

## ***Interactive comment on “Assimilating Compact Phase Space Retrievals (CPSRs): Comparison with Independent Observations (MOZAIC in situ and IASI Retrievals) and Extension to Assimilation of Truncated Retrieval Profiles” by Arthur P. Mizzi et al.***

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RESPONSE TO REVIEWER 1 General Comments: 1. Domain Size: We agree that the study period is such that air barely moves from one side of the domain to the other. But it gets most (if not all) of the way across, so the study period is sufficient to determine whether the assimilation can address errors introduced by the initial and/or boundary conditions. Additionally, we are concerned with the grid interior, and the air

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easily crosses the interior. 2. Bias: We are concerned with three types of bias for the chemistry: (i) bias introduced by the initial and boundary conditions, (ii) bias introduced by the model, and (iii) bias introduced by assimilating bad observations. For the MET experiment the bias is due to initial/boundary condition and/or model errors. For the chemical assimilation experiments, the goal is to reduce the initial/boundary condition errors and thereby improve the forecast. Figure 1 shows that for the L10VMRR, CPSR, and QOR experiments, the assimilation of MOPITT CO retrievals generally reduced the bias and improved the model forecast skill. Despite that increased skill, when we compared the improved forecasts against independent data (the MOZAIC and IASI data) we found that the assimilation had introduced a bias in the upper troposphere. That bias is not due to initial/boundary condition error. Our analysis shows that it is due to the assimilation of biased observations. The improvements in Figure 1 when comparing against IASI are in spite of the bias introduced from assimilating these biased observations. 3. MOPITT Coverage: It is correct that for any particular assimilation cycle a portion of the domain is constrained by MOPITT observations and a portion is not. Such spatial and temporal sparsity is a characteristic of polar orbiting satellite observation platforms. Over time such platforms observe the entire globe. MOPITT observes the entire globe in four days. So that during our study period MOPITT observes the entire domain at least twice. 4. Spin-up Period: The results in Mizzi et al. (2016) suggest that for phase space assimilation experiments the spin-up period for stable verification statistics is two to three days. Our study period is long enough to account for that spin-up period. 5. MOZAIC Data: The MOZAIC data came from ascent and descent soundings on: June 1 and 5, 2008 at Dallas, TX; June 3 and 9, 2008 at Portland, OR; and; June 7, 2008 at Philadelphia, PA. That is ten profiles which cover our study period which ran from June 1 to 9, 2008. The June 1, 2008 profiles were taken during our spin-up period so they were discarded. The discussion of the MOZAIC comparison has been revised to address the reviewer's comments. See P12 L22 to P13 L18. 6. IASI Data: IASI has global coverage nearly every two days, and MOPITT has global coverage every three to four days. Therefore, there were times and locations where

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the MOPITT and IASI observations were coincident. We agree that assimilation results for such coincident location are likely be better than those for non-coincident locations. We are interested in whether the assimilation improves model performance in a bulk sense and do not believe that the results from the coincident (or nearly coincident) locations dominate those bulk verification scores because the areas of overlap are small compared to the domain size. 7. Significance of Skill Differences for MET, VMRR, L10VMRR, CPSR, and QOR: The differences between MET and VMRR and for MET and L10VMRR are not significant. Those between MET and CPSR (or QOR) are likely to be significant. For the experiments that are common to those in Mizzi et al. (2016), we found that the magnitude of the differences is comparable to those found in Mizzi et al. (2016) who found the significance to be as described above based on applying the Student t test to the difference of two means. We revised the figures to include error bars and discussed this in captions and text. See Figs. 2, 7, and 8. 8. Information Content before and after Compression: When using a Singular Value Decomposition (SVD) to compress data, there is a difference between discarding modes with zero singular values and discarding modes whose singular values are non-zero but small relative to the leading singular values. Arguably, due to round-off error in a digital world it may be difficult to distinguish between the zero and small non-zero singular values. However, due to the nature of the retrieval process and because one of the reported products is “degrees of freedom of signal” (DOFS) – the trace, we know the number of non-zero singular values for an SVD of an averaging kernel with an accuracy of one because the DOFS is reported as a real number, and it is arbitrary whether to round the fractional part up or down. Now assuming that we know the number of non-zero singular values, we can perform the compression by discarding the zero singular values and associated singular vectors without loss of information. That is what is done in the compression step of the CPSR transform. If we had discarded modes whose singular values are non-zero but small, then there would have been information loss. 9. If the Information Content before and after Compression Is the Same, Why Do CPSR and QOR Produce Improved Skill? The CPSR experiment can have better verification

