Author response to comments from Referee 3:

We thank the referee for their insightful review. Referee remarks are shown in red and our responses are given in black font. Changes to the manuscript text are given in blue font.

In this study, the authors extract a transport matrix model (TMM) from the ocean component of the UVic Earth System Climate model, and use the TMM to spin-up the biogeochemical component of the model. The biogeochemical state of the TMM model is then compared to an identical simulation using the online circulation model. Overall, this paper provides a useful comparison of the TMM spin-up method to the online method, and shows that the TMM faithfully represents most aspects of the online model, at a fraction of the computational time.

This paper is well written and appropriate for publication in Geoscientific Model Development. However, there are quite a few points in the paper that need to be expanded on and/or clarified in a revised manuscript. These are listed below.

Page 4, line 2: "time-stepped with a simple Euler method": Euler forward or Euler backward?

We use Euler forward. The text has been changed accordingly.

Page 4, line 3: "Fourier filtering at high latitudes": Some more discussion of this would be useful. Why is this filtering applied? What is the underlying cause of the noisiness? What numerical grid scheme (e.g. Arakawa B, C etc.) does the model use?

UVic's ocean model is based on MOM2 and uses an Arakawa B grid. Reviewer 1 also raised the issue of Fourier filtering. We have modified the relevant paragraph in Sec. 2.1 to the following:

Lastly, UVic ESCM applies Fourier filtering in the zonal direction at high latitudes to remove grid-scale noise. The efficiency of the TMM arises from the fact that the discretized advection-diffusion operator has a limited stencil, i.e., only couples nearby points, giving rise to a sparse matrix. Fourier filtering on the other hand couples all points in the zonal direction, greatly reducing the sparsity of the transport matrix and hence the computational efficiency of the sparse matrix-vector products at the heart of the TMM. While the cost of a sparse matrix-vector product is implementation- and hardware-dependent and non-trivial to analyze (e.g., Gropp et al., 2000), it roughly scales with the number of non-zero elements per row. With a 3^{rd} order upwind scheme, there are a maximum of 5 x 5 x 5 = 125 non-zero elements per row. With Fourier filtering that becomes nx x 5 x 5, where nx is the number of zonal grid points. In UVic ESCM, nx=100, implying that the TMM would be roughly nx/5=20 times slower with Fourier filtering turned on. We therefore turn off polar filtering for the passive tracers used to extract the TMs. The numerical treatment of temperature and salinity by the model is not altered.

Page 4, line 7: "Monthly mean TMMs were extracted": More information about how this was done is needed here. What time-step was used in the online model to create the TMs? How was the monthly averaging done?

We thank the reviewer for raising this point, which we failed to address in the original manuscript. We have added the following paragraph to Sec. 2.1:

The neglect of polar filtering and staggering of advection and diffusion terms necessitates using, for stability, a slightly smaller time step in offline simulations with the TMM compared with the online model. In the latter, the default time step is 1.25 days, a choice dictated by the need to synchronize the ocean and atmospheric models. (The biogeochemical terms in UVic ESCM are time-stepped internally within the biogeochemical module using a much smaller time step such that there are 3 biogeochemical steps per ocean step.) No change was made to this during extraction of the TMs, i.e., the model physics and active tracers were integrated using the default time step. Since the explicit TM is extracted as a tendency, the time step for offline explicit advection-diffusion can be subsequently set to any desired value. However, the implicit TM has a time step embedded within it. By default it would also be 1.25 days, but embedding a different time step is quite straightforward: during extraction of the implicit TM, we simply pass the desired time step as an argument to the subroutine that solves for implicit diffusion. We have found an offline time step of 8 hours (28,800 s) to be a good compromise between stability and accuracy. It is also very similar to the biogeochemical time step of the online model (27,000 s).

To clarify the issue of time averaging, the last paragraph of Sec. 2.1 has been changed to:

Using the linear UW3 advection scheme, the coupled physical-biogeochemical model was spun-up to equilibrium for 13,000 years with a fixed, pre-industrial atmospheric CO_2 concentration of 277.4 ppm. The model was run for one additional year with the transport matrix extraction switched on. During this run, explicit and implicit TMs were computed at each time step, and accumulated over the course of a month before being averaged and written out. These monthly mean TMs were subsequently used for the offline simulations. For comparison, we also carried out a similar spin-up of the physical and biogeochemical model using the default FCT advection scheme (see Appendix A).

Page 4, line 20 ff.: These few lines of description are not sufficient. More information is needed here to better describe how the biogeochemical model is coupled to the TMM. Equations and/or pseudocode would be appropriate so that one does not have to download and wade through the code.

