1}, and annual mean input of 149 m a^{-1}. The peak melt input corresponds to approximately 1,000 m^3 s^{-1} for the entire domain. Note that the values used here are unrealistically high, and are designed intentionally to show stable behavior of the system across a variety of input magnitudes, even when subjected to extreme forcing. Figure 5 shows time series plots of this “seasonal” input forcing over one full annual cycle, with the corresponding minimum, mean, and maximum gap height and head. Snapshots of the subglacial hydrology distributions at intervals through the annual cycle are shown in Fig. 6, and an animation of this simulation is included in the supplementary material. As melt increases, the maximum gap height increases, corresponding to growth of the subglacial system and emergence of self-organized efficient channels. The maximum gap height increases with increasing meltwater input until the peak of the melt season, then decreases simultaneously as melt input decreases (note that we use zero englacial storage in this simulation, so there is no lag due to water storage in the system). The hydraulic head initially increases with increased input (meaning an increase in subglacial water pressure as additional water is added to the system), then decreases as efficient low-pressure channels form, then increases again as melt starts to decrease and the channels collapse. We hold the sliding velocity constant, but in reality ice sheet sliding velocity generally increases with increased water pressure (i.e. lower effective pressure) and decreases with lower water pressure. With two-way coupling between the subglacial system and ice dynamics (e.g., Hoffman and Price, 2014; Koziol and Arnold, 2018), the sequence of hydraulic head or basal water pressure variation seen here would likely result in a mid-to-late summer decline in sliding velocity, after which the sliding velocity would increase again. Subsequently, as melt input decreases to the winter minimum, the hydraulic head decreases to low values, which would correspond to a decrease in sliding velocity. As shown in Fig. 6, for the early and late parts of the year, the system essentially behaves as a one-dimensional system, because the melt inputs are not large enough to take the system into a regime where channelization can occur. During the melt season, when inputs increase substantially, self-organized, regularly spaced channels emerge, seen in Fig. 6 as having lower heads than their immediate surroundings in the y direction. These channelized structures collapse and disappear entirely as the meltwater input drops off and returns to the winter minimum. The simulation results shown here demonstrate the ability of our modeling framework to represent both stable regimes, where the subglacial system takes on a relatively smooth quasi-one-dimensional configuration, and unstable regimes with self-organized efficient pathways when high meltwater inputs and discharge trigger the transition to channelization.

To examine mesh dependence in this case of self-organized channelization, Fig. 7 presents gap height and head distributions on three unstructured meshes with typical edge lengths of 50 m, 100 m, and 200 m. At 100 m resolution, the channelization effects are obvious, with similar spacing as on the finer 50 m mesh. At 200 m resolution, the channels are still apparent but the effects are more smoothed than in the finer meshes. In the early and late parts of the cycle, the different meshes are in good agreement for sheetlike drainage. This is shown in a more quantitative way in Fig. 8 with y-averaged quantities for for Day 1 (sheetlike drainage everywhere), Day 200 (peak melt input and extreme channelization), and Day 250 (near the end of the melt input cycle as channelization collapses). We see that the different mesh resolutions converge well for sheetlike drainage, but they show variation with channelization. These local differences are more pronounced in the quantities calculated over elements (gap height and degree of channelization), while differences are relatively small in the smooth pressure distributions calculated at mesh vertices.

4 Discussion

The flexible geometry and flow regimes of the SHAKTI model allow for various drainage configurations to arise naturally. We conserve mass and energy in all parts of the domain, in contrast to several existing models that neglect or redistribute the role of melt opening in sheetlike drainage systems. Previous studies found that with similar equations, including the melt term in a distributed system leads to an inevitable instability and runaway growth, which has been acknowledged as the spark that initiates channelization (Schoof, 2010; Hewitt, 2011). In our formulation, even including melt from internal dissipation, we are able to achieve stable configurations of subglacial geometry, basal water flux, and pressure fields with steady and transient input forcing. Channelized pathways with lower water pressure than their surroundings form from moulin inputs (Figs. 2 and 3) as well as self-organized configurations with high distributed melt input (Fig. 6). A feature of our formulation that contributes to this behavior is the way we calculate the basal water flux (approximate momentum equation, Eq. 5), which allows for a transient, spatially variable transmissivity that transitions naturally between laminar and turbulent flow regimes locally, while allowing both types of flow regime to coexist in the model domain, as well as flow that exhibits attributes along the wide transition between laminar and turbulent flow. To show this more clearly, Fig. 9 presents the distribution of Reynolds number through the initiation of channelization for days 145-175 of the transient example in Section 3.3. On Day 145 (just before the onset of increased melt input, see Fig. 5), the Reynolds number is low throughout the domain, corresponding to laminar flow. On Day 155, Reynolds number has increased, particularly near the outflow at the left, moving into the turbulent regime. As the self-organized channelized structure emerges through Days 165 and 175, Reynolds number becomes increasingly higher in the channelized pathways than their surroundings. If we were to use a purely laminar or purely turbulent flux formulation, the flow would not be accurately represented across this range of Reynolds numbers. If the flux is simulated as laminar everywhere (using a very small value of ω in Eq. 5, so that $\omega Re << 1$ and the flux is always linearly proportional to the head gradient), channelization does still occur with high inputs, but the flow mechanics are not correctly represented for regions with large

Reynolds number. If we force the flux to be turbulent everywhere (by using a large value for ω in Eq. 5, so that $\omega Re >> 1$ and the flux is always proportional to the square root of the head gradient), the nonlinear iteration to solve Eq. (15) becomes stuck in a large oscillation for the same model problems which behave well when we allow for laminar, transitional, and turbulent flow. The concept of laminar-turbulent transition is well established in hydraulics and fluid mechanics, and our representation of the nonlinear flux-gradient relationship (Eq. 5) is consistent with this concept and is also consistent with experimental studies of Zimmerman et al. (2004).

