

## F. SAITO (Referee #2)

*This paper describes the numerical ice-sheet model GRISLI ver 2.0, in particular, with the application for the Antarctic ice sheet simulation. I think this paper is fairly well written with some exception below, and can be accepted with minor revision.*

*One thing better to include is technical (numerical) procedures and properties adopted in the model. Since the source code is not opened to public, information how to solve the model equation is useful for those who has not contact with the model. For example, how to solve the Eq.(2), (thickness evolution)? Explicit, implicit or others? How to solve the linear equations, direct, alternate-direction, conjugate gradient, or other method? How to solve the non-linear ice-shelf equation (Eq.12)? Linearize them? Or the velocity-dependent viscosity ( $\bar{\eta}$ ) at the previous time step is used? How to determine the convergence of the solutions where the iterative solver is involved? Such details are all necessary to evaluate the numerical accuracy of the model, if they want. I suppose they are more or less the same as the original version of the model (Ritz et al. 1997,2001), but repetition (or at least citation of the old papers) are needed for completeness of the model description.*

This is perfectly in line with Referee #3 general comment and we acknowledge that the initial version of the paper provided only few technical information. In the revised version, we now explicitly show the staggered grid in Fig. 2. We have also added two additional figures in the supplementary material showing: i) the boundary conditions when extending the front to the edges of the geographical domain and ii) the matrix used to solve the elliptic equation. We have also considerably expanded Sec. 2.3, adding more information on the numerical resolution of the equations:

“The mass balance equation is solved as an advection-only equation with an upwind scheme in space and a semi-implicit scheme in time (velocities at the previous time step are used). The numerical resolution is performed with a point-relaxation method with a variable time step. The value of this time step is chosen to ensure that the matrix becomes strongly diagonal dominant to achieve convergence of the point-relaxation method. The criteria is thus a threshold that is inversely proportional to the fastest velocity on the whole grid. Note that this smaller time step is solely used for the mass conservation equation and subsequent variables (e.g. surface slopes, SIA velocity) while the rest of the model uses a main time step, typically ranging from 0.5 to 5 years depending on the horizontal resolution.

To solve it, the ice shelves/ice streams equation (Eq. 14) is linearised. The viscosity is computed using an iterative method starting from the viscosity calculated from strain rates from the previous time step. As this equation is the most expensive part of the model, the iteration mode is not always used depending on the type of experiment (for instance not crucial when the objective is to reach the steady state). In this case the viscosity of the previous time step is used. The linear system is solved with a direct method (Gaussian elimination, sgbsv in the Lapack library ([www.netlib.org/lapack](http://www.netlib.org/lapack))).

For the temperature equation (Eq. 19), we solved a 1D advection-diffusion equation for each model grid point. The resolution is performed with an upwind semi-implicit scheme (vertical velocity and heat production at the previous time step is used). The ice thermal conductivity is computed as the geometric mean of the two neighbouring conductivities (Patankar, 1980). Because the horizontal diffusion is neglected, the only horizontal terms concern horizontal advection and are computed

with an upwind explicit scheme. The heat production is computed at the velocity (staggered) grid points and is then summed up to the temperature (centred) grid points.”

*Another thing better to rewrite is the lateral boundary condition of the ice shelf, which is still unclear to me. As described in Sec 2.3, the ice shelf is extended towards the edges of the model domain. As far as I understand, to remove row  $i$  from the matrix corresponds to set horizontal velocity as zero at the front.*

No, it does not mean that horizontal velocity at the front is zero, it means that it has an undefined value.

*I am not sure this is what the authors expect. I suggest to rewrite the second paragraph of the Sec.2.3 to clarify how to formulate the matrix in the model.*

We now show the matrix in the supplementary material and we have been more specific in the revised version, which now reads:

“The resolution of the elliptic system (Eq. 14) is the most expensive part of the model. This is further amplified by the way we prescribe boundary conditions. As in Ritz et al. (2001), the ice shelf region is artificially extended towards the edges of the geographical domain. This artificial extension does not have any consequence on ice shelf velocity since added grid points (that we call “ghost” nodes) are prescribed with a negligible ice viscosity (1500 P a s). The front is then parallel to either  $x$  or  $y$  (Ritz et al. 2001) and thus the boundary condition there is easy to implement (see also Fig. S1 in the supplement). The boundary condition at the real ice shelf front is implicitly done by solving (Eq. 14). However this method increases substantially the size of the linear system solved in (Eq. 14). To go around, a simple reduction method is implemented. Eq. 14 can be written as  $\tilde{\mathbf{A}} \tilde{\mathbf{u}} = \mathbf{B}$  where  $\tilde{\mathbf{u}}$  is a vector alternating  $u_x$  and  $u_y$  components for all the velocity grid points,  $\tilde{\mathbf{A}}$  is a band matrix (very sparse) and  $\mathbf{B}$  is a vector corresponding to the right hand terms in Eq. 14. Every line of  $\tilde{\mathbf{A}}$  and  $\mathbf{B}$  are scaled so that the diagonal terms of  $\tilde{\mathbf{A}}$  are equal to 1. If, for a given velocity node, all the non diagonal terms of the column are very small compared to 1, this means that this node is actually not used by any other velocity node and this line of the matrix can be removed. The threshold to neglect nodes is related to the value of the integrated viscosity of “ghost” nodes. In practice, given its size, the matrix  $\tilde{\mathbf{A}}$  is not actually fully constructed, only the non zero sub/sur diagonals are. An illustration of the matrix is shown in Fig. S2 in the supplement. ”

*In addition, I definitely agree to the specific comments 3 and 4 by the referee #1. The authors should clarify the formulation and the procedures to compute basal hydrology and the back-force coefficient.*

The basal hydrology is now presented in much more details, including the prognostic equation for the hydraulic head,  $h_w$ , from which the effective pressure is computed. We have also clarify in the revised version how the back-force is calculated (please see also our response to the 4<sup>th</sup> point of referee #1).

