Reviewer #1 comments and response:

We thank the reviewer for his/her helpful comments and corrections. Below are our responses to the particular comments and corrections.

<u>Comment 1</u>: "Abstract: the authors report runtimes 10 to 57 times faster. I think this is a bit misleading, as these numbers are when comparing the higher order elements with the reference 25 layers-P1 elements simulation. Improvement in runtimes are interesting if they allow to reach a similar accuracy. As the 25 layers-P1 elements simulation is used as a reference, the respective accuracy of the solutions is not known. However, it is shown that compared to the same reference, a 10 layers-P1 elements simulation falls within the same criteria of 1%. Improvement in runtimes is then only a factor 5. I think this is the more correct number to report in the abstract."

<u>Response:</u> We agree with the reviewer that the stated runtime speed increases can be misleading in this context. In section 4.2 we highlight the associated runtime speed increases between the 25 layer P1 model and the 4/5 layer P2/P3 models, which is referenced in the abstract. Following the reviewer suggestion, and because we deem the 10 layer P1 model to fall within the criteria set in section 4.2, the abstract would be better suited to associate runtime speed increases between the 4/5 layer P2/P3 model and the 10 layer P1 model. When doing so, the runtime speed increase between a 5 layer P3/4layer P2 and a 10 layer P1 model are 5 to 7 times faster. Considering these changes, we updated the abstract accordingly.

"Results indicate that when using a higher-order vertical interpolation, runtimes for a transient ice sheet relaxation are upwards of 5 to 7 times faster than using a model which has a linear vertical interpolation, and thus requires a higher number of vertical layers to achieve a similar result in simulated ice volume, basal temperature, and ice divide thickness."

<u>Comment 2</u>: "Page 2 , lines 95-96: it is said that the stress balance requires less vertical resolution. I'm not sure that this is a well-established result, as for areas with high friction near the base, there is also very sharp gradient of the stresses and strain rates, also requiring higher resolution for the stress balance."

<u>Response</u>: We agree that this is not a well-established result in the literature. In our experience with ISSM, the stress balance is not as sensitive to changes in vertical resolution compared with the thermal computation. To test how sensitive the stress balance computation was in our GrIS model to changes in vertical resolution, we began by collapsing our 25 layer P1 model. By collapsing the 25 layer model, the ice viscosity parameter (B) is depth averaged, and therefore does not depend on depth. We next extruded our collapsed model to 25 layers and 5 layers, and ran a stress balance computation.

We have attached the results of that experiment (Figure 1. A, B, C). When comparing the stress balance surface velocity differences (A) between the 25 layer and 5 layer model, the area averaged difference is 0.22%, with the maximum difference being 6.2%. The area averaged difference in basal velocities (B) are 0.012%, with a maximum difference of 2.6%. Lastly, the area averaged difference in basal shear (C) is 0.54 m/yr with a maximum of 87 m/yr. From this experiment we conclude that the differences associated with the vertical resolution in the stress balance computation are minor when compared to those differences in the thermal computation. We agree that in complex regimes (high friction and large gradients in stress and strain), a higher vertical resolution should be better at capturing features associated with the stress balance. From our stress balance experiment, the larger differences between the 5 and 25-layer model tend to occur in these complex environments. Given the nature of our paper, and the benefit of using higher order vertical finite elements to improve the speed of model run targeted for paleoclimate experiments, we conclude that the stress balance is captured well when compared with the thermal computation, which relies on higher order vertical finite elements to achieve a similar accuracy as a model with a higher vertical resolution.