The transient example in Section 3.3 illustrates one possible pattern of idealized seasonal evolution of the subglacial drainage system, where channels emerge with increased melt and collapse to a sheetlike system again in the winter. The higher water pressure during the melt season would imply increased sliding velocity in a two-way coupled system, with a decrease in mid-to-late summer with well established channelized drainage, followed by an increase as the efficient system initiates its shutdown, and a decrease as meltwater input returns to the background winter rate. This seasonal pattern is reminiscent of observations of some Greenland outlet glaciers (Moon et al., 2014), and subglacial hydrology may indeed play a key role in shaping the seasonal velocity behavior of some glaciers, both land-terminating and marine-terminating. In future work on real glacier topography, we aim to investigate other velocity signatures, such as those that experience an annual minimum velocity in the late melt season, which is thought to be a result of highly efficient channel development (Moon et al., 2014) or those with high winter sliding velocities, which may be indicative of hydraulically isolated or poorly connected regions of the bed that maintain high water pressure through winter (e.g., Hoffman et al., 2016; Downs et al., 2018; Rada and Schoof, 2018).

4.1 Model limitations

This paper is intended to present a description of the SHAKTI model formulation with demonstrative simulations under simple scenarios. Application to real glaciers remains for upcoming work, but we wish to clearly address limitations of the model and acknowledge challenges faced by this and other subglacial hydrology models.

Time-stepping is an important factor in numerical models such as SHAKTI. To illustrate the influence of time step size, Fig. 10 presents evolution of maximum head in the single moulin example (see Section 3.1 and Fig. 2) for different time step sizes. In this example, the model converges properly to the same steady configuration for time step sizes dt=0.25 h to dt=3 h. Note that as the time step increases, small stable fluctuations are seen. With dt=4 h, however, the model never converges to the solution, but instead enters a large stable oscillation between incorrect values. For larger time steps than dt=4 h, the nonlinear iteration itself has difficulty converging due to a blow-up in dissipation rates, which leads to an oscillation because of our cap on water pressure to not exceed ice overburden pressure. The appropriate time step size is dependent on particularities of a simulation such as topography, ice thickness, and meltwater input rates. Due to the highly nonlinear nature of the equations, it is unfortunately not straightforward to establish a criterion for stable model behavior. As a general guideline we suggest conducting an initial test with a time step of 1 hour and adjusting accordingly. Implementing adaptive time stepping in SHAKTI could ease this process for users, but currently the adjustments are done manually. Note that the time steps required in subglacial hydrology models are typically much smaller than time steps frequently used in long-term ice dynamics simulations, which may be on the order of years or decades.

We calculate basal gap height over each element, which means that the geometry is dependent on mesh size. It is not our aim to necessarily capture each individual cavity or channel cross-section, but rather to obtain the effective geometry over each element and its effect on the pressure field, which has an important influence on ice sheet sliding velocity. In Sections 3.2-3.3, we examined mesh sensitivity in example simulations (see Figs. 3 and 7). With very large elements (km scale), the effects of channelized drainage may be smoothed out. For large-scale simulations, a variable mesh should be used with coarser resolution in the ice sheet interior away from the margins and finer resolution at lower elevations where the bulk of meltwater is produced and enters the subglacial system (where channelized networks are likely to form and sliding velocities are higher). The typical edge length scale should be selected according to the particular application, depending on the resolution of bed topography, sliding velocities, modeling goals, as well as practical concerns of computing power. As a rough guideline, to capture the formation of channelization in decent detail, we suggest an edge length of 150 m or less in the domain area of most interest (e.g., the few km nearest the terminus of a glacier).

As stated in Section 2.2, the current formulation does not handle either high water pressures that exceed overburden (we cap water pressure at overburden pressure and do not represent uplift) or low water pressures where the system would transition to free surface flow (we assume the subglacial gap is always filled with water and allow unphysical negative water pressures to be calculated in the presence of steep slopes). The sample simulations presented in Section 3 do not involve either of these extreme pressure ranges in their solutions, so the results included here are unaffected by the upper limit imposed on water pressure or by allowing negative water pressures in lieu of transitioning to a partially filled system.

The examples in Section 3 do not involve complex bed topography, which is beyond the scope of this initial model description paper. The model has been successfully tested on real ice and bed geometry, however, and results will be included in forthcoming work.

Under thick ice with low meltwater input, the nonlinear iteration may have trouble converging to a head solution, entering a stable oscillation. This can frequently be resolved by decreasing the time step and/or employing under-relaxation to help the nonlinear iteration converge.

The SHAKTI model is not currently coupled to ice dynamics in a two-way manner. We prescribe a constant ice sliding velocity, and this sliding velocity does not evolve according to the influence of subglacial water pressure. With this one-way coupling, we are able to infer only qualitatively how the ice velocity would be affected by the changing subglacial system. In upcoming work, we plan to implement two-way coupling with the ice dynamics of ISSM to test different sliding laws and the behavior of the fully coupled system.

5 Conclusions

In this paper, we presented the SHAKTI model formulation with simple illustrative simulations to highlight some of the model features under different conditions. The model is similar to previous subglacial hydrology models, but employs a single set of “unified” governing equations over the entire domain, including opening by melt from internal dissipation everywhere, without imposing a distinction between channelized or sheetlike systems. The geometry is free to evolve; efficient, low-pressure channelized pathways can and do form as the subglacial system adjusts and facilitates transitions between different flow regimes. We find that with high meltwater input (via moulins or distributed input), self-organized channelized structures emerge with higher effective pressure (i.e. lower water pressure) than their surrounding areas. As meltwater input decreases, these channelized drainage structures collapse and disappear.

