

Review of the GMDD manuscript ‘The Matsuno baroclinic wave test case’
Authors: Ofer Shamir, Itamar Yavoby, and Nathan Paldor

Summary:

The manuscript introduces a test case for shallow water models that is built upon analytic solutions of the linearized shallow water equations on the equatorial beta-plane, originally published by Matsuno (1966). The new test case has some similarity to the Rossby-Haurwitz test case, a standard test case for spherical shallow water models and a solution of the non-divergent barotropic vorticity equation. However, the Rossby-Haurwitz wave becomes unstable over longer integration periods in shallow water models, as e.g. shown in Thuburn and Li (2000), and its default parameter settings lead to high-speed gravity waves. The new test case approximates the solutions of the shallow water equations for small-speed gravity waves, and therefore mimics the baroclinic modes of the atmosphere and oceans. The flow pattern of the new test case is also more stable than the barotropic Rossby-Haurwitz flow pattern. It might therefore present an interesting addition to the suite of shallow water test cases on the sphere.

Example solutions for the test case are explored for westward-traveling Rossby waves (planetary waves) and eastward-propagating inertia-gravity waves (EIG) for both atmospheric and oceanic conditions with mean shallow water heights of $H = 30$ m (atmosphere) and $H = 0.5$ m (ocean).

These example solutions are generated with a tropical channel version of a shallow-water model in spherical coordinates. The analysis is based on Hovmoeller diagrams and temporal and spatial spectra. Fortran, Matlab and Python files are provided as supplemental material that can be used to compute the initial conditions and analytic solution at any time and any location.

General comments:

In general, the manuscript is very well written, informative and a very valuable addition to the suite of shallow-water test cases. However, some questions and corrections need to be addressed before it can be recommended for publication.

1) The manuscript claims (e.g. at the bottom of page 4) that this test case can be used for tropical-channel shallow water models (as presented in this manuscript) and global-scale models. From the manuscript it is not entirely clear that the test will work for global models due to the use of the equatorial beta-plane approximation in the derivation for e.g. the transformations of (x,y) and the wavenumber k . The modeling community (as a ‘customer’ of this test case) generally works with global shallow water models and tropical-channel model in spherical geometry are extremely rare. It therefore would have been more valuable (or convincing) to present example solutions for a global shallow water model instead of a tropical-channel model. Can the tropical-channel shallow water model also be configured as a global model to demonstrate that the test case works for the whole sphere? Please provide extended explanations or ideally results from a global model.

2) Model developments with regular latitude-longitude grids have become very rare over the last decade. More typical grids are now cubed-sphere, hexagonal or icosahedral grid with built-in grid irregularities. The manuscript states that the solutions of this test case are very stable for at least 10 wave periods, which is demonstrated on a regular lat-lon grid. This triggers the question whether this statement will hold for today’s models with non-latitude-longitude grids. Another question is whether small perturbations of the initial conditions will disrupt or shorten the

stability of the test case. Please provide information on these aspects.

3) As detailed below (points 5-7), the description of the initial conditions is incomplete. In addition, the analytic equations (Eq.(3)) differ slightly from the implementation in the Fortran, Matlab and Python codes. The test is therefore not usable by others in its current form, and the manuscript/codes need to be corrected.

Technical comments:

- 1) Page 1, line 9, also page 2, line 32: Please describe the model as an ‘equatorial channel’ model.
- 2) Page 1 line 12, page 2 line 2, page 5 lines 3&7: Generalize the description of the grids. A test case for only ‘latitude-longitude’ grids will have rather limited use. I think you meant to say that given the location of a latitude and longitude, the initial conditions and analytic solutions can be computed on any grid.
- 3) Page 2, line 24: It is incorrect to say that the term ‘baroclinic’ is associated with density variations in the vertical directions. A flow with identical density and pressure variations (e.g. for isothermal conditions) is still barotropic. Density and pressure variations need to vary independently of each other.
- 4) Page 3, line 15: What is meant by ‘reduced gravity’. The initialization of the test case uses the regular Earth’s gravity. Modify.
- 5) Page 3, line 17, also page 5&6 section 3.1: The wave mode $n=1$ is selected which leads to three distinct real roots in Eq. (2). Two of these roots are selected for the example results, but no equations are given for the Rossby wave root and EIG root. Without this information, the description of the initial conditions is incomplete. Add this information to Section 3.1.
- 6) Page 4, Eq. (3) and text: The manuscript fails to explain the meaning and definition of ψ_n . What is the relationship between ψ_n and the normalized Hermite polynomials H_n ? Without the definition of ψ_n the description of the initial conditions is incomplete.
- 7) Page 4, Eq. (3): Eq. (3) seems to be correct, but the Fortran/Matlab/Python scripts use a wrong $u_{\hat{}}$ calculation. E.g. the Fortran code in line 200 needs to read $\text{sqrt}(g*H_0)$ instead of just ‘ $g*H_0$ ’.
- 8) Page 4, line 10: State that the amplitude A needs to have units of m/s.
- 9) Page 4, line 25: you imply that the planar wavenumber k is unitless, so that that spherical wavenumber $k/(a \cos\phi_0)$ has units of 1/m. Please comment and clarify. Correct typo, should be ‘replaced’.
- 10) Page 8, line 1: What is meant by the ‘transport form’ of the SWEs? This seems to imply the ‘advective form’. However, the provided equations are in ‘conservation form’.
- 11) Page 8, line 10: Explicitly state whether the example model uses diffusion or smoothing/filtering operations for the computations, and if yes, which ones. Should users of the test case try to omit all diffusion/filtering operations in their models when using this test case? E.g. the provided shallow water code contains provisions for a temporal Asselin filter.
- 12) Page 9, Fig. 2: The value for the symbol ϕ_f is not provided. Add this information.
- 13) Page 9, Fig. 2: It is highly unusual and confusing to see and interpret the flipped Hovmoeller diagrams. Typically, Hovmoeller diagrams list the position along the x-axis and time along the y-axis. I recommend flipping the axes in Fig. 2 to make the interpretation of the Hovmoeller diagrams easier.
- 14) Supplemental material: Please add Fortran/Matlab wrapper codes that will enable the user to

create/test the initial conditions. In addition, the codes should not expect to receive regular longitude and latitude arrays, but should be callable for any longitude and latitude position.