*Some minor points (PmLn corresponds to the line number  $n$  in page  $m$ )*

*Units: use `\unit{}` macro.*

Thanks for the suggestion, we have done so.

*P1L6: 'or Tsai et al....' may be better?*

Changed.

*P2L16: 'right' might not be a right word for this context. How about 'practical tool for...'*

This has been replaced by "the most suitable tool for".

*P3L19 Eq(1): Divergence, not Gradient (need dot).*

We have followed this convention consistently in the text.

*P3L22 Eq(2) and after:  $\bar{u}_x$  is better than  $\bar{u}_x$ .*

Changed.

*P3L26 Eq(3) and similar array equations: Use  $\displaystyle$ .*

We have followed your suggestion.

*P4L1 or around: Need definition like  $\sigma_i = \tau_{ii}$ , otherwise the paper misses the equation for longitudinal stress components, since Eqs 5, 6, 8, 9 are described with  $\tau_{ij}$ .*

We agree, this has been added.

*P4L11 Eq(7) or (9): The enhancement factor should be inserted. Otherwise  $E_{SIA}$  in P12L1 is confusing.*

We have now added the equation for  $B_{AT}$  in which the enhancement factor appears (renamed as  $S_f$  for consistency with Ritz et al., 1997 and to avoid confusion with the activation energy  $E_a$ ).

*P4L16 Eq(8): Need range of  $i,j$ .*

Information added ( $i,j=\{x,y,z\}$ ).

*P4L25 Eq(9): No explicit formulation of  $B_{AT1}$  and  $B_{AT3}$ . Are they documented in Dumas (2002)?*

We have added the equation for  $B_{AT}$  and expanded the description of how they are calculated for the Glen viscosity and the linear viscosity. We hope that the new version of the manuscript contains all the necessary information.

*P5L10 Eq(10):  $i$  in  $\rho_i$  conflicts with row  $i$ .*

The density of ice is simply referred as  $\rho$  consistently in the whole manuscript.

*P5L9: (for  $i = x,y$ ) not (for  $i=x,y,z$ ). When  $i=z$  in Eq(11), the coefficient 2 must disappear.*

Thanks for noticing, this has been corrected now. Vertical velocity is mentioned later in the manuscript.

*P5L10 Eq(11): no definition of B. B may conflict with B\_{AT}, which is better to avoid.*

We have added the definition of B (bedrock elevation). We stick to B and B\_{AT} for consistency with Ritz et al. (2001).

*P5L21: S is already defined in L9.*

Thanks for noticing, the definition here has been removed.

*P5L23: (see also 2.3 numerical feature)*

Changed in the text to:

“[...] see also Sec. 2.3 on the numerical features”

*P5L26: The basal drag is very small but not necessarily zero. All we can do is to neglect it.*

We agree, we now simply say that we neglect it.

*P7L12 Eq(16): No definition of H\_g. Typo of H\_{gl}?*

Thanks for noticing, we meant H\_{gl}.

*P8 Sec2.1.6. Need to mention how to compute the vertical velocity. I suppose vertical velocity is not directly computed as Ritz et al. (1997).*

Vertical velocity is indeed computed as in Ritz et al. (1997). We added this information in the text : “ $u_z$  is the vertical velocity, computed as in Ritz et al. (1997) (wt in Eq. 14).” This velocity is used for 3D advection (temperature and tracers).

We have also added more information on the link between temperature and viscosity:

“The viscosity for the velocity grid points is the horizontal average of the viscosity on the centred grid and not the viscosity computed from the horizontal average of the temperature. This is preferable for regions with mixed frozen and temperate basal conditions.”

*P8L10: ‘zero’ requires the unit. I prefer to write as ‘the melting point’.*

True, it has been changed.

*P8L25 Eq(22) Write \exp (backslash before exp) following LaTeX convention.*

Done.

*P9L27, Better to cite Le Meur and Huybrechts after ELRA sentence also.*

Done.

*P10L26: Better to avoid to use A and B for matrix and vector, which conflict with the rate factor or bedrock elevation.*

Most of the letters in the alphabet relate to a variable used at some point in the model. We think that A and B, being the first letters, are more generic. We have nonetheless added a tilde (~) on top of them for a better distinction with respect to the rate factor and the bedrock elevation.

*Table 1: Unit of the acceleration should be m/s. Really same values for ice and mantle thermal conductivity?*

Thanks for noticing this. We changed the unit of the acceleration and we corrected the ice thermal conductivity.

*Figure 1: If z at the ice bottom is always zero as the figure, you need to reformulate all the governing equations using z.*

Sorry for this, z should have been B at the ice bottom and  $S=B+H$  at the top. The figure has been updated.