To understand the overall mass balance and behavior of glaciers and ice sheets, it is crucial to understand different observed seasonal velocity patterns, and the corresponding enigmatic drainage systems hidden beneath the ice. Combined with advances in remote and field-based observations, and modeling of other processes involved in the hydrologic cycle of ice sheets and glaciers (such as surface mass balance, meltwater percolation and retention, and englacial transport of water), subglacial hydrology modeling may help close a gap in ice dynamics models to inform predictions of future mass loss and sea level rise. Forthcoming work will focus on application of the SHAKTI model to real glaciers and coupling the model to an ice dynamics model (ISSM, into which SHAKTI is already built).

Code availability. The SHAKTI model is freely available as part of the open source Ice Sheet System Model (ISSM), which is hosted in a subversion repository. https://issm.jpl.nasa.gov/download/

Author contributions. HR and AS formulated the model equations. AS wrote the stand-alone versions of the finite volume and finite element models. MM built the parallel model into ISSM and assisted AS with further model development. AS performed simulations and compiled the manuscript with contributions from HR and MM.

Competing interests. The authors declare that they have no conflicts of interest.

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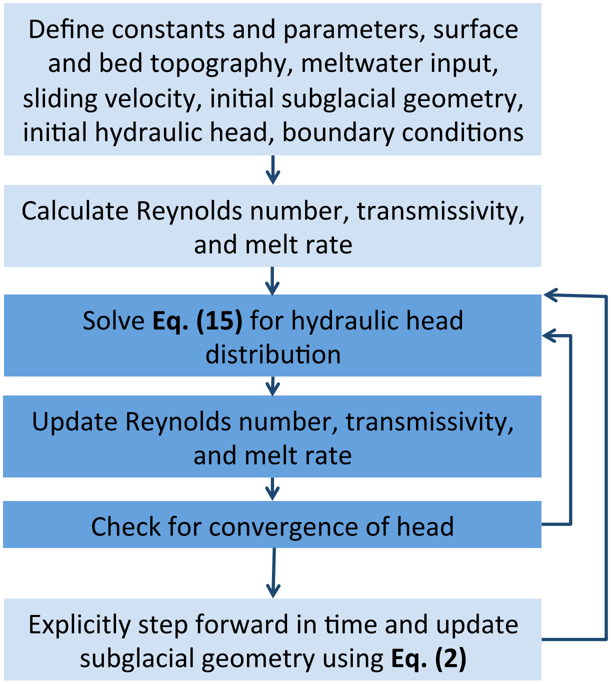

