

## Response to Christian Mätzler

### Comment 1

The statement (p. 2, l. 10-11) on the influence of the atmosphere is not adequate because atmospheric effects can be quite significant and sometimes dominant. The authors can circumvent the problem by defining the boundary conditions at the snow surface. Instead of an illumination by constant cosmic background, the illumination also contains an atmospheric contribution, leading to a frequency-dependent sky brightness temperature  $T_{\text{sky}}$ . A main advantage of microwave radiation is that scattering in the atmosphere is negligible (except for precipitation). The introduction of Kirchhoff's Law on thermal emission, using emissivities and scene reflectivity (snow & substrate) together with an appropriate figure would improve the understanding. In addition the link with active radiation would become more apparent.

The statement was moderated by "most frequencies" but for clarity reason it completely is removed. Most applications (which is the topic of this introductory paragraph) uses satellite data at frequencies (typically 19 and 37 GHz) where the atmosphere has a weak effect and is often neglected. The remaining of the comment seems to refer to Figure 1. We have added a symbol for the incoming beam and explained in the legend that it can be the radar beam or the atmospheric contribution. It is not clear why Kirchhoff law should be introduced here. The snowpack is usually non-isothermal (as the atmosphere) which makes the notions of emissivities and Kirchhoff law not fully adequate. This is the reason why we need to solve the radiative transfer equation with the thermal emission term, not just compute the reflectivity of the scene.

### Comment 2

On p. 4, l. 27 it would be helpful to have references for Python and LGPLv3 License

We have added web link as the language and the license do not exist as citeable document.

### Comment 3

Equation (1) on p. 5 is the well-known radiative transfer equation for plane-parallel media (S. Chandrasekhar, Radiative Transfer, 1950), here in the Rayleigh-Jeans Approximation. Unfortunately, in this form, it is only valid if the refractive index  $n=1$ . Since snow is a refractive medium with  $n>1$ , the equation needs modifications. For isotropic snow, the adaptation is simple. The specific intensity  $I$  has to be changed to its reduced value  $I_1 = I/n^2$ , (C 1) see e.g. the Fundamental Theorem of Radiometry, in Mobley, C.D., Light and Water (1994), or Hilbert, D., Die Begründung der elementaren Strahlungstheorie, Physik. Zeitschrift XII, 1056- 1064 (1912). For anisotropic media, see e.g. Bekefi, G., Radiation Processes in Plasmas, New York, Wiley (1966). In a non-scattering and non-absorbing medium  $I_1$  is a conserved quantity, but not  $I$ . Likewise, the source term  $\alpha T(z)$  is to be divided by  $n^2$  to get  $\alpha_1 T(z)$ , where

$$\alpha_1 = \alpha/n^2 = 2\nu k/c^2 (C 2)$$

Here  $c_0$  is the speed of light in vacuum. Thanks to this correction, the emitting source term is a constant quantity in an isothermal environment, a requirement of thermodynamics. This is not true for  $\alpha T(z)$  in a layered medium with  $n(z)$  changing with height.

It is possible that the authors made the necessary adaptation without being aware of, meaning that numerically, everything is OK. Still the formulation should be corrected. The adaptation is automatically taken into account in the formulation of temperatures (Rayleigh-Jeans) and brightness temperatures, instead of radiances. The following page (extract from lecture notes) gives some more details.

We have changed the text by renaming "specific intensity" to "reduced specific intensity" and given

the definition. The reference to Mobley 1994 has been added.

As stated in the text, the code is implemented with  $I=Tb$  and  $\alpha=1$  so that this mistake has no impact in the code.

#### Comment 4

Equation (4), last integral: integration interval must be changed to  $\mu'$  from 0 to +1.

Yes, thank you to spot this mistake

#### Comment 5

In Equations (2) to (4) the variable  $\mu$  appears as being the same in all layers. This is incorrect. The incidence angle (and thus  $\mu$ ) changes due to refraction from layer to layer. Refraction should be formulated explicitly and taken into account. Otherwise the connection fails at layer interfaces. Note that upon refraction the solid angle of beams is changing too

We have made the refraction explicit by adding a function to convert the angles between layers and introducing the S function (for details cf manuscript) to apply the Snell-Descartes law.

#### Comment 6

p. 7, l. 16-17: The depolarisation factors are defined with respect to the 3 main axes of the ellipsoid with  $A_1+A_2+A_3 = 1$ . Equation (6) gives the mean value of the squared-field ratio for an isotropic distribution of such ellipsoids. The situation with all  $A_i=1/3$  corresponds to spherical scatterers.

We have changed the sentence "In SMRT version 1.0, only isotropic microstructures are considered which implies  $A_j= 1/3$ ."

into

"In SMRT version 1.0, only spherical scatterers are considered which implies  $A_j= 1/3$ ."

#### Comment 7

p. 12, l. 4: after

this illustration I expect a short description of what it means. Illustrative results are missing

We have added a description and the numerical result.

Comment 8 p. 12, l. 12: in addition, the temperature of the substrate is required (for the passive mode).

This information is added

Comment 9 p. 13, l. 8-9: Improve sentence to „Different configurations can be explored by adapting the code provided as open source (see data availability)“, and explain the missing part more clearly, using an additional sentence. Examples would help.

We have reformulated the sentence. The provided codes are well documented, changing the frequency or the snowpack properties is straightforward.

Comment 10 p. 14, l. 10: change „scattering coefficient“ to „brightness temperature“ (which is actually shown in Figure 5).

corrected.

Comment 11 p. 17, l. 2-3: clarify „fixing density and SSA“ in Figure 8. The caption to Figure 8 indicates a fixed radius of 0.1 mm.

The sentence is rephrased: “Each curve is obtained, for a given density, by optimizing  $\phi_{exp}$  to obtain equivalence between”

Comment 12 p. 18, l. 25: delete „constructive“ or add „and destructive“ before „interferences“ and add „for short phase differences“

corrected as suggested.

Comment 13 p. 20, l. 11-12: what do you mean with „jupyter notebooks“ ? And explain the acronym „DORT“

we have reformulated and added a reference for jupyter notebooks. DORT in the Annex title has been expanded.

Comment 14 p. 20, l. 20-22: The description of the treatment of streams in different layers is much too short to be understood here. It is related to my Comments 3 and 5 (above). Please improve this text and estimate the potential errors introduced by one or the other method.

We have added a precise description in the Appendix (page 22) using the newly introduced function S (your comment 5)

Comment 15

Tests should be made to check how accurate the radiative-transfer code is. One simple check is by assuming an isothermal environment ( $T_{sky} = T_{snow} = T_{substrate} = T$ ), and then computing internal brightness temperatures in all different directions and at different positions. If any of these results differ from T, an error is indicated. Choose situations without, with weak and with strong volume scattering, and interface reflections, respectively.

Internal numerical tests in SMRT has been added to check this situation of isothermal model [https://github.com/smrt-model/smrt/blob/master/smrt/test/test\\_physics\\_law.py](https://github.com/smrt-model/smrt/blob/master/smrt/test/test_physics_law.py).

We have also added a test on Kirchoff law ( $1 - \text{reflectivity} = \text{emissivity}$ ) for opaque and isothermal surface. These tests are run for high and low scattering media and for thick and shallow snowpacks.

The results are accurate within a 0.01K tolerance for the isothermal situation and 0.002 in emissivity for the kirchoff law.