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scores compared to VMRR and L10VMRR because the: (i) correlations are greater, and/or (ii) transformed observation errors are smaller. We think it is primarily due to smaller observation errors. They are smaller due to the compression step of the CPSR transform. They cannot be smaller due to the diagonalization step because that is a variance maximizing rotation. If the compression step had no filtering effect on the errors, then the variance resulting from the diagonalization step would be no smaller than that from the compression step. The QOR experiment produces the same results as the CPSR experiment because the linearly dependent portions of the averaging kernel do not contribute to the assimilation results (this point is discussed in greater detail below and revised in the text). See P11 L21 to P12 L23. 10. Construction of the Super-Observations: The paper states that the super-observations were constructed as follows, we: (i) sort the retrievals, retrieval priors, averaging kernels, and retrieval error covariances into bins that are  $\sim 90$  km square, (ii) calculate the binaverage for each of those variables, and (iii) assimilate the bin-average retrievals. We use an arithmetic average (as opposed to error covariance weighted average) when calculating the super-observation and do not apply a correction to the retrieval error covariance super-observation because we are interested in the impact of the reported errors and can apply an error tuning factor to adjust the errors and balance the observation fit as needed. Other studies e.g., Eskes et al. (2003), Miyazki et al. (2012 a and b, 2015), and Barre et al. (2016) have used similar super-observation strategies. We believe that the description of this process here and in the text of the revision is sufficient to describe our methodology. See P6 L7 to P6 L17. 11. Why Do the RJ3 Experiments Perform Worse against IASI? We answer this question in the discussion of Table 3 and Fig. 10. When one discards retrievals at a certain level (for discussion consider the top three levels as done in the paper), it impacts the amount of remaining information to be assimilated (Table 3 – the difference between the first and second row) and the sensitivity of the resulting averaging kernel (see Fig. 10 – the difference between columns (a) and (b)). Those changes combine to remove most of the beneficial impacts from assimilating the MOPITT observations. In effect assimilating retrievals at MOPITT’s

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top three levels positively impacts the middle and lower troposphere through the averaging kernel. When those retrievals are not assimilated those positive impacts are not realized. See P19 L24 to P21 L14. 12. Why Do We Conclude that CPSR and QOR Perform Better against MOZAIC than MET? The paper has been revised to address this comment. See P12 L25 to P13 L21 and the revised Fig. 2. Specific Comments: 1. Introduction: It reads like a duplication of the abstract. Yet, the introduction has a very different purpose, including how the current research fits into the work that is being done by other researchers. Currently, it mostly explains the relation between this work and Mizzi et al, 2016. A wider context is needed to provide the reader with a more general background of this research. We have revised the introduction to provide a wider context as requested. See P1 L11 to P4 L9. 2. page 5, line 8-14: How many MOZAIC profiles have been used for validation? Eight profiles were used. This is discussed in Section 5.1 (page 10, lines 8-11). That seems like the appropriate place because this section is the general introduction of the observational data used for validation, and Section 5 discusses specific application of the validation data to this paper. 3. Page 5, line 18: which state variables would otherwise be influenced by the CO data? There are two types of correlations that this localization is concerned with: (i) correlations that have been observed in the field e.g., CO and O3 are known to be correlated, and (ii) correlations in the ensemble. In the ensemble Kalman filter, the update depends on the correlation in the ensemble. Some of those are real correlations i.e., they are observed in the field (or thought to be real based on the chemistry) and some are spurious. Historically, in chemical data assimilation, all correlations except those between the observed species and the corresponding state variable are turned off through localization. In this paper, we used the customary localization for chemical data assimilation. 4. Section 4.1: the difference between regular and L10 retrievals is explained, but a more explicit link should be made to with is done in experiment VMRR and L10VMRR. The section about QOR and CPSR should return to this discussion since the equations that are shown there suggest L10VMRR are used although this is nowhere mentioned. Revised, see P8 L7 to P8 L18. 5. Page 6, line 20: 'Another

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reason is to include pre-processing methods that enable us to not assimilate selected retrievals' I don't quite understand what is meant here. Revised, see P8 L17 to P8 L18. 6. Section 4.2: It remains unclear from the description if any filtering of dominant eigen- vectors is applied to QOR, and, if so, based on what criterion. For the QOR and CPSR experiments there is no filtering of the dominant singular vectors (i.e., filtering those whose singular values are non-zero). One problem is identifying the non-zero singular values. The paper is revised to make this more clear. P9 L8 to P9 L9 and P10 L13. 7. Page 9, line 2: 'rank of A is greater ... i.e.,  $n - k \geq q$ ' But this assumes that the elements that are removed are in the null space of A, which need not be the case (for example, if the purpose of leaving out layers is bias correction). This does not assume that the discarded elements are in the null space of A. The dimension of A is n and the rank of A is k. After discarding q elements of the retrieval profiles the dimension of the revised A (call it A) is  $n - q$  by n. The rank of A i.e., the number of non-zero singular vectors will be less than or equal to k. Generally,  $k \ll n$  so that discarding q elements of the retrieval profile has little or no impact on the rank of A i.e., as you observed the discarded elements of the retrieval profile are in the null space of A. If the discarded elements are not in that null space, then the rank of the truncated A is less than that of full A. The paper is revised accordingly. See P10 L19 to P11 L1. 8. page 10, line 4: The right order is VMRR -> MET -> L10VMRR ... Corrected. 9. page 10, line 8: 'We have investigated our results and concluded that they are correct ...', What was done? In preparation of this paper, we revised the computer code used by Mizzi et al. (2016). The revised code incorporates the QOR code into the CPSR code as opposed to using separate codes as was done in the earlier paper. When writing this paper, I had forgotten that we had similar results between CPSR and QOR in the earlier paper. So, it is true that I spent time double-checking the code etc. to confirm that these results were correct. During preparation of the revisions I reviewed the earlier paper. Therefore, this has been revised in this paper. See P12 L10 to P12 L22. 10. page 10, line 9-23: The two 'explanations' that are given discuss why CPSR and QOR result may be similar or different, but they don't explain what is referred to