Figure 1. Schematic of computational procedure used to solve the model equations

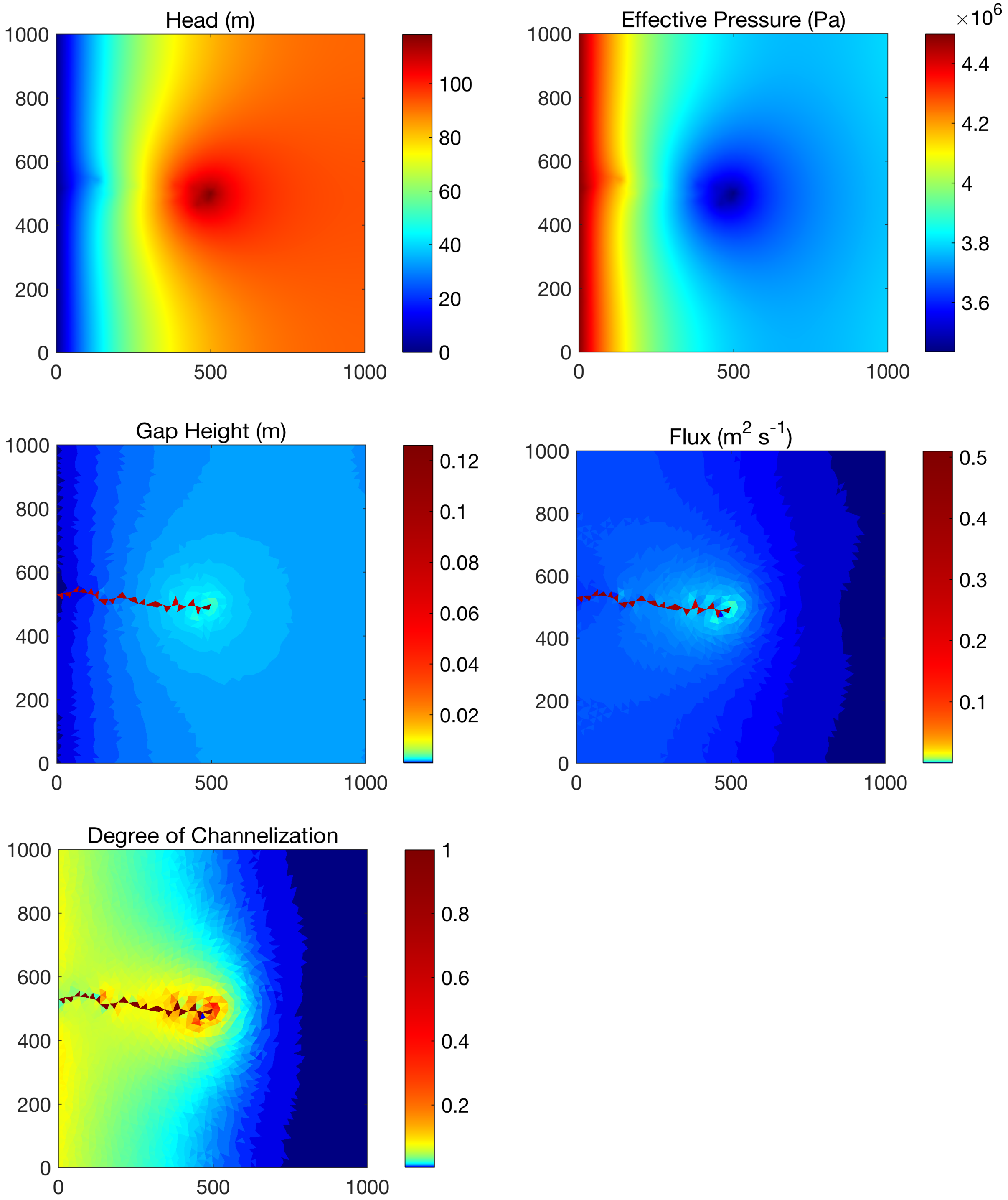

Figure 2. Steady configurations of hydraulic head, effective pressure, gap height, depth-integrated basal water flux, and degree of channelization for steady input of 4 m^3 s^{-1} into a moulin at the center of a 1 km square domain. Ice thickness is 500 m, with surface and bed slope of 0.02. A clear efficient pathway forms from the moulin input to the outflow at the left edge of the domain.

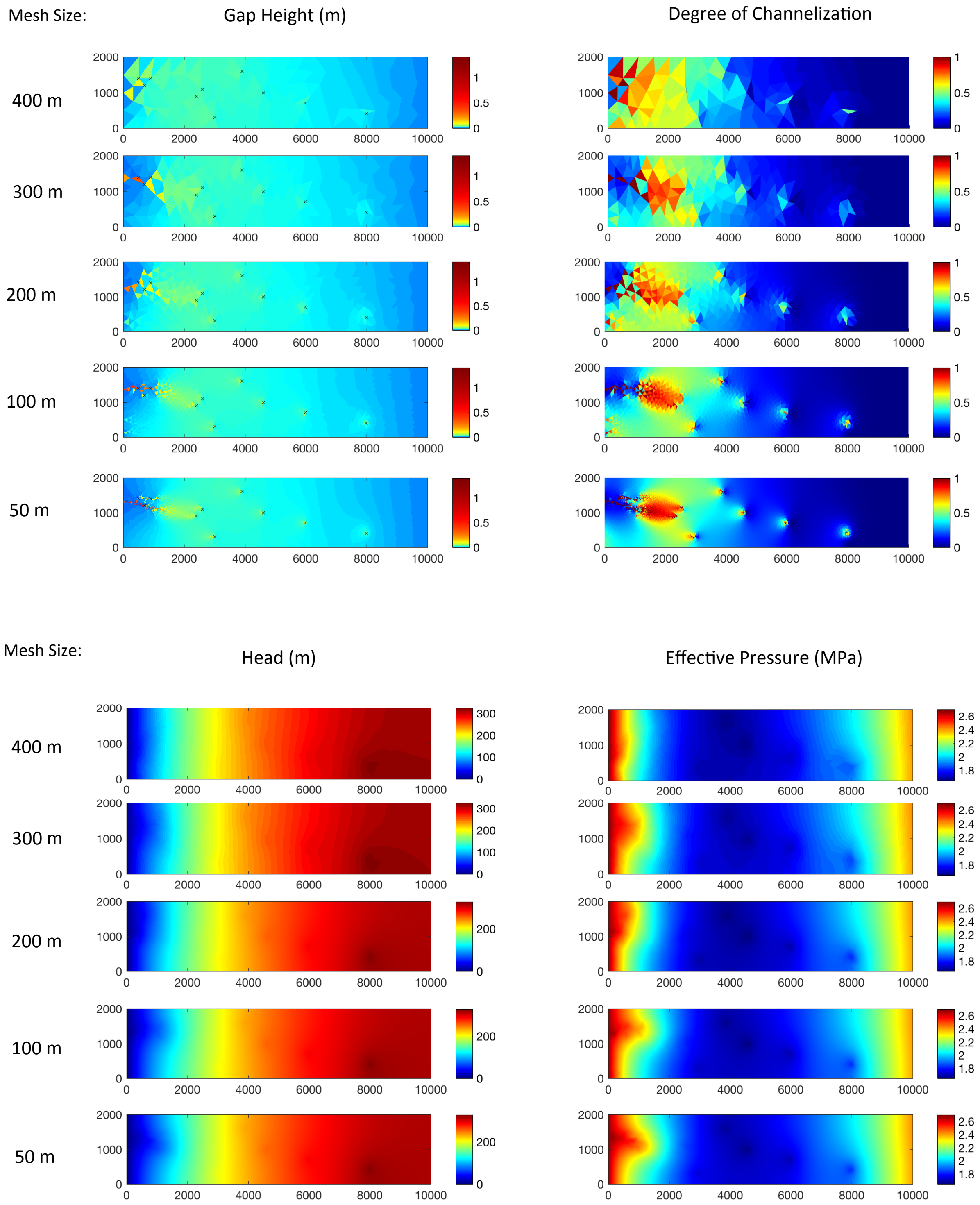

Figure 3. Steady-state distributions resulting from steady input of 10 $m^3\ s^{-1}$ into 10 moulins. As a qualitative evaluation of mesh dependence, results are shown for typical element side lengths ranging from 50 m to 400 m. Moulin locations are indicated on the gap height plots as black markers. Rather than each moulin forming an independent channel, the various inputs warp the pressure field and interact to produce a principal efficient drainage pathway.

