Comments to the manuscript of Picard, Sandells, and Löwe, 2018: SMRT

General comment:
This manuscript describes excellent work for the advancement of microwave remote sensing of snowpacks. The present version needs some corrections and improvements as described below. Furthermore I see a need for numerical validations, e.g. as proposed in Comment 15.
This review does not include the Appendices (except for p. 20).

Special comments for the revision:

Comment 1
The statement (p. 2, l. 10-11) on the influence of the atmosphere is not adequate because atmospheric effects can be quite significant and sometimes dominant. The authors can circumvent the problem by defining the boundary conditions at the snow surface. Instead of an illumination by constant cosmic background, the illumination also contains an atmospheric contribution, leading to a frequency-dependent sky brightness temperature T_{sky}. A main advantage of microwave radiation is that scattering in the atmosphere is negligible (except for precipitation). The introduction of Kirchhoff's Law on thermal emission, using emissivities and scene reflectivity (snow & substrate) together with an appropriate figure would improve the understanding. In addition the link with active radiation would become more apparent.

Comment 2
On p. 4, l. 27 it would be helpful to have references for Python and LGPLv3 License.

Comment 3
Equation (1) on p. 5 is the well-known radiative transfer equation for plane-parallel media (S. Chandrasekhar, Radiative Tranfer, 1950), here in the Rayleigh-Jeans Approximation. Unfortunately, in this form, it is only valid if the refractive index n=1. Since snow is a refractive medium with n>1, the equation needs modifications. For isotropic snow, the adaptation is simple. The specific intensity I has to be changed to its reduced value

$$I_1 = I/n^2, \qquad \text{(C 1)}$$

see e.g. the Fundamental Theorem of Radiometry, in Mobley, C.D., Light and Water (1994), or Hilbert, D., Die Begründung der elementaren Strahlungstheorie, Physik. Zeitschrift XII, 1056-1064 (1912). For anisotropic media, see e.g. Bekefi, G., Radiation Processes in Plasmas, New York, Wiley (1966). In a non-scattering and non-absorbing medium I_1 is a conserved quantity, but not I. Likewise, the source term $\alpha T(z)$ is to be divided by n^2 to get

$$\alpha_1 T(z), \text{ where } \alpha_1 = \alpha/n^2 = 2\nu k/c_0^2 \qquad \text{(C 2)}$$

Here c_0 is the speed of light in vacuum. Thanks to this correction, the emitting source term is a constant quantity in an isothermal environment, a requirement of thermodynamics. This is not true for $\alpha T(z)$ in a layered medium with n(z) changing with height.
It is possible that the authors made the necessary adaptation without being aware of, meaning that numerically, everything is OK. Still the formulation should be corrected. The adaptation is automatically taken into account in the formulation of temperatures (Rayleigh- Jeans) and brightness temperatures, instead of radiances.

The following page (extract from lecture notes) gives some more details.

Slightly inhomogeneous medium

Now we assume that the medium is slightly inhomogeneous, but scattering and absorption are absent (reflection and scattering are negligible if the gradient of the real part of the refractive index is sufficiently small: $|\nabla n'| << k$, and absorption is negligible if the imaginary part is $n''=0$). The rays are no longer straight lines, but follow the rules of geometric optics (Snell's Law, Fermat's Principle of the shortest path, Eikonal Equation). It can be shown (Hilbert, 1912; Mobley, 1994) that the following quantity is conserved:

$$I_{1\nu} = \frac{I_\nu}{n'^2}\text{; thus } \frac{dI_{1\nu}}{ds} = 0\text{; for the Stokes Vector } \mathbf{I}_{1\nu} = \frac{\mathbf{I}_\nu}{n'^2};\ \frac{d\mathbf{I}_{1\nu}}{ds} = 0 \qquad (9.3)$$

Note that $n'=n$ because $n''=0$. For illustration and verification of (9.3), we investigate the situation of a one-dimensionally inhomogeneous medium where the refractive index deceases in a transition region with increasing height (Figure 9.2).

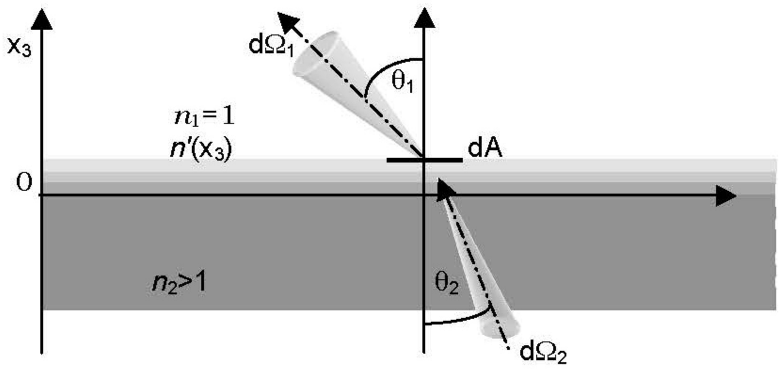

Figure 9.2: Power conservation for a refracted ray passing from one medium in another through dA. Reflection is avoided by a soft transition $\left|\frac{dn}{dx_3}\right| << \frac{1}{\lambda}$