Following both Reviewer #1 and #2 suggestions we adjust (in revised version) Line 115 from:

"Although the stress balance computation does not require a high vertical resolution, the thermal model usually does in order to capture sharp thermal gradients near the base of the ice."

to (additional changes in the color red):

The majority of the computational demand for an ice sheet model resides within the stress balance computation. Although the thermal model requires many vertical layers in order to capture sharp thermal gradients near the base of the ice, stress balance tests performed with ISSM (not shown here) on models with 25 layers and 5 layers show the area averaged differences in the surface and basal velocities to be 0.22 % and 0.012% respectively. Therefore, for the purposes of the experiments outlined in this study, we consider that the stress balance computation does not require a high vertical resolution. As a consequence of the high number of vertical layers needed for the thermal computation, however, more runtime is needed during the stress balance computation than is necessary.

Comment 3: "Sec. 2.4. Figure 2 compares an exponential function captured by vertical elements with different polynomial interpolation. We understand that the figure is for 1 P3 element (i.e. 4 layers of nodes) or 3 P1 elements (i.e. 4 layers of nodes); but what is the corresponding number of P2 elements?"

Response: It seems the confusion surrounding Figure 2 is a mistake on our part regarding the wording of the caption. The figure on the left shows 3 prismatic elements, and on the right, we show the exponential profiles captured by the different finite elements for these 3 elements. We have changed the wording of the figure caption,

From:
"On the left is an example of P1xP3 prismatic elements. On the right is an example of exponential profile captured by P1, P2 and P3 finite elements. With higher order finite elements in the vertical, sharp gradients in temperature are captured more precisely than with a linear (P1) interpolation."

To:

"On the left is an example of 3 prismatic elements used to capture an exponential profile. On the right is an example of exponential profile captured by P1, P2 and P3 finite elements. With higher order finite elements in the vertical, sharp gradients in temperature are captured more precisely than with a linear (P1) interpolation."

<u>Comment 4</u>: "Sec. 3.1: it is said that P1 elements are used for the stress balance. What is the default number of integration points. Does it allow to capture the temperature profile, affecting the viscosity, within the element?"

<u>Response:</u> We use Gauss-Legendre integration points in the vertical that capture polynomials of degree up to 9, which is adequate for the integrals that we have here.

<u>Comment 5</u>: "Sec. 3.1. All the introduction is about using higher order models for the stress balance, however most of the EISMINT comparison is done with the Shallow ice model, and we learn this very late in the results section. It should be said here that the 100 000 years experiment is done with the SIA, justifying the comparison with EISMINT2, and that the BP model is used only to do 100 years relaxations."

<u>Response:</u> Both reviewers expressed a need for more clarity in defining what type of model was used. We have therefore made adjustments in section 3 and 3.1. In section 3 (Model description and experimental setup) we make clear the SIA was used in the single dome experiment and BP used in the steady-state solution (Text added below is in the color red).

"We first test the precision of the higher-order vertical interpolation using a simplified single dome ice sheet experiment that uses the SIA, following experiment A of the European Ice Sheet Modeling INiTiative (EISMINT2) experiments (Payne et al., 2000). We then apply a similar setup to a GrIS wide model, where the steady-state thermal solution is computed using the BP model. Specifics regarding model setup and the relevant experiments are discussed below."

In section 3.1 (second sentence), we add:

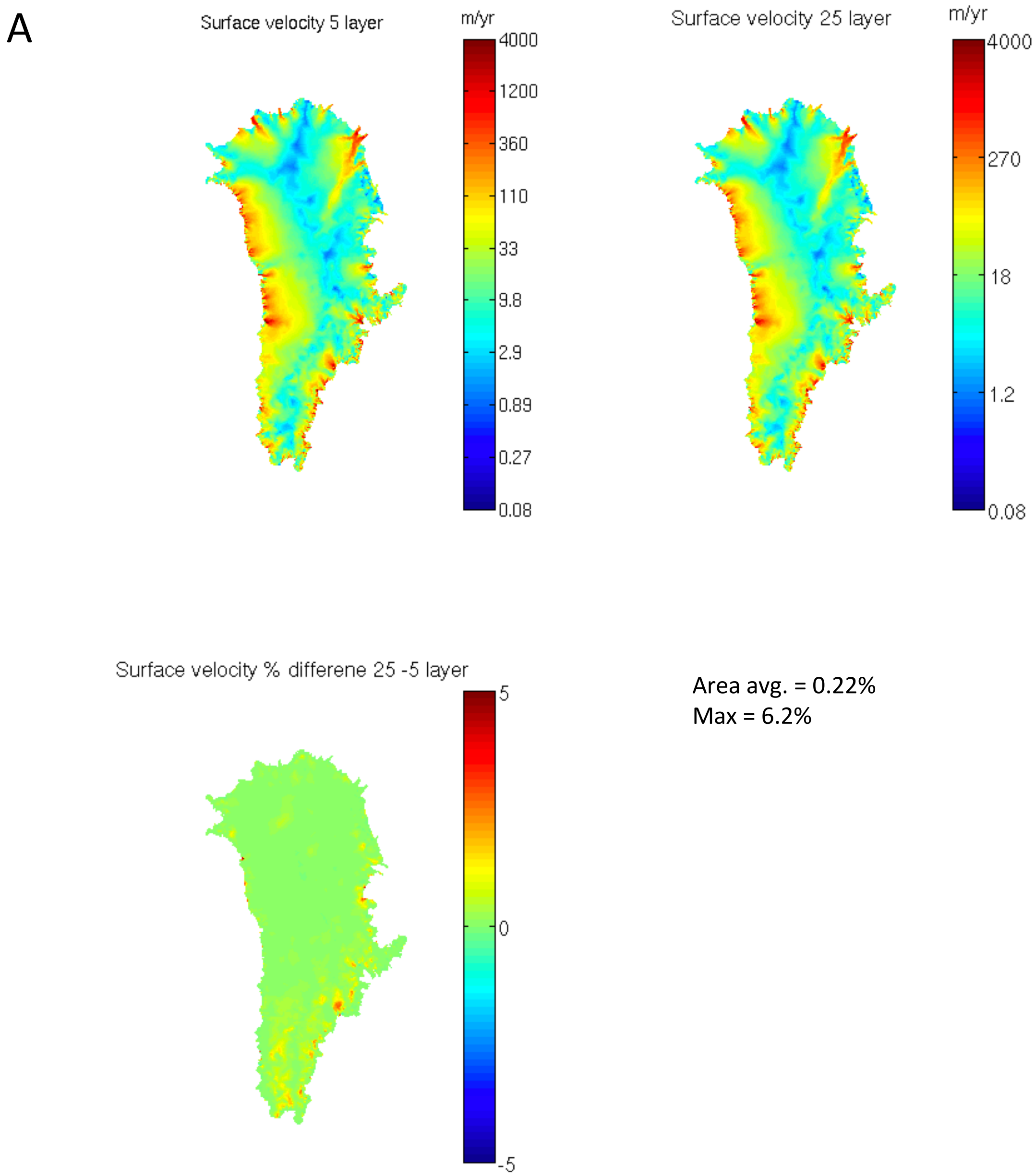
"We perform all of our single ice dome experiments using the SIA on models with horizontal grid resolution of 20 km x 20 km, with a model domain of 1500 km x 1500 km"

<u>Comment 6</u>: "Page 5, line 227. It is said that the elements are finer in areas of steep topography and ice flow gradients. I think that the refinement is based on the second derivatives, not the gradients, so elements are finer where changes in slope and ice flow gradients are high?"