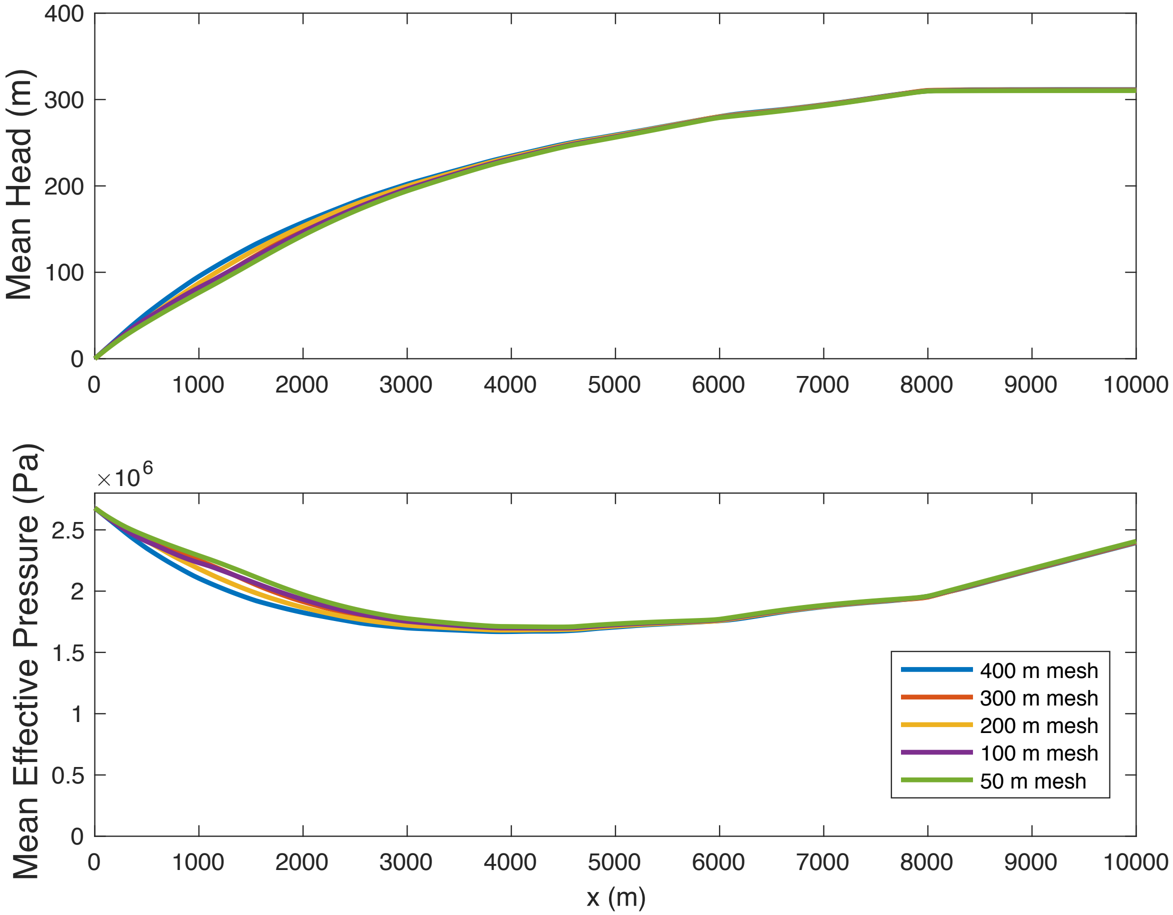

Figure 4. Mean head and effective pressure (averaged in y direction) for the 10-moulin example (Fig. 3) using unstructured meshes with typical element side lengths ranging from 50 m to 400 m.

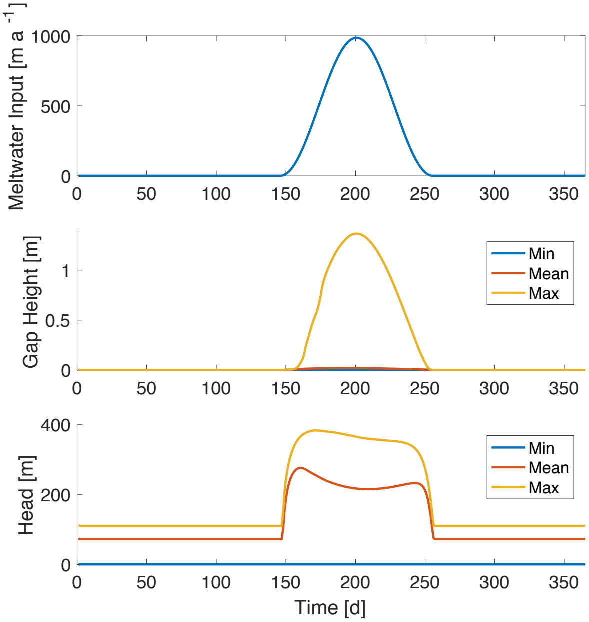

Figure 5. Seasonal cycle of distributed meltwater input over one annual cycle, with gap height and head evolution time series. As meltwater input increases, the maximum gap height increases, then decreases simultaneously with the decrease in input. As meltwater input increases, the head increases, then decreases as efficient drainage pathways are established (corresponding to lower water pressure in the efficient pathways, as well as lower head in the unchannelized upstream regions as shown in Fig. 6). As melt decreases, mean head increases again as the efficient pathways start to collapse, then decreases as melt returns to the winter minimum.