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as 'the discrepancy' in line 9. As discussed in the preceding comment, the paper has been revised because there is no discrepancy. However, this discussion of why the CPSR and QOR results are similar is still apt. The paper states that: [c]onsequently, the CPSR and QOR experiments yield similar results because: (i) the QOR experiment apportions the error and assimilates the linearly dependent modes (which have little or no impact), while (ii) the CPSR experiment apportions the error and does not assimilate the linearly dependent modes. 11. page 10, line 17: You mean that the observational error covariance is still singular when transformed to the SVD space? No, in the QOR experiment, the averaging kernel is rotated with the leading singular vectors from the observation error covariance matrix. The averaging kernel matrix is singular and so is the rotated averaging kernel matrix. Generally, for this work the observation error covariance matrix was not singular. Revised P9 L9 to P9 L10. 12. page 11, line 13: A more quantitative discussion is needed here of the bias that is found, versus what has been reported MOPITT v5. Deeter et al. (2013) reported a positive bias in the MOPITT CO retrievals of ~14% in the upper troposphere. Martinez-Alonso et al. (2014) suggested that the MOPITT CO retrievals were not biased. Based on those papers it is uncertain whether the retrievals but subsequent researchers e.g. Barre et al. (2016) have treated the MOPITT retrievals as biased in the upper troposphere. The results in Fig. 1 for the CPSR and QOR experiments suggest that the MOPITT retrievals in the upper troposphere are positively biased by at least 8% (likely more because the assimilation adjusts the analysis to lie between the prior and the assimilated observation). 13. page 12, line 15: How about the opposite: MOPITT's lower tropospheric sensitivities influencing the upper troposphere through Ak smoothing? Please provide additional explanation. We do not understand the comment in light of the cited text. 14. page 13, line 15: 'Those results suggest ... most sensitive to CO in the lower troposphere' Why is that? The text above describes, but doesn't explain anything. Deeter et al. (2007) report that MOPITT CO retrievals have sensitivity to (i.e., can observe) CO in the lower troposphere. Our results from assimilating MOPITT CO do not show improvement in the lower troposphere. The analysis of Figs. 2 and 3 shows why the assimilation results

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do not have improvements in the lower troposphere. Fig. 2 confirms our conceptual understanding of MOPITT CO retrievals that the DOFS needs to be in the neighborhood of 2.0 to have sensitivity to CO in the lower troposphere. However, due to linear dependencies in the averaging kernel profiles that make up the composite profiles in Fig. 2, it is possible that sensitivities to the lower troposphere are masked for DOFS in the 1.0 and 1.5 figures. Figure 3 looks at the sensitivities for the linearly independent averaging kernel profiles. Figure 3 (second row) shows that for all DOFS categories, the linearly independent averaging kernel profiles have sensitivity near the surface. But when the error covariance is considered (Fig. 3 third row), that lower tropospheric sensitive disappears. Without that sensitivity, the assimilation cannot adjust CO in the lower troposphere. See discussion P14 L1 to P16 L14. 15. page 14, line 20 - 21: I don't really see this in the Figure. The text has been revised to make this more clear. P16 L16 to P17 L9. 16. page 16, line 8: L10VMRR-RJ3 does account for the observation error covariance, but you just don't diagonalize / reduce its rank, right? L10VMRR-RJ3 is the same as L10VMRR except that retrievals above 250 hPa are not assimilated. Neither of these experiments account for the observation error covariance. Diagonalization of the observation error covariance matrix is done by discarding the covariance terms (leaving only the error variance). This is the conventional approach to diagonalization of retrieval-based error covariance matrixes. That form of diagonalization does not reduce the rank of the rotated error covariance matrix. FYI: the diagonalization rotation is not a compression/rank reduction step. 17. page 17, line 12: 'The CPSR-RJ3 experiment skill improvement ...' How about the significance of this? The goal of Fig. 7 is to show that conventional method for assimilating a truncated retrieval profile (L10VMRR-RJ3 EX) and the phase space method (CPSR-RJ3 EX) give similar results when compared to assimilating the full retrievals (L10VMRR and CPSR respectively). The significance of the difference between L10VMRR and L10VMRR-RJ3 and that of the difference between CPSR and CPSR-RJ3 was not determined. However, the figures have been revised to include error bars generated using the ensemble samples. The L10VMRR difference may not be significant, but the difference between CPSR and

