Reviewer #2 comments and response:

We thank the reviewer for his/her helpful comments and corrections. Below are responses to the particular comments and corrections.

Comment 1: "Introduction) It would be helpful to expand on the descriptions of the types of physics in order to more strongly make the case that Blatter-Pattyn is both desirable and expensive. I don't think you need any equations, but the hierarchy could be more explicit in terms of physical assumptions and their mathematical/numerical consequences. For a modeling journal like GMD, more detail than for a "regular" journal seems appropriate."

Response: We have adjusted the paragraph (in the revised version, lines 93-115) following the reviewer's recommendation (addition to prior text is in the color red) as follows:

With model-data comparisons of past ice sheet changes becoming more common, however, some applications may benefit from using an ice sheet model of increased complexity, particularly when comparisons of past margin behavior is of interest. Ideally, full stokes (FS) models provide a comprehensive 3D solution to the diagnostic equations. FS models, however, are prohibitively expensive computationally and are mainly relegated to modeling experiments no more than a few hundred years. As described above, SIA models represent the highest degree of simplification of the FS equations, in which the vertical shear stress is the only non-zero stress component in the force balance equations. Although advantageous due to its computational efficiency, SIA models cannot simulate ice streams, grounding line dynamics, and floating ice shelves. On the contrary, shelfy-stream or shallow-shelf approximation models (SSA; MacAyeal, 1989) were developed to be implemented in ice shelve regions where longitudinal stresses dominate, however, these models cannot represent slow flow in the interior of the ice sheet where vertical shear is non negligible. Higher-order models (Blatter, 1995; Pattyn, 2003; herein referred to as BP for Blatter-Pattyn) on the other hand, that include membrane stresses and elements of the vertical shear stress have been a hallmark in the ice sheet modeling community over the past decade, being favored for their ability to model both the fast and slow components of ice flow, while being computationally cheaper than FS models.

Comment 2: Line 96) "stress balance computation does not require a high vertical resolution" could use a reference."

Response: Because this is not a well-established metric, there is no known reference for the original statement made. However, from our experience with ISSM, users have found that the number of layers does not directly hinder an accurate stress balance computation.

In order to make this point without overstepping the boundaries of the existing literature, we refer the reviewer to the answer for **comment 8** and **comment 2 from reviewer #2**. Based on these tests we have changed our text in the introduction (revised version, lines 115 -122) to read (additional text in the color red):

"The majority of the computational demand for an ice sheet model resides within the stress balance computation. Although the thermal model requires many vertical layers in order to capture sharp thermal gradient near the base of the ice, stress balance tests performed with ISSM (not shown here) on models with 25 layers and 5 layers show the area averaged differences in the surface and basal velocities to be 0.22 % and 0.012% respectively. Therefore, for the purposes of the experiments outlined in this study, we consider that the stress balance computation does not require a high vertical resolution. As a consequence of the high number of vertical layers needed for the thermal computation, however, more runtime is needed during the stress balance computation than is necessary."

<u>Comment 3:</u> "Section 2) This could use a reference to your favorite finite element method book for the definitions of the elements."

<u>Response:</u> We have added a reference and sentence (revised version, line 224):

For more information about the finite element method, we direct the reader to Zienkiewicz and Taylor (1989).

Zienkiewicz, O.C. and Taylor R.L. The finite element method. Vol. I. Basic formulations and linear problems. London: McGraw-Hill, 1989. 648 p. Vol. 2. Solid and fluid mechanics: dynamics and non-linearity. London: McGraw-Hill, 1991. 807 p. [School of Engineering, University of Wales. Swansea, Wales]

<u>Comment 4:</u> "201) Totally OK to only run one of the EISMINT2 experiments, but explain very briefly why you picked A. Presumably because it's the initial spin-up?"

<u>Response:</u> We ended up choosing experiment A to help determine how our different set of experiments would perform under the initial relaxation, rather than for the remaining EISMINT 2 experiments which primarily tested the response of each model to various changes in forcings. Because we were concerned with the outcomes of the experiments using different vertical resolutions and difference vertical finite elements, our interest was in the spread of the various experiments in ISSM compared to the suite of models in the EISMINT2 experiment A relaxation.

To clarify in section 3.1, we added: Rather than performing all of the experiments associated with the EISMINT2 benchmarks, we choose to limit the analysis to only experiment A, where models begin from the same initial state.