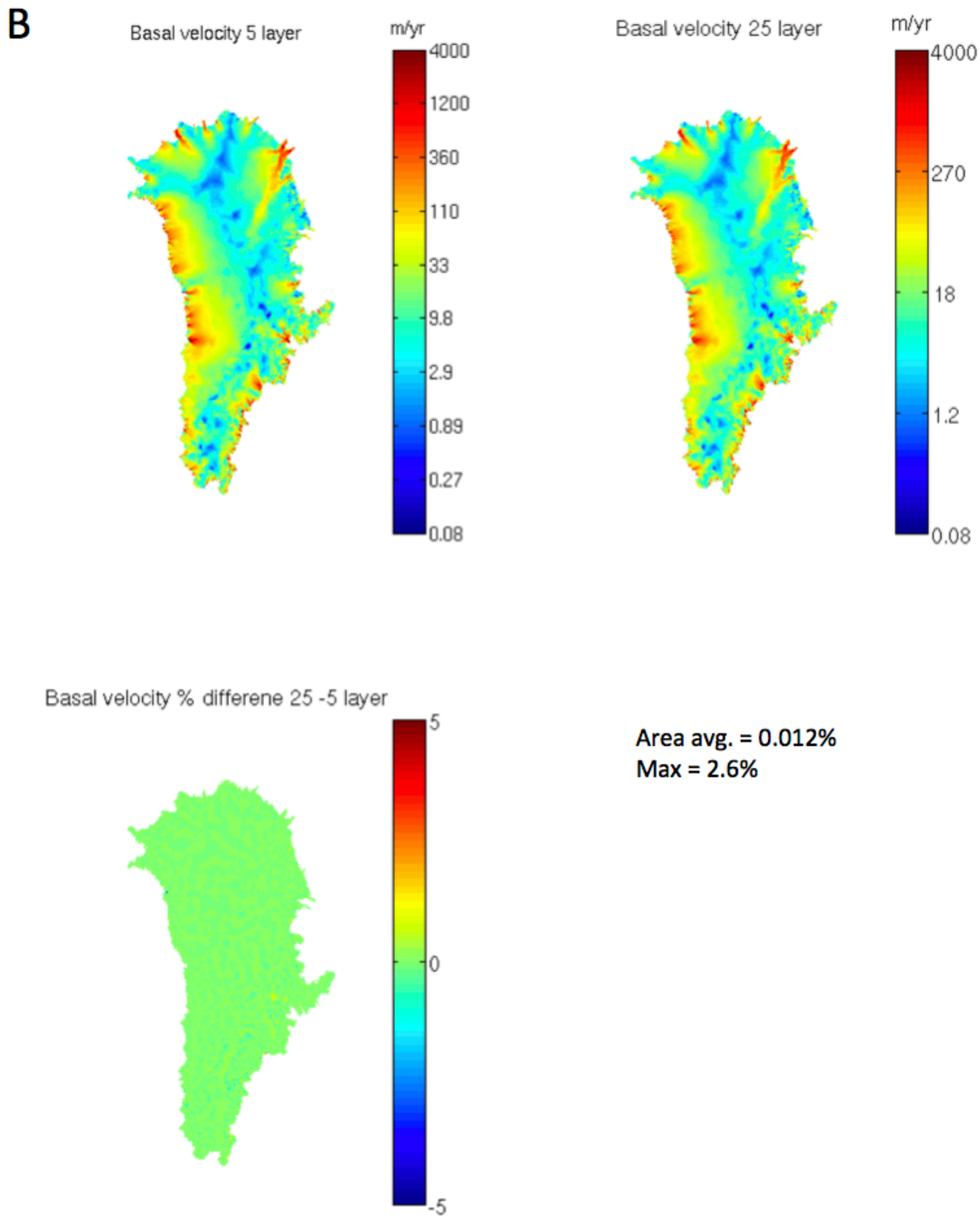
<u>Response:</u> We have fixed the wording of the sentence to reflect this (changes in red). In the revised version it is on line 286.

"The GrIS wide model relies on anisotropic mesh adaptation, whereby the element size is refined as a function of surface elevation (Howat et al., 2014) and InSAR surface velocities from Rignot and Mouginot (2012), becoming finer in areas in regions where the second derivative of these two quantities is higher."

