Response to referee 1

Interactive comment on "The "ABC model" (Vn 1.0): a non-hydrostatic toy model for use in convective-scale data assimilation investigations" by Ruth Elizabeth Petrie et al.

Anonymous Referee #1

We would like to thank referee 1 for his/her comments. The referee's comments that require attention to the paper are reproduced below preceded with "**Referee comment**", the authors' responses are preceded with "**Authors' response**", and our actions are preceded with "**Authors' changes**". Changes are indicated in blue in the paper (the new tables have only the caption in blue, and the new appendices have only the titles in blue).

- **Referee comment**: What I am missing here is a discussion that would relate findings from the Section 5 to findings in other Sections (2, 3, and in particular 4). Some of the results presented in the Section 5 are somewhat contra-intuitive and should be more elaborated. The authors describes mainly WHAT happens in terms of the selected diagnostic quantities when the parameters are changing. More insight is needed in WHY this happens. . .
  - **Authors' response**: Our purpose of the earlier sections of the paper was to understand how the parameters relate to different aspects of the system, and in order to choose suitable A, B and C parameters for the numerical work, as well as a suitable time step. Despite efforts to understand why the apparent degree of imbalance changes in the way it does as A is modified, we have not been able to explain this (see penultimate change below though). We have increased the cross referencing to earlier sections.
  - **Authors' changes**
    - Some more description is now given in Sect. 4.7 regarding the choice of time step.
    - There are also references back to previous sections from Sect. 5 as follows.
    - In Sects. 5.3.1 (discussion of the reference parameters), 5.3.2 (changes of A), and 5.3.3 (changes of B) we now compare the gravity wave speeds found from the linear analsysis in Sect. 4 with the propagation speeds found in the non-linear model runs. This involved the inclusion of a new table showing that these speeds are consistent (Tab. 3).
    - In Sect. 5.3.3 (discussion of changes to B), we do refer back to the scale analysis equations in Sect. 2.2.
    - End of Sect. 5.3.2 (discussion of changes to A) and conclusions section, we comment on the possible (lack of) validity of using the balance diagnostics (48) to compare the degree of imbalance between different parameter values.
    - Other related specific points are responded to below.
- **Referee comment**: For example, in Section 4 the authors motivate the choice of the time step (1s, with a sub-time step 0.5s) in the split explicit forward-backward scheme, which is a first order scheme, by the estimate of the highest frequency of the acoustic wave (subsection 4.6) corresponding to the reference parameters (A=0.02 s-1, B = 0.01, C=10000 m2 s-2). In section 4 the authors also analyse how the values of parameters A,B and C influence speed of gravity and acoustic waves (subsection 4.5). In Section 5 the authors conduct 3 pairs of the additional experiments with increased/decreased values of parameters A, B and C . It is not totally clear from the description of these experiments if some adjustments to the time step of integration were done due to change in the speed of waves propagation. The following questions emerge :

- **Referee comment**: 1) The choice of the reference parameters is such that the speed of the gravity and the acoustic waves becomes comparable. The change in parameters values will change the relative ratio between the speed acoustic and gravity waves. For example in A+ experiment acoustic waves will be slower than the gravity waves...What implication will this have on the precision of numerical solution?
  - **Authors' response**: This is correct and we had not completely accounted for this.
  - **Authors' changes**: We have found the maximum wave frequency over all of the possible parameter values and have chosen a new suitable time step for the model which will be suitable for all runs made ($\Delta t = 0.1$s now, instead of $\Delta t = 1$s in the original). Please see revised Sect. 4.7.
- **Referee comment**: 2) Obviously the dissipation of energy for the energy conserving system is due to the lack of precision of the numerical solution. What is the main cause of the numerical error?
  - **Authors' response**: Shortening the time step improves the conservation of energy only marginally, but having a smaller grid length improves conservation significantly.
  - **Authors' changes**: This is now discussed in Sects. 5.3.2, 5.3.3, Fig. 7, and briefly in the conclusions.
- **Referee comment**: 3) B+ experiment results in the largest error in the conservation of energy. This is the experiment with the largest non-linear term (advection) and with the highest horizontal speed for acoustic wave. B+ experiment results in the increased vertical transport. To what extend increased vertical transport is related to the numerical noise?
  - **Authors' response**: There is indeed an association with vertical transport and imprecision. This raises the question whether changes in (im)precision will result in unreliable balance results (and hence possibly explain the counter-intuitive results). We do not believe that this is the case as the integrations at higher resolution result in dramatically less loss of energy, but the balance results are not qualitatively changed.
  - **Authors' changes**: This is discussed in the last part of Sects. 5.3.2, and 5.3.3.
- **Referee comment**: 4) In all experiments higher values of A+, B+ or C+ (and higher speed for wave propagation) results in a more geostrophically imbalanced field (figures 6,8 and 9, panel d) to the right), while decreased values of parameters A-, B- and C- (and lower speed for wave propagation) all results in somewhat lower geostrophical imbalance that grows slower in time (figures 6, 8 and 9, panel d) to the left). At the same time all three parameters control different features of the flow (static stability, degree of non-linearity and compressibility). What mechanism lies behind this effect? Can a too large time step produce a less balanced field?
  - **Authors' response**: These results do indeed seem counter-intuitive, and we have still not been able to explain them, despite an in depth investigation.
  - **Authors' changes**: We have considered that the unexplained balance results may be due to imprecision (as above – either due to inability to support the wave motions in some of the runs, or the differing degrees of vertical motion – see Sect. 5.3.2). We believe tentatively that the results are not due to imprecision (by doing experiments with different grid-lengths), although we are as yet unable to explain the apparently anomalous effect of A on the degree of imbalance – again Sect. 5.3.2. As stated above a possible cause of these 'unexplained' results is that the balance diagnostics (48) may not be suitable for comparison between