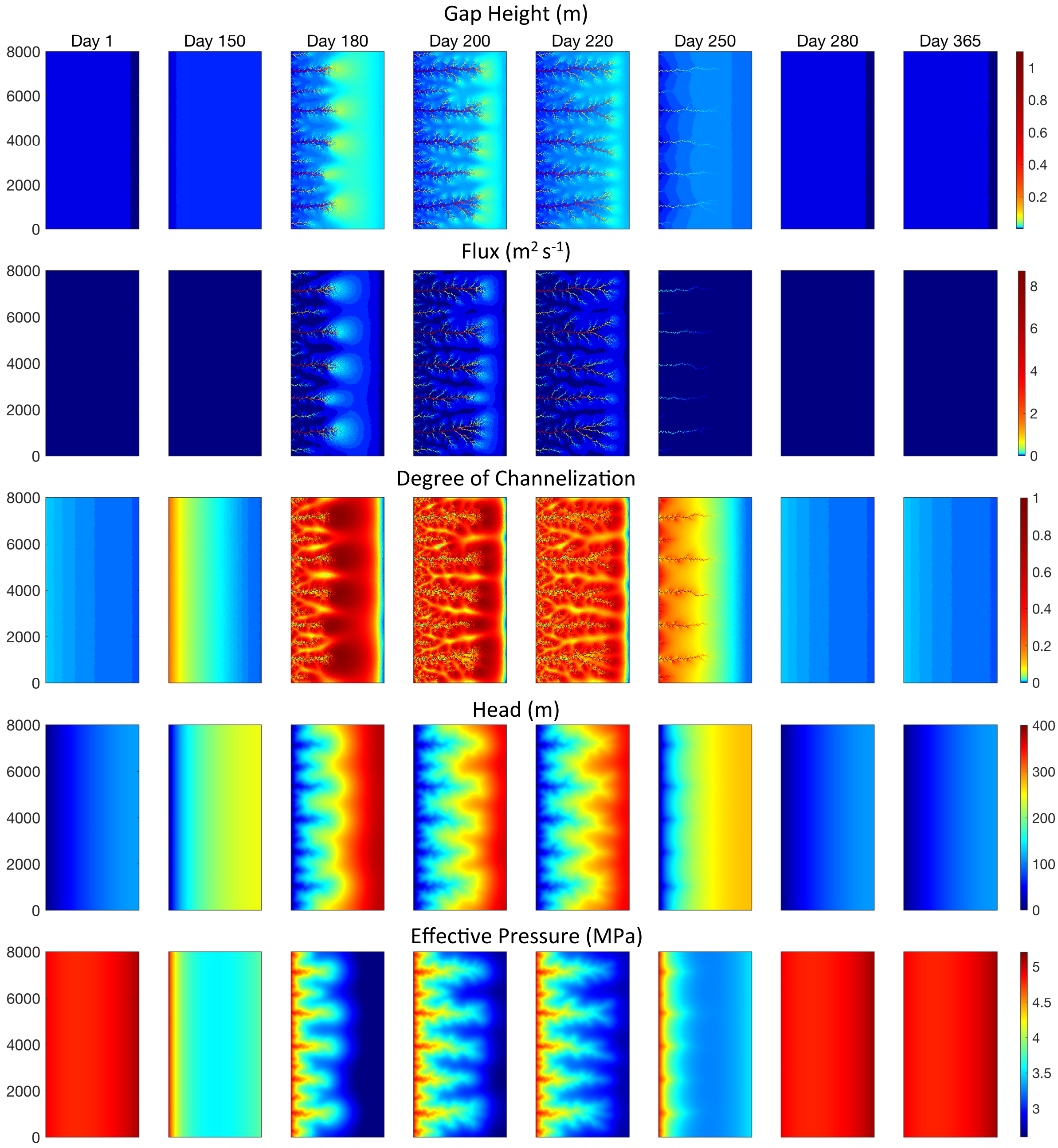

Figure 6. Seasonal evolution with distributed meltwater input as shown in Fig. 5 on a 4 km by 8 km domain over one full annual cycle. Self-organized efficient drainage pathways form from the outflow (left edge of the domain) as melt input increases, persist through the melt season, and collapse again as melt input decreases, returning to a steady sheet configuration. The efficient pathways show lower head (i.e. higher effective pressure) than their surrounding areas in the y direction.