<u>Comment 5:</u> "Section 3.1) Although you do say you're comparing to an experiment for SIA models, it should be more clear in the text that you're using SIA."

<u>Response:</u> Both reviewers expressed a need for more clarity in defining what type of model was used. We have therefore made adjustments in section 3 and 3.1. In section 3 (Model description and experimental setup) we made clear the SIA was used in the single dome experiment and BP used in the steady-state solution (Text below in the color red is what was added).

"We first test the acuracy of the higher-order vertical interpolation using a simplified single dome ice sheet experiment that uses the SIA, following experiment A of the European Ice Sheet Modeling INiTiative (EISMINT2) experiments (Payne et al., 2000). We then apply a similar setup to a GrIS wide model, where the steady-state thermal solution is computed using the higher order BP model. Specifics regarding model setup and the relevant experiments are discussed below."

In section 3.1 (second sentence), we added:

"We perform all of our single ice dome experiments using the SIA on models with horizontal grid resolution of 20 km x 20 km, with a model domain of 1500 km x 1500 km"

Comment 6: "Section 3.2, first paragraph) It should be more clear what your thermal steady-state computation is, i.e. a simultaneous solution of heat transport and momentum equations coupled by viscosity. Also, the description of the effective viscosity can probably just say it's a function of strain rates/velocity gradients and a temperature dependent hardness. The variable name B isn't necessary when the equation isn't shown."

Response: The thermal steady state computation is done iteratively (e.g. thermal computation, stress balance, thermal computation, stress balance, etc.) until a user defined criterion is met (in this case, fixed at 10 iterations).

For the effective viscosity, we have decided to add the equation (line 275 in revised version):

$$\mu = \frac{B}{2\dot{\epsilon}_e^{\frac{n-1}{n}}}$$

"Where B is the ice hardness, n is Glen's flow law exponent and $\dot{\epsilon}_e$ is the effective strain rate. The ice hardness, B, is temperature dependent following the rate factors given in Cuffey and Paterson (2010, p. 75), while basal drag is empirically determined following a viscous flow law outlined in Cuffey and Paterson (2010)."

Comment 7: "Section 4.1) It's worth pointing out that Experiment A is a comparison of models without a known analytic solution, some of which produced more plausible (to me, anyway) solutions than others (e.g., W, X and Y). Comparison to the EISMINT2 results that don't show stability problems may be more reasonable. Also, it seems possible that your 25-layer results are better than the EISMINT2 models."

Response: We agree that no known solutions exist for the EISMINT2 experiments, and therefore have added the following sentence, line 335 in revised version: "It is important to note that no known analytic solution was provided in the EISMINT 2 experiment A comparison." And we have adjusted the sentence following to read: "Similar to Rutt et al. (2009), however, we compare our simulated values to the mean and the standard deviation of the values for experiment A in the EISMINT2 experiment to assess the relative spread."

The reviewer brings up a good point that comparison to all models may not be necessary as many of the models experienced a thermal instability in the radial symmetry of the basal temperature. We have made a table (similar to Table 1 – attached below) to show how our simulations compare to the EISMINT2 experiment A models **W, Y, and Z**, which had a radially symmetric basal temperature during the spinup procedure. In general, our conclusions discussed in the paper remain the same, which leads us to favor including all models associated in the EISMINT2 experiment A rather than only using models **W,Y, and Z**. The conclusion is that when using fewer vertical layers, those models using the higher order vertical finite elements tend to match the model mean for the EISMINT2 experiment A models. This general conclusion is similar when using the mean of all models as well as the mean of only models **W,Y, and Z**.

<u>Comment 8</u>: "Section 4.2) Given that a major selling point of the new method is making BP affordable, I'd like to see some direct description of the dynamical results at some point. How do your calculated velocities compare for different numbers of layers and vertical elements? Is vertical shearing captured well? You only discuss experiments for which SIA is reasonable. I have some concern about whether the reduction in the number of levels will work for transient runs with more realistic geometry than the ice dome that include areas in which BP is necessary."