Figure 1

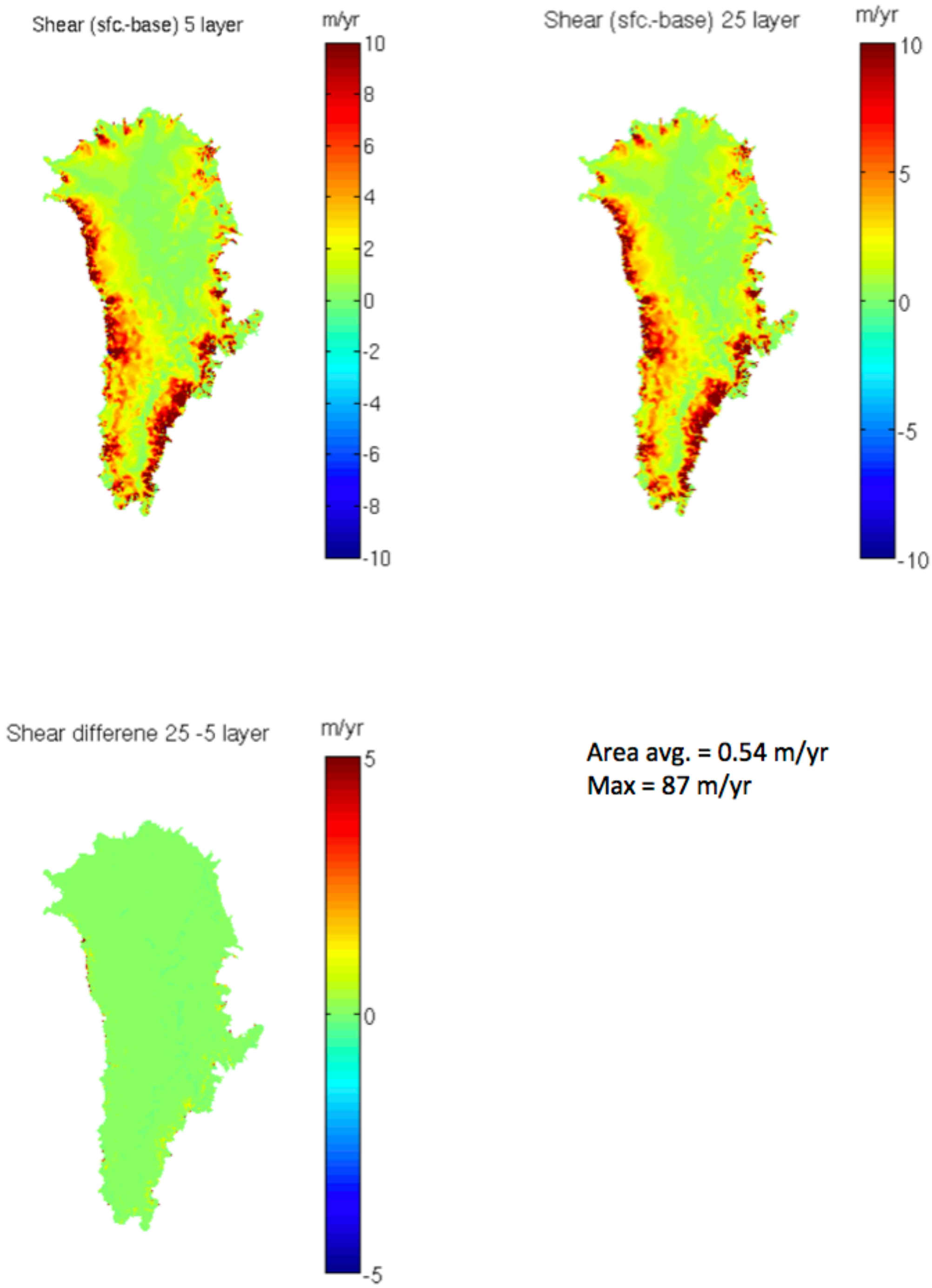


Area avg. = 0.22%
Max = 6.2%



Area avg. = 0.012%
Max = 2.6%

C



Area avg. = 0.54 m/yr
Max = 87 m/yr