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CPSR-RJ3 are likely significant. But the significance of those differences is not a key point here because: (i) we are interested in whether the extension of the CPSR method to truncated retrievals gives the expected results, (ii) it gave expected results in the upper troposphere but not in the middle and lower troposphere, and (iii) the unexpected results mean that we need to revise the extension and/or develop other methods for assimilating truncated retrievals. 18. page 17, line 21: 'Reject Top Three'. Besides the point that this explanation of the meaning of the "RJ3" experiment comes rather late, it is also not clear what justifies this method of MOPITT bias correction – given earlier publications about the nature of the bias. An explanation of RJ3 has been added to the text in Section 5.2. This is not a bias correction algorithm. It is a phase space method for not assimilating retrieval observation that are thought to be bad observations. See P18 L12 to P 18 L15. 19. Figure 4 - 5: The discussion in the text is hard to follow, because it requires comparing figure 4 and 5. Why not show model – data differences, before / after assimilation in one Figure? The discussion in the text has been revised to ease the comparison of the figures. See P16 L16 to P17 L9. 20. Figure 10: Why do these plots show results for all levels, whereas they represent experiments in which retrieval results for certain levels are not taken into account. How can these levels nevertheless show up? As explained in Section 4.3, the averaging kernel starts out as a square matrix whose dimensions depend on the dimension of the retrieval profile. In the paper, the dimension of the retrieval profile is  $n$  so the dimension of the averaging kernel is  $n \times n$ . If we do not assimilate  $q$  elements of the retrieval profile, then the adjusted averaging kernel has dimensions  $(n - q) \times n$ . The adjusted averaging kernel maps the true atmospheric state (observed on the  $n$  levels of averaging kernel profile) to the  $n - q$  levels of the truncated retrieval profile. Figure 10 displays the  $n$  levels of the averaging kernel profile. All  $n$  levels are present because even though the retrievals at the  $q$  levels are discarded, the retrievals at the  $n - q$  levels that are assimilated are dependent on the true atmospheric state at all  $n$  levels of the averaging kernel profile. Technical Details: 1. Title: misses a closing parenthesis (maybe better to remove the details enclosed in parenthesis anyway) Corrected. 2. page 9, line 3: remove 'changes

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in' Corrected. 3. page 9, line 4: 'retrieval' i.o. 'observation' Corrected. 4. page 12, line 15: 'artifact' i.o. 'artifice' Corrected. 5. page 12, line 25 and onwards: 'sensitivity' i.o. 'variability' Corrected. 6. page 12. line 7: 'Fig. 3' i.o. 'Fig. 2' Corrected. 7. page 16, line 4: 'Section V.C.' Corrected. 8. Figure 4, title of the lower left panel: CPSR – MET Corrected. 9. Figure 8, 9, titles of panels in the bottom row: What is 'Del-Fcst'? Corrected. RESPONSE TO REVIEWER 2 General Comments: 1. Expansion of the Introduction: The Introduction has been revised. See P1 L11 to P3 L24. 2. Additional Information for the IASI CO Retrievals: Revised. See P6 L20 to P7 L3. 3. Comparison with Observations Lacks Rigor: Figures 1, 6, and 7 have been revised to address this concern. Figures 4, 5, 8, and 9 are intended to be qualitative to show the types of changes chemical data assimilation can make. The discussion of these figures has been revised to make the comparison more clear. See P6 L20 to P7 L3. 4. Displaying Singular Vectors without Singular Values: As this comment relates to Fig. 3, we have added Table 2 which contains the singular values. As it relates to Fig. 10, Table 3 (the former Table 2) contains the singular values. The text has been revised accordingly. See P14 L21 to P14 L24; P20 L6 to P20 L23. 5. Comparison with Other Papers Assimilating MOPITT CO: The paper references some other papers that assimilated MOPITT CO. Those were related to global forecast models. There are no papers (that we know of) that have assimilated MOPITT CO in a regional model. Generally, it is not appropriate to compare the results of chemical data assimilation in a global model with that in a regional model due to the presence of lateral boundary conditions in the regional model. If for the sake of discussion, we made such a comparison, we would find that the magnitude of the improvements of the CPSR experiment compared to the MET experiment in the domain-averaged vertical profiles are comparable. See e.g., Barre et al. (2015). However, that statement is not true for the VMRR and L10VMRR experiments. The lack of an impact in those experiments is a result of our short study period and not tuning the observation errors. But as explained in the paper at P12 L 4 to P12 L9 we do not view that as a deficiency in the experimental design. We are interested in the assimilation of CPSRs. If they show an impact during a shorter study