<u>Response:</u> This comment echoes concerns also shared from **Reviewer #1.** We are attaching the same analysis that was used to respond to **Reviewer #1 comment 2** (Figure 1. A, B, C)**.**

To test how sensitive the stress balance computation was in our GrIS model to changes in vertical resolution, we began by collapsing our 25 layer P1 model. By collapsing the 25 layer model, the ice viscosity parameter (B) is depth averaged, and therefore does not depend on depth. We next extruded our collapsed model to 25 layers and 5 layers, and ran a stress balance computation.

We have attached the results of that experiment (Figure 1. A, B, C). When comparing the stress balance surface velocity differences (A) between the 25 layer and 5 layer model, the area averaged difference is 0.22%, with the maximum difference being 6.2%. The area averaged difference in basal velocities (B) are 0.012%, with a maximum difference of 2.6%. Lastly, the area averaged difference in basal shear (C) is 0.54 m/yr with a maximum of 87 m/yr. From this experiment we conclude that the differences associated with the vertical resolution and the stress balance computation are minor when compared to those differences in the thermal computation. We agree that in complex regimes (high friction and large gradients in stress and strain), a higher vertical resolution should be better at capturing features associated with the stress balance computation. From our stress balance experiment the larger differences between the 5 and 25-

layer model tend to occur in these complex environments. Given the nature of our paper, and the benefit of using higher order vertical finite elements to improve the speed of model run targeted for paleoclimate experiments, we conclude that the stress balance is captured well when compared with the thermal computation, which relies on higher order vertical finite elements to achieve a similar accuracy as a model with a higher vertical resolution.

Although our tests show that the stress balance velocities are captured well using 5 layers, we understand that in transient experiments this may change. This is a larger question that we strongly regard to be acceptable for a different study separate than this one.

Comment 9: "347) "we began by using the relaxed model simulations that have thus far only used the shallow ice approximation. . ." Please be a little more clear that you mean the single dome calculations from 3.1/4.1."

Response: To clarify, we add: "we began by using the relaxed model simulations that have thus far only used the shallow ice approximation for the single dome experiments in section 4.1"

Technical corrections:

Line 76: Made the requested change and cut "a"
Line 77: Made the requested change and cut "for"
Line 80: Made the requested change from "towards" to "to"
Line 109: Made the requested change and added "more"
Line 129: Made the requested change and added a multiplication sign "×"
Line 143: Made the requested change and added spaces before and after the "=" sign.
Line 177: Made the requested change from "As" to "While"
Line 192: Made the requested change from "was" to "were", and changed "thermomechanically" to "thermomechanical"
Line 256: Made the requested change to "with respect to both"
Line 263: Made the requested change and cut "and above"
Line 357: Made the requested change from "criteria" to "criterion"