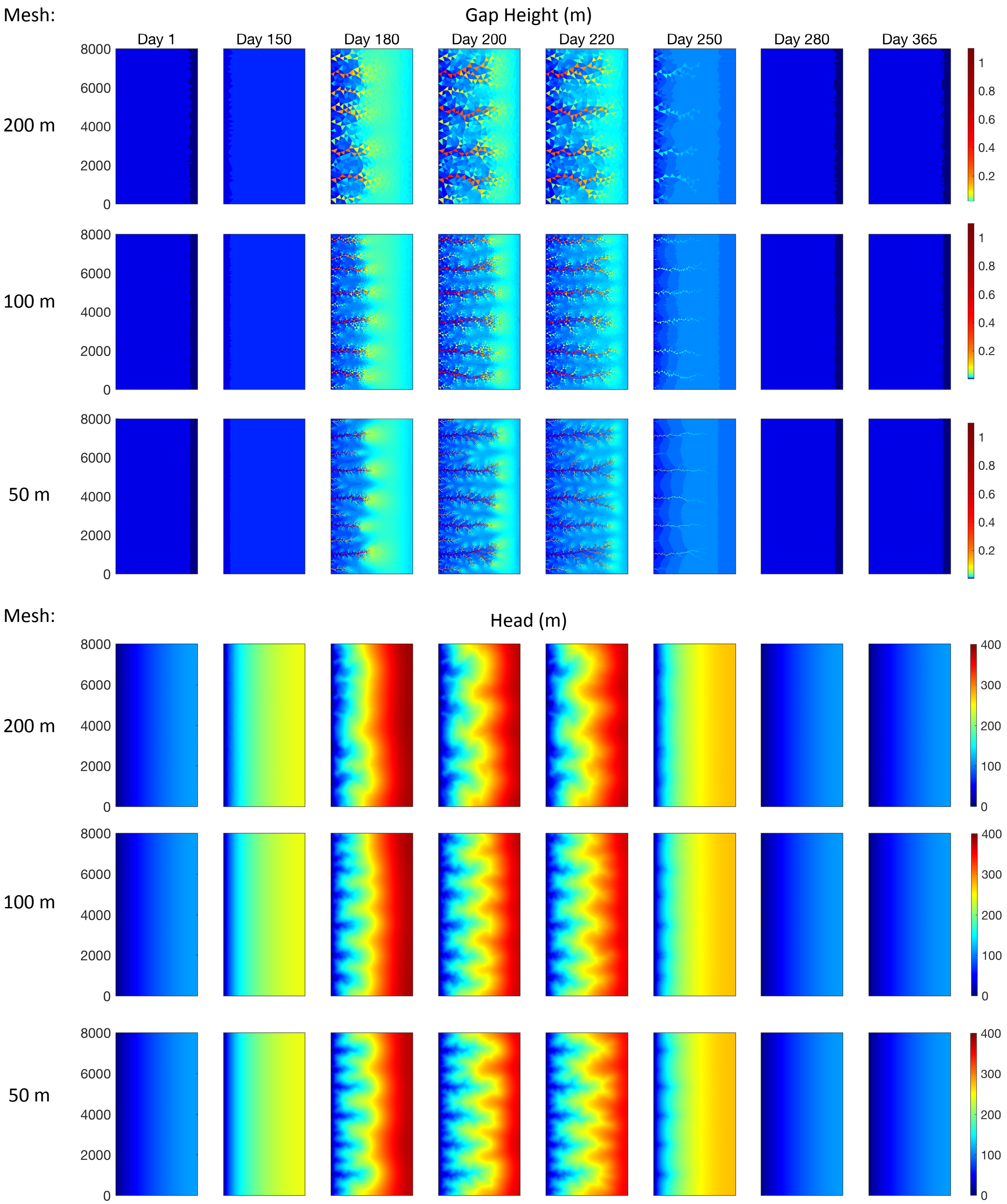

Figure 7. Mesh dependence shown for the transient example with distributed input (see Section 3.3 and Figs. 5 and 6) with typical element edge lengths of 50 m, 100 m, and 200 m.

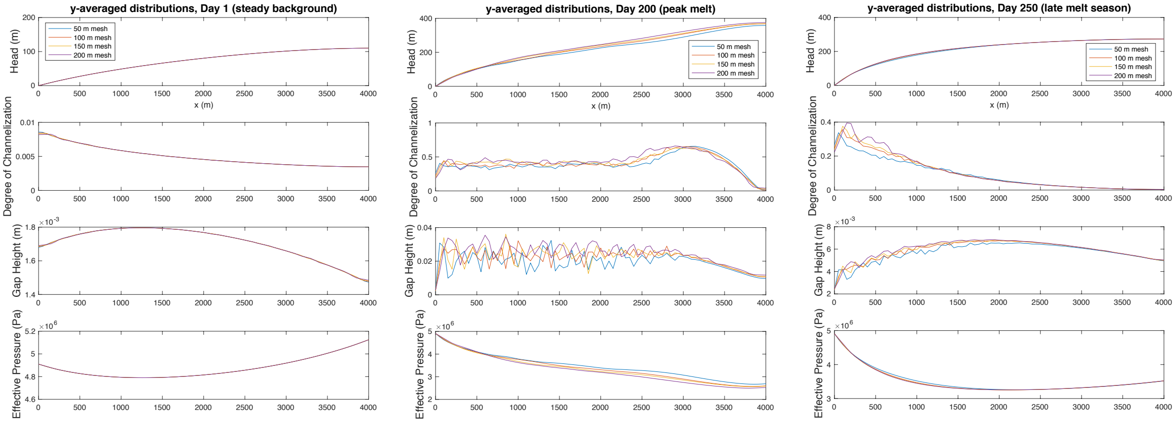

Figure 8. Mesh dependence shown with y-averaged quantities for the transient example (see Section 3.3 and Figs. 5-7) for three selected days. The model has very little dependence on mesh size with sheetlike drainage (Day 1). With channelization (Day 200 at the peak of the input and Day 250 with some channelization), mesh size leads to variability in the highly channelized regions. The local differences are more pronounced in the quantities calculated over elements (gap height and degree of channelization), while differences are relatively small in the smooth pressure distributions calculated at vertices of the mesh.