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period but more conventional methods that do not account for redundant information or error correlations fail to show an impact, then that failure identifies deficiencies in the conventional methods. The paper's point is to compare the CPSR assimilation results with independent observations, extend the CPSR algorithm to truncated retrieval profiles, and explain why assimilating truncated profiles may give unexpected results. The paper has been revised accordingly. See P11 L3 to P11 L 7. 6. The CPSR "Take Away" Message: In addition to the computational and storage efficiencies associated with the CPSR method, the "take away" message from Mizzi et al. (2016) and this paper is that the CPSR approach is the more accurate way to assimilate retrieval profiles and that failing to account for the observation error covariance and averaging kernel linear dependencies can lead to unexpected results (as illustrated in the paper by the VMRR and L10VMRR experiments which suggest that it is necessary to: (i) tune the observation error variance and (ii) use a longer study period to get an assimilation impact. The CPSR and QOR results in this paper highlight those deficiencies with the conventional method. The paper has been revised to make these "take away" messages more clear. See P11 L3 to P11 L 7; P16 L12 to P16 L14. 7. Figure 1 Suggests that Assimilating Biased Retrievals Performs Better than Not Assimilating Biased Retrievals. Is that a Reflection on How Fig. 1 Was Prepared? Are There Other Papers that Have Assimilated Truncated Retrieval Profiles? We agree that is one interpretation of Fig. 1, and it is a limitation of using a bulk verification statistic where the skill reduction in the upper troposphere is offset/dominated by the skill improvement in most of the middle and lower troposphere. 8. Another "Take Away" Message Is that the Assimilation of Raw Retrievals Shows Little Impact. The Mention of Other Papers Showing an Impact is Necessary: This is not an intended message in this paper. The paper has been revised to clarify this point. See P11 L3 to P11 L 7. Specific Comments: 1. P1L18: The choice of 'results confirm' suggests that a computational assessment is performed and included in this paper, which is not the case. It may be a matter of rephrasing and or expanding, in the introduction, on the computational benefit indicated in Mizzi et al. (2016) in use of CPSR. Not sure of your concern here. Mizzi et al. (2016) show that

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when assimilating CPSRs there is a computational cost reduction due to the reduced number of observations to be assimilated. In this paper, the assimilation of CPSRs as applied to full retrieval profile necessarily has the same computational cost reductions as found in Mizzi et al. (2016). 2. P1L23-24: Point (ii) is not specifically shown in this paper. Agreed, the associated text is removed throughout. 3. P2L10: This line is a summary line of a result of Section 5.1. Might best be removed by referring to issues and concerns to be addressed in the paper and not the results themselves. Agreed, the referenced text is revised. 4. P2L12: "In the second part of the paper" refers to what section? As well it assumes a first part which has not been specified explicitly (this referring the P2L10 above). It is suggested to begin this sentence (if kept) instead with 'Therefore, we . . .' Revised. 5. P2L13: "The rest of this paper" would best be replaced by "This paper" considering P2L12 above and that the results section is also alluded to below. Revised. 6. P2L14-18: Sections 2, 3, 4, and 5 instead of II, III, IV and V. This applies to one or two more places in the paper. Revised. 7. P1L16: ' . . . and an extension of CPSRs' (added 'an') Revised. 8. P1L17: Might be worthwhile to refer here to the content of the two subsections in Section 5 and Section 2 We are unsure what is meant by subsections in Section 5 and Section 2. 9. P6L16 and P6L18: The two lines referring to Gaussian/non-Gaussian distributed errors seem to contradict each other somewhat. The Gaussian distribution refers to the L10VMRRs and the non-Gaussian distribution refers to the VMRRs (which have a lognormal distribution). 10. P6L20-21: Not clear on the value/meaning of this last sentence. Revised. 11. P6-7: Equation numbering not aligned (as would be from use of 'rightjustified') Revised. 12. P6L18-19: Point (ii) could refer to Eq. (2) and QOR to make even clearer the relationship between QOR and CPSR. Revised. 13. P6L20-P7L1 and P7L4-P7L8 are somewhat repetitive. Maybe part P6L20- P7L1 could be removed with some changes for an introduction to what follows. P6L20-P7L1 says the QORs were discussed in Mizzi et al. (2015) and explain why they are being included in this paper. P7L4-P7L8 explains where QORs come from and presents their definition. The text has been revised to facilitate revisions to the Introduction. 14. P8L20-24: As pointed out earlier, one

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could point to Eq. (2) and QOR for this part. Section 5: Revised. 15. P9L13: How about the MET and CO assimilation not being coupled (or being localized?) as per P5L18. Revised. 16. P9L14 and top of Figure 1: What are the units? Maybe unitless because both are referring to  $\log(\text{vmr})$ ? Do these sum up the contributions from all vertical levels? Out of curiosity, how large are these values relative to the observation and background error standard deviations? This might be useful to compare with the RMSE. The comparisons are done in retrieval space and the units are ppb. The figures have been revised to include units. Yes, the metrics are computed by summing the contributions from all vertical levels. When tuning the assimilation system, we balanced the RMSE and the total spread, so they are comparable. 17. P10L1: Due only to discarding the observation error cross-covariances and not also due (at least partly) in removing the a priori effect? Just wondering? A comparison to other papers also assimilating MOPITT CO might be pertinent here. That is correct. The a priori effect is removed for both experiments. We are not sure what comparisons you have in mind. The VMRR and L10VMRR experiments are similar to what has been done in other papers, but our study period is shorter which partly explains why they get an assimilation impact and we do not. But as explained earlier, the CPSR and QOR results in this paper show that need for a longer study period and tuned observation errors highlights the deficiencies with conventional methods. In other applications, we have run experiments similar to the VMRR and L10VMRR experiments and found significant improvements from assimilation of MOPITT retrievals. Those results are not shown or discussed because they are not relevant to the goals of this paper. 18. P10L4 and Figure 1: Would be better to split Fig. 1 in Fig. 1 (for upper panels) and Fig. 2 (for lower panels) Revised. 19. P10L4: Use of arrows might not be best. Revised. 20. P10L4-P10L23: Would some or much of this have been stated in Mizzi et al. (2016)? If so, might be best to reduce the text. This was not discussed in Mizzi et al. (2016) because there we had found that QOR and CPSR gave similar results. See response to Reviewer 1 at Specific Comment 10. 21. P10L25-P11L8: There is mention of the increased bias with MOZAIC from CPSR and QOR, this supported also by IASI CO in