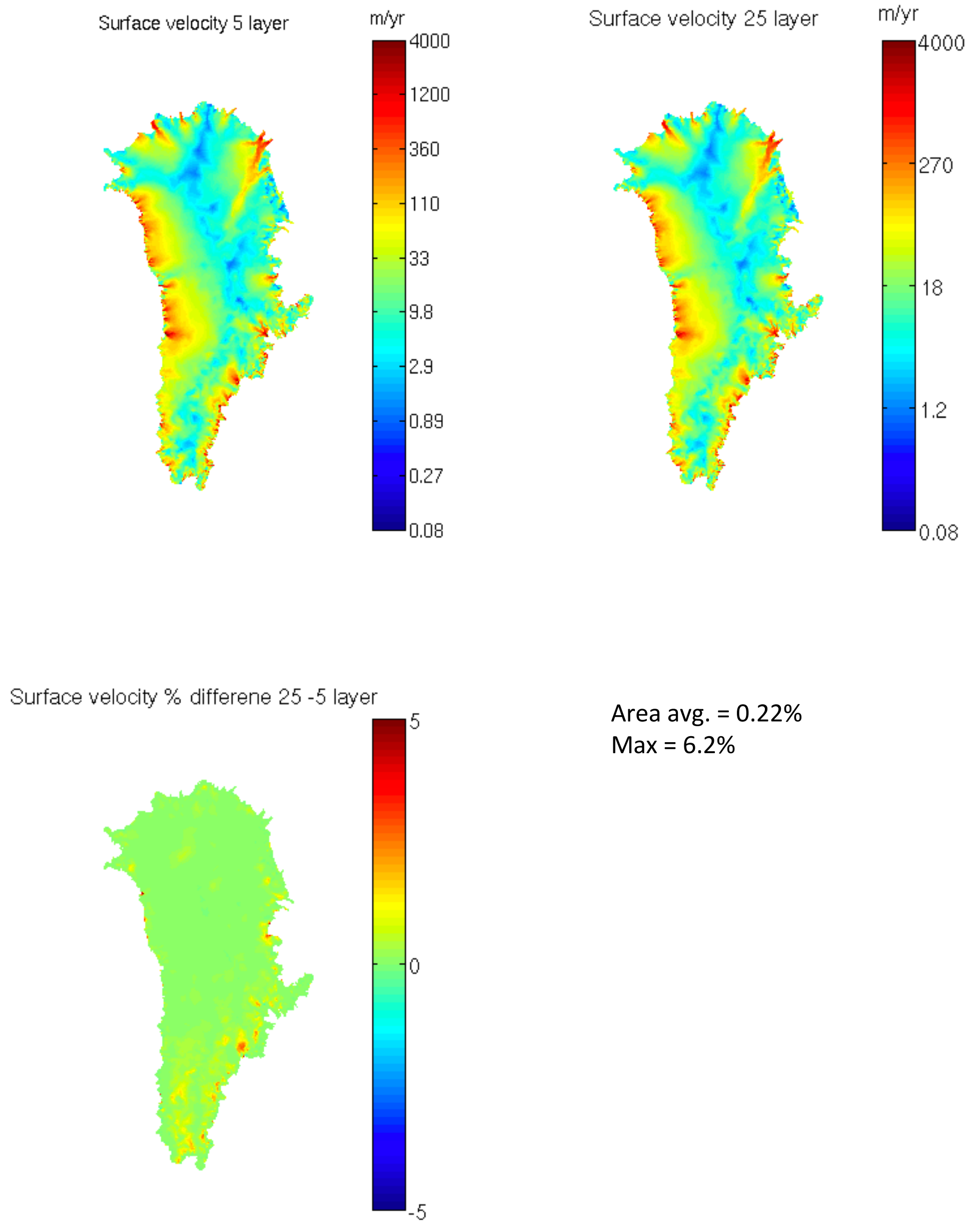
Table 2: We removed the units from the header.
Figure 3: We changed the last sentence of the figure caption from "Only those models that fall within 2% of the simulated ice volume for the 25 layer P1 model are labeled" to "Only those models that fall within 2% of the simulated ice volume for the 25 layer P1 model are labeled and colored as shown in their respective legends"

Table 1 using EISMINT2 models W,Y, and Z

	Volume (10^6 km^3)	Ice divide basal temp (K)	Ice divide thickness (m)
Eismint 2 exp. A (mean value) *Payne et al., 2000*	**2.134 ± 0.03**	**256.28 ± 0.80**	**3676.5 ± 55.65**
25 layer P1	2.144	254.723	3767.0
3 layer P1	2.344	247.229	4093.2
4 layer P1	2.265	250.240	3960.4
5 layer P1	2.231	252.351	3876.5
6 layer P1	2.209	253.285	3844.4
7 layer P1	2.192	253.793	3823.0
8 layer P1	2.179	254.115	3806.7
9layer P1	2.171	254.337	3794.5
10 layer P1	2.165	254.480	3785.4
3 layer P2	2.264	249.873	4023.2
4 layer P2	2.169	252.598	3838.1
5 layer P2	2.146	253.717	3785.8
6 layer P2	2.138	254.225	3764.8
7 layer P2	2.131	254.488	3753.9
8 layer P2	2.124	254.532	3747.1
9 layer P2	2.123	254.634	3743.6
10 layer P2	2.122	254.656	3741.3
3 layer P3	2.245	250.019	4002.0
4 layer P3	2.160	252.689	3826.4
5 layer P3	2.145	253.581	3779.3
6 layer P3	2.143	253.895	3765.0
7 layer P3	2.138	254.213	3756.5
8 layer P3	2.131	254.334	3750.3
9 layer P3	2.129	254.436	3748.5
10 layer P3	2.127	254.600	3746.2