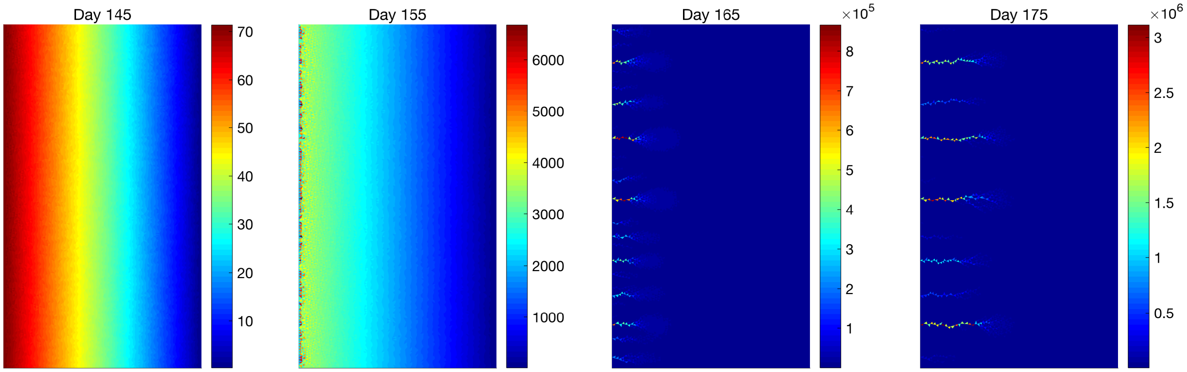

Figure 9. Reynolds number evolution during the onset of channelization in the transient example with distributed input (see Section 3.3 and Figs. 5 and 6). Initially, the entire domain has low Reynolds number, corresponding to laminar flow. As the meltwater input increases, Reynolds number rises into the turbulent regime and becomes clearly higher in the self-organized channelized structures.

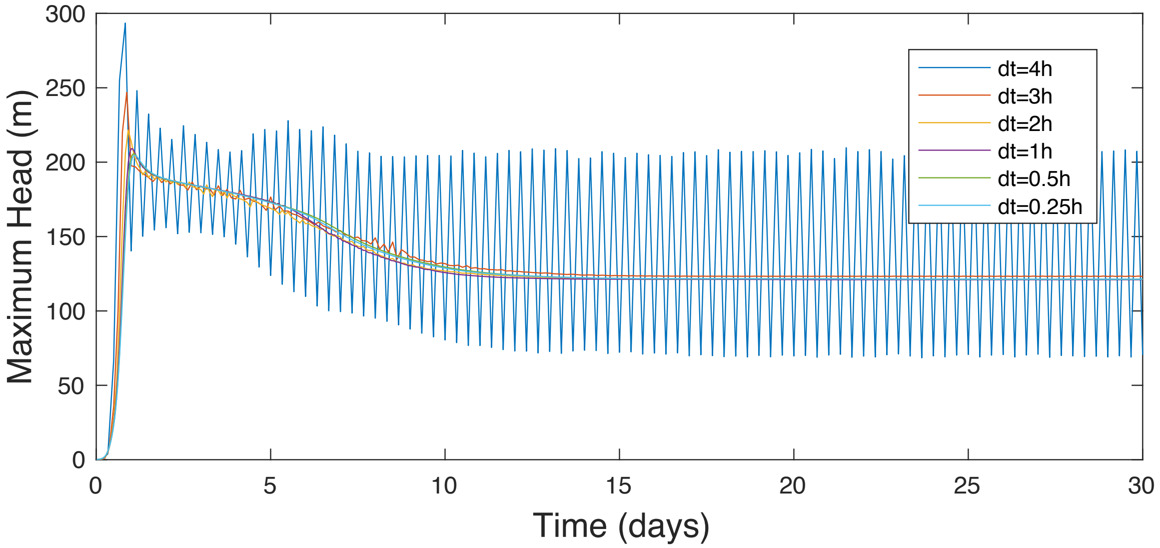

Figure 10. Maximum head evolution to illustrate time step dependence for the steady simulation with a single moulin input (see Section 3.1 and Fig. 2). For dt<4h, the model converges properly to the correct solution, but with dt=4h it enters a large, stable oscillation and never converges.

Table 1. Variables used in model equations

Symbol	Units	Description
b	m	Subglacial gap height (average over element)
b_e	m	Englacial storage volume per unit area of bed, $b_e = e_v(h - z_b)$
t	s	Time
$\boldsymbol{q}$	$\mathrm{m}^2\ \mathrm{s}^{-1}$	Gap-integrated basal water flux, $\boldsymbol{q} = \frac{-b^3 g}{12\nu(1+\omega Re)}\nabla h$
$\dot{m}$	$\mathrm{kg\ m}^{-2}\ \mathrm{s}^{-1}$	Internal melt rate
p_i	Pa	Ice overburden pressure, $p_i = \rho_i g H$
p_w	Pa	Subglacial water pressure, $p_w = \rho_w g(h - z_b)$
Re	Dimensionless	Reynolds number, Re = $\lvert\boldsymbol{q}\rvert/\nu$
h	m	Hydraulic head
β	Dimensionless	Parameter to control opening due to sliding over bedrock bumps, $\beta = (b_r - b)/l_r$ for $b < b_r$, $\beta = 0$ for $b \geq b_r$
N	Pa	Effective pressure, $N = p_i - p_w$

Table 2. Constants and parameters

Symbol	Value	Units	Description
ρ_w	1,000	kg m^{-3}	Bulk density of water
$i_{e\to b}$		m s^{-1}	Input rate of meltwater from englacial system to subglacial system
ρ_i	910	kg m^{-3}	Bulk density of ice
A		Pa^{-3} s^{-1}	Flow law parameter
n	3	Dimensionless	Flow law exponent
b_r	0.1	m	Typical height of bed bumps
l_r	2.0	m	Typical spacing between bed bumps
u_b	10^{-6}	m s^{-1}	Sliding velocity (31.5 m a^{-1})
g	9.8	m s^{-2}	Gravitational acceleration
ω	0.001	Dimensionless	Parameter controlling nonlinear transition between laminar and turbulent flow
L	3.34×10^5	J kg^{-1}	Latent heat of fusion of water
G	0.05	W m^{-2}	Geothermal flux
c_t	7.5×10^{-8}	K Pa^{-1}	Change of pressure melting point with temperature
c_w	4.22×10^3	J kg^{-1} K^{-1}	Heat capacity of water
ν	1.787×10^-6	m^2 s^{-1}	Kinematic viscosity of water
e_v		Dimensionless	Englacial void ratio