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Fig. 6 and related to the MOPITT bias (also displayed by compared MOPITT and IASI in Fig. 6 – if IASI has comparatively no or less bias?) IASI CO retrievals are not known to have a systematic bias similar to that discussed for MOPITT CO retrievals in the text. The MOZAIC in situ profile observations are collected at or near urban airports. As such they are thought to be representative of a polluted urban environment. That issue is discussed in the text at P13 L12 to P13 L13 and the reason why we did not plot the lower levels of the MOZAIC profiles in Fig. 2. 22. P11L13-15: Any CO assimilation papers showing or not some impact near/at the surface? Revised. 23. P11L12: 'little or no change' instead of 'little or no improvement' as whether or not there is any improvement is not shown here. Revised. 24. P11L19: The Fig. 2 blow-up histograms are not really needed. It's up to the authors. Might it be best to split the histogram and the lower panels into two separate figures? We felt that the blow up of the histograms helped to reveal the details of the distribution that were relevant to the discussion in lines P14 L1 to P14 L14. We have not separated the histogram and singular profile figures because we want to associate the singular vectors and the DOFS histograms and we want to disassociate the singular vectors and the transformed averaging kernel profiles. 25. P12L1-4 (and beyond): Could differences in the vertical of the CO background (forecast) error variances/covariances also be a contributing factor to some degree, this depending on the assimilation setup? Having some sense of the variation in the vertical of error variances (and error correlations) might be beneficial. Would differences in background error covariances in different papers contribute to explaining differences in results? These figures/results do not depend on the background error covariance. They are an analysis of the terrestrial MOPITT CO profiles (the observations) assimilated during the study period. We agree that the vertical distribution of the observation error variance impacts these results. That aspect is addressed/discussed in the Fig. 3. Differences in the reported background error covariance do not explain these differences because they are independent of the background fields. 26. P12L12: Was any scaling really needed? P12L7-22 (and beyond): See General Comments on the display of the singular vectors. P12-P13: I only skimmed the text for the re-

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view on these pages. P13L17-18: One might question the application of the scaling in the first place. Due to the symmetry of the SVD, the singular vectors produced by the SVD subroutine and -1.0 times those singular vectors are both valid solutions to the SVD. The vertical structure of the singular vectors for each mode depends in part on the vertical distribution of the MOPITT instrument sensitivity. From the literature, we know that MOPITT has sensitivity to CO in the upper and lower troposphere. For ease of interpretation we chose a +/- 1.0 scaling (for the first and second rows as discussed in the paper) that make the singular vector vertical sensitivity consistent with that published in the literature. Then we applied that scaling consistently throughout the discussion in this section. 27. P13L18: e.g. '. . . that, when . . . is considered, the . . .' (while this is likely somewhat subjective, adding some commas here and or similarly elsewhere in the paper might be considered) Revised. 28. P13L20: '. . . and the first . . .' (added 'the') Revised. 29. P13L22: Might the validity of this assertion depend on the singular values? For the diagonalization transform, the modes are ordered by decreasing singular value. The singular value is the compressed observation error variance after accounting for the covariance. When we project the compressed averaging kernel onto the singular vectors of the compressed retrieval error covariance matrix as scaled by the square root of corresponding singular value, the result shows the vertical sensitivity of the transformed averaging kernel removing linear dependences and after accounting for the: (i) error covariance terms, (ii) the magnitude of the observation error variance, and (iii) the vertical structure of the observation error covariance. When the modes are ordered by decreasing singular value, the referenced statement is valid. 30. P14L2-3: Please indicate actual references and elaborate on results where applicable. Revised to add references. 31. P14L3: What is meant by 'do not adjust for the averaging kernel linear dependencies or for the observation error covariance"s"' [Might the latter be in reference to not including error correlations (cross-covariances)?] The CPSR algorithm perform two tasks: compression and diagonalization. The compression task compresses the averaging kernel by removing linear dependencies. The diagonalization task diagonalizes the compressed