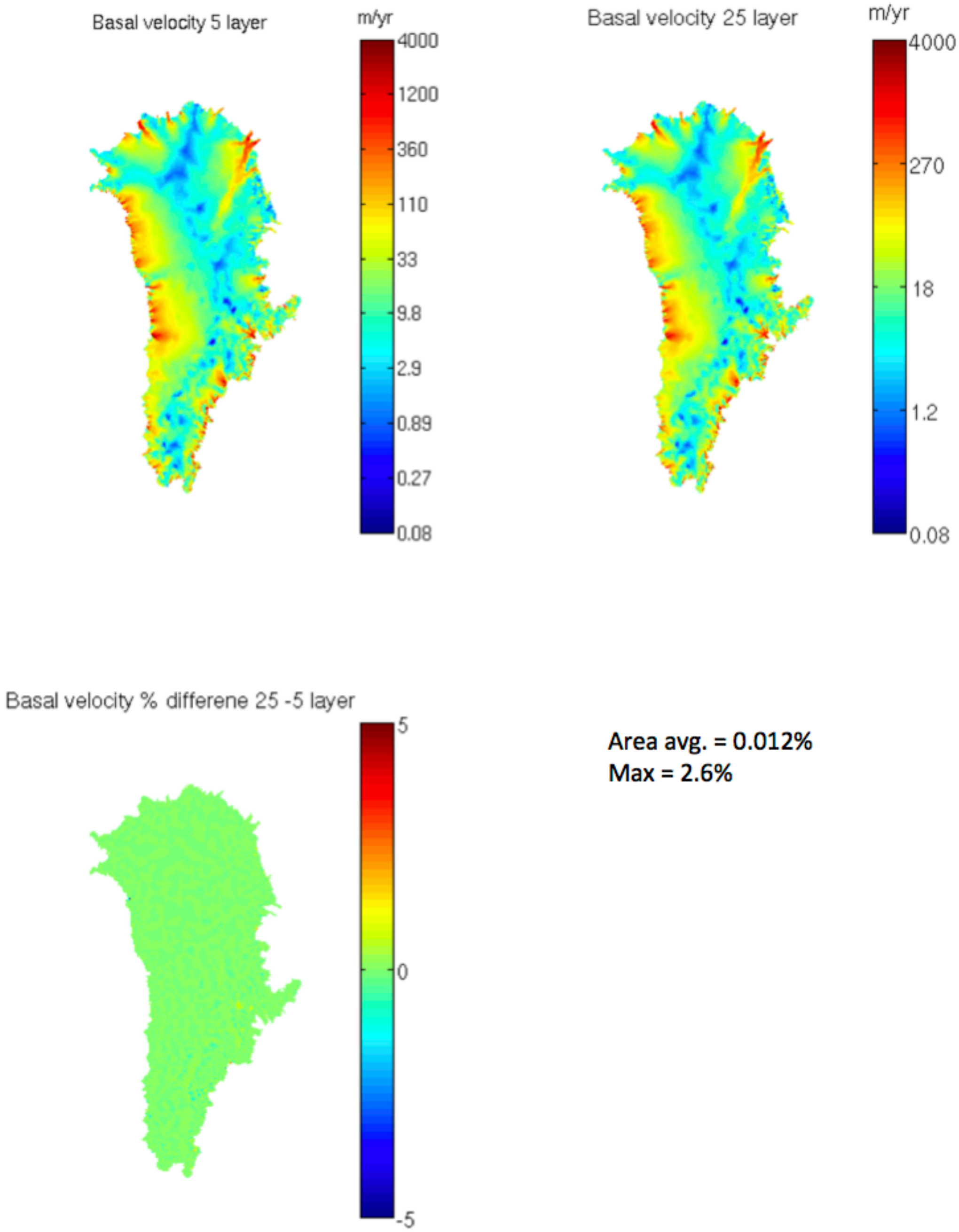
Table 1 - **Reviewer 2 comments**. Ice volume, ice divide basal temperature, and ice divide thickness for each indivicual simulation after 100 kyr. Also shown is the corresponding values for the EISMINT2 (Payne et al., 2000) experiment A simulations **W, Y, Z**, which were noted in Payne et al. (2000) to produce a more radially symmetric basal temperature. The shading indicates those simulations whose values fall within 1 standard deviation (green), 2 standard deviations (blue,) and 3 standard deviations (red) from the EISMINT2 mean values for experiments W, Y, Z.