different parameter sets. See end of Sect. 5.3.2 (discussion of changes to A) and conclusions section. We do think though that the effect of B though can be explained – Sect. 5.3.3.

- **Referee comment**: Section1. Introduction. – p2. l10. Sentence "The DA scheme that combines the observed and the background data should provide an analysis which is approximately consistent with the observations and the model." An expression "approximately consistent" is misleading in this context. Please reformulate the statement.
  - **Authors' response**: Thank you.
  - **Authors' changes**: This statement has been reformulated (Sect. 1).
- **Referee comment**: p2. l30 . Sentence "These methods though suffer from noise in the sampled error covariance matrix and so rely on fixes such as localisation, which is known to destroy balances when they are relevant". I think it is inappropriate to call localisation approach as "fixes" because this is a mathematically sound method that increase rank of the resulting matrix, even though it indeed might destroy balances. Please reformulate the statement or remove ". . . fixes such as ..."
  - **Authors' response**: Thank you.
  - **Authors' changes**: This statement has been reformulated (Sect. 1).
- **Referee comment**: p3. l28 . Sentence "These simplifications permit a large-scale balanced flows and sporadic small-scale non-hydrostatic flows (i.e. convection) to coexist within the frame- work of a simplified and practical model." I do not think that it is appropriate to relate all sporadic small-scale non-hydrostatic flows to convection. Please moderate the statement. One of the biggest problems when modelling on high-resolution is to distinguish between a convective motion and a numerical noise that often happen on similar scales. The toy model environment such as the proposed one could be a very useful framework to study the propagation of the error on mesoscales that comes different sources (the error in the initial conditions, the model error due to deficiencies in the numerical scheme and the interaction of these two sources)
  - **Authors' response**: Yes, we agree that not all small-scale motion is convection.
  - **Authors' changes**: Have replaced "i.e. convection" with "including convection" (Sect. 1).
- **Referee comment**: Section 2. Subsection 2.1.2 "The "ABC model" modifications" – p6 l9 . "Linearizing Eq. (8f) about the basic state . . .." Please explain explicitly the procedure of linearizing equation around the basic state. Some readers might not be familiar with this approach
  - **Authors' response**: Thank you.
  - **Authors' changes**: Appendix A has been added to describe linearisation, with an example of linearising (8f) to give (9). The appendix is referenced from the main text in Sect. 2.1.2 (after (9)).
- **Referee comment**: Subsection 2.3.1 "Conservation of mass" – p8 l21 Please explain or provide the references to what "the divergence theorem" (known more as Gauss's theorem or Gauss- Ostrogradsky theorem in calculus) means
  - **Authors' response**: Yes, thank you.
  - **Authors' changes**: Appendix B has been added to show how the divergence theorem is used to show conservation. The appendix is referenced from the main text in Sect. 2.3.1. The divergence theorem is explained in Appendix B.