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covariance matrix by rotating the compressed quasi-optimal retrieval equation into a coordinate space that maximizes the compressed variance (thereby accounting for the covariance). Since no other chemical data assimilation researcher is compressing the averaging kernel and rotating the compressed observation error covariance matrix, they are not adjusting for the averaging kernel linear dependencies or for the observation error covariance. This is what is meant. 32. P14L11-13 (and remainder of the paragraph): While there is some level of consistency in the coastal regions, it is not that evident that one could say that the analysis and forecasts are 'generally consistent' with the observation. Maybe some re-phrasing would be needed. A quantitative evaluation might help. The text has been revised to make the comparisons and conclusion more clear. 33. P14L15-16: Has (i) been looked at to some degree? The text has been revised and this has been removed. 34. P14L16:17: Has (ii) been verified? No, because there is no way to determine whether the reported emission are too low. 35. P14L17: The changes in the analyses seem rather weak in the central U.S. or thereabouts in comparison what is needed to increase the analysis to levels fairly close to what is seen in Fig. 5. Might a quantitative evaluation help? The areal coverage of IASI is greater than that of MOPITT. In the central US where there are MOPITT observations, there is increased CO so that the magnitudes are in better agreement with IASI. Where there are no MOPITT observations, there is decreased CO so that the magnitudes are in worse agreement with IASI. 36. P14L20: Does this refer to the central U.S. or is an overall assertion? It is not so clear from the figures if for the central U.S.. Either way, a quantitative evaluation (by regions maybe) might be more meaningful to justify this assertion (and those above). This discussion has been revised to make it slightly more quantitative. We are reluctant to make it too quantitative because the point of Figs. 4, 5, 8, and 9 is to show examples of: (i) the types of horizontal impacts one gets from chemical data assimilation, and (ii) how those impacts correspond to the observations. The text has been revised to make this more clear. 37. P14L25: Might it be worth to mention/discuss the level of similarity and differences between 'SS' and 'RS' profiles? The figures have been revised to remove the state space profiles. 38. P15L3-6: 'for

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pressures less than about 500 hPa, the MOPITT CO assimilation with CPSR draws the forecast and analysis further away from IASI while the opposite occurs for larger pressures.’ Could refer to the comparison to MOZAIC in Fig. 1 to support the comparison with IASI in the upper levels. For pressures less than 250 hPa, the CPSR experiment draws the forecast and analysis further away from IASI in Fig. 6. The text has been revised accordingly. 39. P15L11-21: An alternative would be for a version of this ‘summary’ to instead be in the ‘Summary and Conclusions’ section. It’s up to the authors. We decided to leave this here as an intermediate conclusion. 40. P15L14: It is not really that the ‘phase space’ observations error variances is reduced as oppose to the transformation allowing to account for the otherwise neglected ‘re-trieval space’ error correlations. The CPSR experiment has better verification scores compared to VMRR and L10VMRR because the: (i) correlations are greater, and/or (ii) transformed observation errors are smaller. We think it is primarily due to smaller observation errors. They are smaller due to the compression step in the CPSR transform. They cannot be smaller due to the diagonalization step because that is a variance maximizing rotation. If the compression step had no filtering effect on the errors, then the variance resulting from the diagonalization step would no smaller than that from the compression step. 41. P15L16-17: Part of (ii) is actually a repetition of (i). Some change in the sentence is needed. Revised. 42. P15L17: As part of (ii), has the statement ‘linearly dependent portion of the transformed retrievals do not . . .’ (repeated earlier as well) been verified, noting that background error covariances (and its non-zero error correlation coefficients) contribute to determining the distribution of information for strongly overlapping averaging kernels (likely requiring more computational effort though). Any other references for his part (e.g. Migliorini, 2008 and or 2012 or even Mizzi 2016)? If so, they should also be indicated earlier on in this paper. The statement has been verified because the QOR algorithm is now part of the CPSR algorithm. So, the same computer code is used for the QOR results as is used for the QOR part of the CPSR results. Thus, the similarity of results for the CPSR and QOR experiments implies that the linearly-dependent part of QOR profile remaining after the transformation does not

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contribute to the analysis increment. No other researchers (that we know about) have found this result. 43. P15L21-23: Have other assimilation studies shown this as well – that the resulting CO analyses and forecasts in the upper levels would be biased. This result would be expected considering the literature on the MOPITT CO data – assuming IASI and also MOZAIC CO is less biased. Might be good to indicate that this was not entirely un expected. The only prior study that made this observation (of which we are aware) was cited previously Barre et al. (2016). Note: Barre et al. (2016) did not present results from assimilating the biased retrievals. They discarded them before performing their forecast/assimilation experiments. 44. P15L23. This also applies to the comparison with MOZAIC CO. Same response as in 42. 45. P16L4: Section V.C to be changed. Corrected. 46. P16L8: ‘. . .accounts for the error correlations of the observation error covariance matrix.’ We agree that the error covariance and error correlations are related. In the paper and in the CPSR algorithm we are accounting for the error covariance. We have used that terminology consistently throughout the paper. So, we have not made this change. 47. P16L11: e.g. ‘that, in the upper troposphere, the’ (commas) P16L17: e.g. ‘troposphere, there’ Revised. 48. P16L18: ‘A comparison with’ Revised. 49. P16L21-22: Could be re-phrased. Revised. 50. P17L1: Remove ‘However’, i.e., ‘The forecast . . .’ Revised. 51. P17L3: ‘... United States similar to, though weaker than, the CPSR experiment’ or something similar Revised. 52. P17L5-7: e.g., ‘The upper tropospheric impacts of Fig. 9 show even smaller changes for the CPSR-RJ3 experiment except for the reductions over the southeastern United States. The CPSR-RJ3 experiment therefore further demonstrates, in addition to Fig. 7, the reduction of bias in the upper troposphere through the removal of the biased observation profile elements, this though at the expense of reduced improvements in the lower troposphere.’ Revised. 53. P17L9-11; This should explicitly refer to the upper right-hand side panel with the comparison to IASI CO. This refers to both the MOPITT and IASI comparisons. The text has been revised. 54. P17L11: The improvement is rather small though as compared to CPSR (and QOR) in Fig. 1. This needs to be indicated. Is this related to how this diagnostic is generated, e.g. maybe because of