A

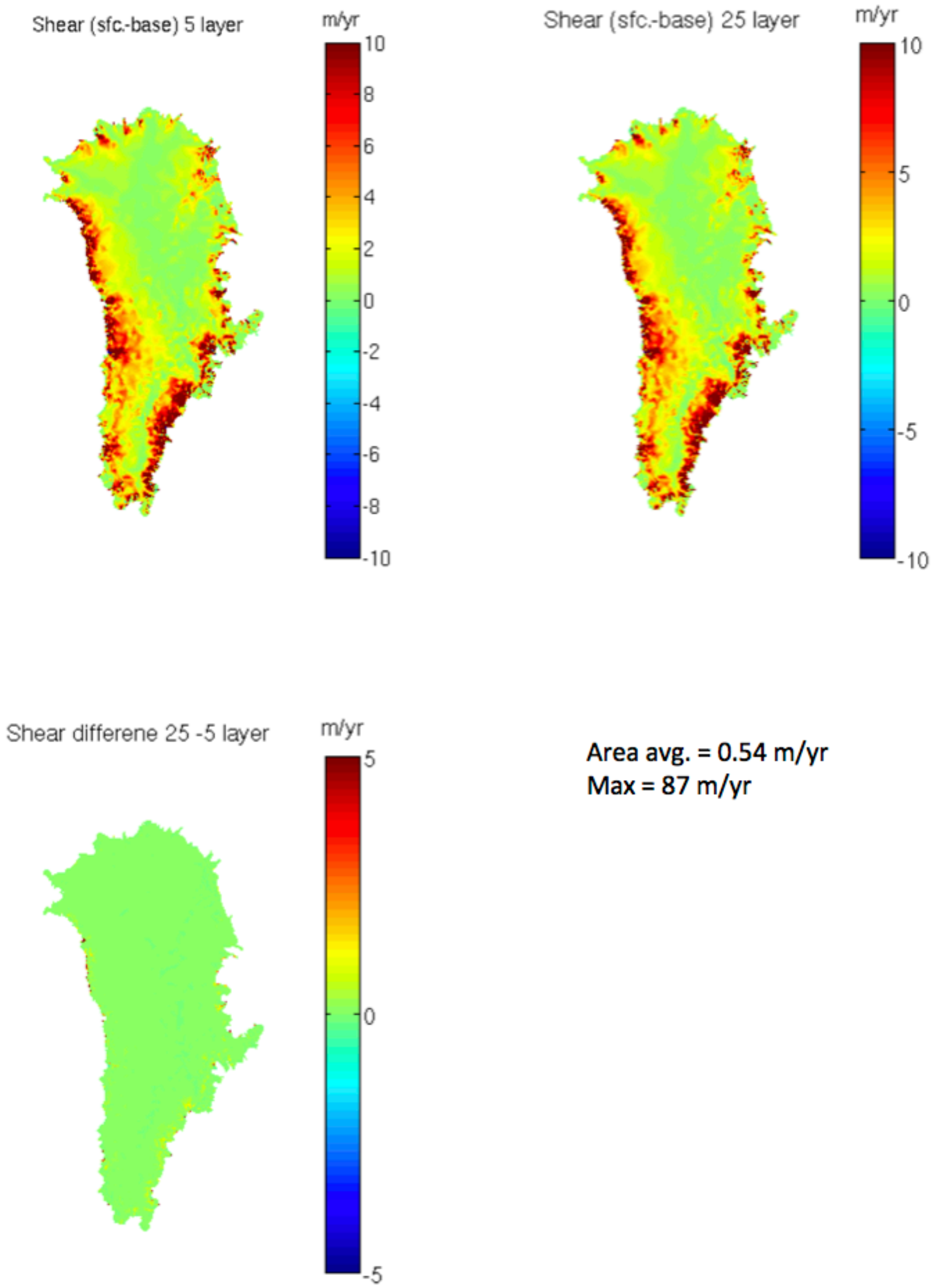


Area avg. = 0.22%
Max = 6.2%

B


Basal velocity 5 layer
m/yr
4000
1200
360
110
33
9.8
2.9
0.89
0.27
0.08
Basal velocity 25 layer
m/yr
4000
270
18
1.2
0.08
Basal velocity % differene 25 -5 layer
5
0
-5
Area avg. = 0.012%
Max = 2.6%

C



Area avg. = 0.54 m/yr
Max = 87 m/yr