Power conservation requires $dP_1=dP_2$, thus

$$I_\nu(1,\theta_1,\phi_1)\cdot\cos\theta_1\cdot d\Omega_1\cdot dA\cdot d\nu = I_\nu(n_2,\theta_2,\phi_2)\cdot\cos\theta_2\cdot d\Omega_2\cdot dA\cdot d\nu \qquad (9.4)$$

From Snell's law we have $\sin\theta_1 = n_2\sin\theta_2$. Furthermore, since $d\Omega_1 = \sin\theta_1 d\theta_1 d\varphi$, $d\Omega_2 = \sin\theta_2 d\theta_2 d\varphi$, and $\cos\theta_1 d\theta_1 = d(\sin\theta_1) = n_2 d(\sin\theta_2) = n_2\cos\theta_2 d\theta_2$, we get

$$\cos\theta_1 d\Omega_1 = n_2^2\cos\theta_2 d\Omega_2 \qquad (9.5)$$

Equations (9.4) and (9.5) lead to (9.3). Equation (9.3) also means that the Planck function is not conserved, but the following quantity is:

$$B_{1\nu} := \frac{B_\nu(\mathbf{r},T_b)}{\left(n'(\mathbf{r})\right)^2} = \frac{2h\nu^3}{c_0^2\left(\exp(h\nu/k_bT_b)-1\right)} = \text{constant} \qquad (9.6)$$

Since the quantities on the right side either are fundamental constants (h, k_b, c_0), a fixed frequency ν, or the brightness temperature T_b, Eq. (9.6) means that T_b does not change along the propagation path. Thus $I_{1\nu} = B_{1\nu}$ and T_b are conserved quantities. This is a first important result, the *fundamental theorem of radiometry* (Mobley, 1994). If the brightness temperature T_b did change, it would violate principles of thermodynamics.

Comment 4
Equation (4), last integral: integration interval must be changed to μ‘ from 0 to +1.

Comment 5
In Equations (2) to (4) the variable μ appears as being the same in all layers. This is incorrect. The incidence angle (and thus μ) changes due to refraction from layer to layer. Refraction should be formulated explicitely and taken into account. Otherwise the connection fails at layer interfaces. Note that upon refraction the solid angle of beams is changing too.

Comment 6
p. 7, l. 16 -17: The depolarisation factors are defined with respect to the 3 main axes of the ellipsoid with $A_1+A_2+A_3 = 1$. Equation (6) gives the mean value of the squared-field ratio for an isotropic distribution of such ellipsoids. The situation with all $A_i=1/3$ corresponds to spherical scatterers.

Comment 7
p. 12, l. 4: after this illustration I expect a short description of what it means. Illustrative results are missing.

Comment 8
p. 12, l. 12: in addition, the temperature of the substrate is required (for the passive mode).

Comment 9
p. 13, l. 8-9: Improve sentence to „Different configurations can be explored by adapting the code provided as open source (see data availability)“, and explain the missing part more clearly, using an additional sentence. Examples would help.

Comment 10
p. 14, l. 10: change „scattering coefficient“ to „brightness temperature“ (which is actually shown in Figure 5).

Comment 11
p. 17, l. 2-3: clarify „fixing density and SSA“ in Figure 8. The caption to Figure 8 indicates a fixed radius of 0.1 mm.

Comment 12
p. 18, l. 25: delete „constructive“ or add „and destructive“ before „interferences“ and add „for short phase differences“

Comment 13
p. 20, l. 11-12: what do you mean with „jupyter notebooks“ ?
And explain the acronym „DORT“

Comment 14
p. 20, l. 20-22: The description of the treatment of streams in different layers is much too short to be understood here. It is related to my Comments 3 and 5 (above). Please improve this text and estimate the potential errors introduced by one or the other method.

Comment 15
Tests should be made to check how accurate the radiative-transfer code is. One simple check is by assuming an isothermal environment ($T_{sky} = T_{snow} = T_{substrate} = T$), and then computing internal brightness temperatures in all different directions and at different positions. If any of these results

differ from T, an error is indicated. Choose situations without, with weak and with strong volume scattering, and interface reflections, respectively.

Comment 16
Figure 5: add (print) the value of the stickiness parameter in the upper graph and the sphere radius in the lower graph.

Typos:
p. 5, l. 31: change „Planck constant“ to „Boltzmann constant“
p. 6, l. 12 & 20: change ‚transmittivity‘ to ‚transmissivity‘ as used in the microwave range or ‚transmittance‘ as used in optics.
p. 6, l. 22: change „materials permittivity“ to „material permittivity“.
p. 7, l. 8: use symbol c_0 for speed of light in vacuum as proposed in (C 2).
p. 9, l. 1: change „will“ to „with“.
p. 13, l. 25: change 256 K to 265 K.
p. 14, l. 16: change „yields“ to „yield“
p. 17, l. 23: delete „other“ (written twice)
p. 19, l. 18: delete „have“ before „highlighted“
p. 19, l. 24: change „numerically equivalence“ to „numerical equivalence“
p. 20, l. 6: change „other representation“ to „other representations“