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a dominance of the lower tropospheric RMSE contributions (as compared to the upper layers)? Yes, this is related to how the metric in Fig. 1 is generated. The text has been revised to indicate that the improvement is slight. 55. P18L5-6: This should refer to the levels with pressures below about 500 hPa. 'Significantly' seems to be an exaggeration based on the curve. I suggest removing 'significantly'. \ Revised. 56. P18L7-13 and Figure 10 (with Table 2): The bottom row of Fig. 10 (even in combination to Table 2) suggests that the 'reject middle three' may have least impact in assimilation. This would be contrary to just looking at the traces in Table 2 which, based on the earlier statement, indicate the 'reject bottom three' provide the least amount of info. Am I missing something? Any discussion or comments. Table 3 (the former Table 2) indicates the amount of variability explained (or the amount of information remaining) after discarding (not assimilating) the referenced levels. For the trace, this table indicates that: (i) the Full Profile contains the most amount of information, (ii) Mode 1 explains  $.9639/1.452=66\%$  of the variability, (iii) Mode 2 explains 33%, and (iv) Mode 3 explains 1%. With that interpretation, rejecting the top three levels has the greatest reduction in the total information and rejecting the bottom three levels has the least reduction. The bottom row of Fig. 11 (the former Fig. 10) shows the vertical sensitivities for each mode. For the Full Profile (column (a)) most of the maximum sensitivity is near 250 hPa and the magnitude is  $\sim 6$ . For rejection of the top three levels (column (b)), the maximum sensitivity is between 350 hPa and 700 hPa, and the magnitude is  $\sim 2$ . It is those changes in the total information and vertical sensitivity that explain the results in Fig. 7 (the former Fig. 6). 57. P19L9: Removing 'likely' seems appropriate as it seems pretty certain. Revised. 58. P19L16: Instead of 'magnitude of the observation errors' is it more the omission of the 'observation error correlations in the assimilation' in comparison the CPSR and QOR effects? We do not agree with this interpretation. See responses to Reviewer I, General Comment 9 and Reviewer II, Specific Comment 40. 59. P19L19: 'Truncated the observation errors' may not be the correct wording considering the above. Please see response to Reviewer II, Specific Comment 57. 60. P19L22-23: Applies also to IASI CO. Even better would be to instead mention

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MOZAIC CO at P20L5. Revised. 61. P20L2: 'because, by accounting for . . . error correlations,' We account for observation error covariance. While it is true that these indicate observation error correlations, we are explicitly accounting for the covariance (not the correlations). 62. P20L21-P21L3: Different contradictory statements in this sentence related to the impact at the surface. Also, one could mention the approximate proportion of cases where surface impact may occur. One might also consider the background error variances in also contributing to the level of impact at near the surface (on top of the averaging kernels themselves (and obs error covariances)) We were unable to identify the contradictory statements in the lines listed which are in the Code and Data Availability section. 63. P20L10: 'confirming the applicability of the CPSR . . .' Revised. 64. P20L13: 'Excluding the assimilation of some elements of the observation profiles can ...' Revised. 65. P20L16: 'to address the reduced impact from not assimilating retrieval profile levels' ('reduced' instead of 'remote' and . . .) Revised. 66. Table 1: Might be better to follow the form of Table 1 in Mizzi et al. (2016) Revised. 67. Figures: Font sizes for panels with y-axis as pressure are on the edge of being too small or are too small. Please check. Revised. 68. Figures 6 and 7: For clarity, might be best to drop the 'SS' results (at least for Fig. 7 if not both). That is unless the one intends to mention and discuss in the text, for Fig. 6 for example, the level of similarity and differences between 'SS' and 'RS'. Revised. 69. Figure 10: 'except that this figure' (added 'that') Revised. 70. Figure 7 (lower panels): Unless this is a visual clarity issue, it seems that the Met EX RS results near the surface differ between the CPSR panels and the L10VMRR panels, while they would be expected to be the same. Please check. The Met EX RS results are not the same for the L10VMRR and the CPSR experiments. For the L10VMRR experiment the retrieval equation (the retrieval equation is used to map the state space CO profile into retrieval space) is the full equation. For the CPSRRJ3 experiment, the mapping is based on the truncated retrieval equation (the equation after discarding the biased elements, rows, and columns as appropriate).

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