- **Referee comment**: Subsection 2.3.6 "Total combined energy and its conservation" – p10 l5. Here the notation "the divergence theorem" is used again without any definition or explanation...
  - **Authors' response**: Thank you.
  - **Authors' changes**: This is included in Appendix B.
- **Referee comment**: Section 4. Subsection 4.1"Linearization" – p13 l19-21 . "The non-linear model equations are linearized about the reference state and a state of rest. It is convenient to write the model equations in terms of velocity potential and streamfunction. The Helmholtz theorem gives: . . .." Please explain do what "the reference state" and "a state at rest" mean, what does "the linearization of the equation" mean and why this procedure is performed; Please provide references or explain "the Helmholtz theorem"; To obtain equations (39) from equations (15) the flow was first split in divergent and rotational parts (15 a,b $\rightarrow$ 39 a,b) and then expressed in terms of velocity potential and streamfunction. Equations 15 a,b,c,e were linearized around a state at rest and equation 15 d was linearised around the reference state
  - **Authors' response**: Thank you.
  - **Authors' changes**: The "reference state" text has been removed, and the meaning of "a state of rest", "linearisation" and the reason why linearisation is done is now explained in appendix A (and referenced from the main text at the start of Sect. 4.1). The Helmholtz theorem is now explained and referenced in Sect. 4.1.
- **Referee comment**: p 14 Please explain more clearly in words what procedure is performed here and what is the meaning of the analysis in spectral space. For reader it might be difficult to follow derivations. Please explain the decomposition of the flow onto orthogonal modes.
  - **Authors' response**: Thank you.
  - **Authors' changes**: Some extra text has been added at the start of Sect. 4.
- **Referee comment**: Subsection 4.6 – p17 l 18. Please explain what is the meaning of Courant number and it is "sufficiently small"
  - **Authors' response**: Thank you.
  - **Authors' changes**: Some extra text at the end of Sect. 4.7 has been added, including mention of the CFL condition.
- **Referee comment**: Subsection 5.2. "Intermittent convection-like behaviour" – p20 l3 . "An obvious indication of presence of convection is vertical motion and . . . ". I think it is important to refer here to Eq 44 indicating that for pure linear systems the balance part of the flow does not have vertical wind component.
  - **Authors' response**: Thank you.
  - **Authors' changes**: This has been done (Sect. 5.2). Note that Eq. (44) is now Eq. (45).
- **Referee comment**: Subsection 5.3 "Systematic exploration of model behaviour over parameter space" Figures 6, 8 and 9, panel d) The plots are very difficult to read, another line style/colours/symbols need to be used. Legends should be moved away because they destroy information on the plots.
  - **Authors' response**: Agreed.

- **Authors' changes**: The text on these figures has been made larger, the figures have been made larger, and the balance diagnostic plots have been improved (the legends have been moved away from the lines, a colour scheme has been introduced, and a uniform scale has been used for all geostrophic and (separately) for all hydrostatic imbalances.

- **Referee comment**: Figure 7. It is difficult to read the plot. Another style/colours/symbols need to be used
  - **Authors' response**: Agreed.
  - **Authors' changes**: Change made (in addition to an added energy vs time plot for runs of the model with increased resolution).

- **Referee comment**: Technical Comments – p11 l16 . Should be "$\delta t = \Delta t/n$" instead of "$\delta t = \delta t/n$"
  - **Authors' response**: Agreed.
  - **Authors' changes**: Change made (Sect. 3.3.1).

- **Referee comment**: p15 l22. Should be "σ_{a}" instead of "σ_{g}"
  - **Authors' response**: Agreed.
  - **Authors' changes**: Change made (Sect. 4.5).

- **Referee comment**: p15 l24 . Should be "small-scale" instead of "scall-scale"
  - **Authors' response**: Agreed.
  - **Authors' changes**: Change made (Sect. 4.5).

Response to referee 2

Interactive comment on "The "ABC model" (Vn 1.0): a non-hydrostatic toy model for use in convective-scale data assimilation investigations" by Ruth Elizabeth Petrie et al.

Anonymous Referee #2

We would like to thank referee 2 for his/her comments. The referee's comments that require attention to the paper are reproduced below preceded with "**Referee comment**", the authors' responses are preceded with "**Authors' response**", and our actions are preceded with "**Authors' changes**". Changes are indicated in blue in the paper (the new tables have only the caption in blue, and the new appendices have only the titles in blue).

- **Referee comment**: Having read the other review, I strongly agree with the request for a further analysis of the sample integrations in section 5, and their relation with the findings of the previous sections.
  - **Authors' response and changes**: please see report for reviewer 1.
- **Referee comment**: 1. Understandably, there are a lot of symbols. I believe it would be useful to have a list of symbols as an appendix. While reading the paper, I had to go back sometimes between sections to know the differences in variables, e.g. p vs p_0 vs p_{00}, or when are the variables calligraphic, when do they have a star, etc.
  - **Authors' response**: Thank you, agreed.
  - **Authors' changes**: A table has been added to the end of the introduction section.
- **Referee comment**: 2. It was a little difficult to read the axes in some of the figures. In fact, could some of the panels be done larger? For example, a figure has 4 panels stacked vertically with a lot of white space to the sides, while a 2x2 grid would show them better.
  - **Authors' response**: Figure 6 does have four panels stacked vertically as stated. We could make this into a 2x2 grid, but we would like to maintain the correspondence with Figs. 8 and 9 so that the nth row of each Fig. can be compared directly.
  - **Authors' changes**: Figures 6, 8 and 9 have been made larger and the bottom-most panels of each has been made clearer. Figure 7 has also been made larger and clearer (and now has two panels, to show how energy is numerically not conserved when the grid resolution is increased – to answer the other reviewer's comments).
- **Referee comment**: Typos: There are some typos both in the text and in the equations. For instance \delta t = \delta t /N. These have been identified by the other reviewer so I am not repeating them.
  - **Authors' response**: Thank you.
  - **Authors' changes**: Corrections made (please see report for reviewer 1).