We appreciate the reviewer's point about having to wade through the code to see how we interface the biogeochemical model to the TMM. But we note that there aren't any special "TMM equations" beyond eqn. (1), which shows how the biogeochemical term (q) is incorporated into the TMM framework. In a sense that is all there is to it. Obviously of course there are a lot of implementation details that are very specific to each biogeochemical model. But this makes it difficult to come up with any sort of meaningful "pseudocode" as the reviewer asks for. Instead, we have expanded the discussion in Sec. 2.2 in a way that we hope addresses the reviewer's concerns. It now reads as:

To apply the TMM code to a particular biogeochemical problem essentially requires providing a routine that takes as input vertical ``profiles" of tracer concentrations at a

horizontal location at the current time step (along with corresponding variables such as layer thickness, temperature, wind speed, etc at that location), and returns profiles of the biogeochemical tendency term, q. In practice, coupling an existing biogeochemical model such as the one in UVic ESCM to the TMM framework involves writing a ``wrapper" routine that serves as an interface between the TMM driver (written in C) and the biogeochemical code (typically written in some dialect of Fortran).

While the specific implementation of the wrapper will depend on the details of the biogeochemical model, in general it performs three main tasks. First, it copies required data from TMM arrays (that are passed as input arguments to the wrapper routine) to those of the biogeochemical model. Second, it calls the actual routine that computes biogeochemical source/sink terms (q). Normally, this routine would be called from the time-stepping loop of the model in which the biogeochemical model is embedded. Third, as these tendency terms are stored in arrays in the biogeochemical model, the wrapper copies them to arrays that are passed back to the TMM driver (as output arguments to the wrapper routine). To simplify this exchange of data, the horizontal grid on which the biogeochemical model (and ocean model in which it is embedded) is declared to have a size of 1x1. In essence, the biogeochemical model is treated as a 1-dimensional column model. In the case of UVic ESCM, where the code for physical and biogeochemical models are deeply intertwined, a few minor, additional changes to the original code were also necessary. Most of these changes were required in order to make available the full set of diagnostics accumulated by UVic ESCM. See Sec. 5 for information on where to download the code from.

Page 4, line 30: Why does the MOC weekend when switching from the FCT to UW3 advection schemes? Some discussion of this is needed.

It is believed that the strength of the MOC increases with the vertical diffusivity (at least in models). In a numerical model that diffusion arises not only from the explicitly modeled diffusion term (and corresponding prescribed diffusivity), but also from the implicit diffusion inherent to every numerical advection scheme. As is stated in the Methods section on page 3, 3rd paragraph, UW3 is less diffusive than FCT, which may be why the MOC weakens when switching from FCT to UW3. However, while an important and interesting point, addressing it in more detail than already stated in the manuscript would take us beyond the scope of the present study.

Page 5, line 25: Why is alkalinity sensitive to small changes in oxygen?

This section is now an appendix. The sentence has been expanded to read:

The largest differences are in nitrate and alkalinity, both of which are sensitive to small differences in oxygen via biological processes discussed in Section 3.1.

Page 5, "Mean state" section: Define the "mean state" of the TMM model and the online model. Are they directly comparable? Presumably the "mean state" of the TMM model is the annual average of the seasonally-cycling model which represents year 13001 in the online model âA T is that correct? Is the mean state of the online annual average of the 13001st model year? Or is it the multi-annual average of some

range of years âA T and thus would include natural inter annual variability as well? This needs a careful description, and if the two "mean states" are not directly comparable, this should be discussed.

The UVic ESCM has no inter-annual variability and the reviewer is correct that in the online model the mean state is that represented by the final year of the 13,000-year spin-up. In the TMM the mean state is represented by the final year of a 5,000-year spin-up. The first two sentences of Sec. 3.1.1 now read:

We first compare the annual-mean state of the online (UVIC_UW3) and offline (UVIC_TM) simulations, taken from the final year of the corresponding spin-up. A fully stable UVic ESCM simulation with annually repeating seasonal forcing has no inter-annual variability.

Figure 9: Oxygen: It would be useful also to show the suboxic/hypoxic volume for the TMM and online models. Do they match up well? And related to this, the water-column denitrification rate in each model âA T how does it compare? This is an important biogeochemical process that is highly sensitive to the details of the oxygen distribution.

It is important to know if the TMM version of the model captures the behavior of the online model.

Figure 9 (now Figure 4) now includes suboxic regions at 300 m depth. A sentence is added to the first paragraph of Section 3.1 (page 6):

While the global annual average rate of denitrification is higher in the online model $(5.63 \times 10^{-13} \text{ mol N m}^{-3} \text{ s}^{-1})$ than in the offline model $(5.55 \times 10^{-13} \text{ mol N m}^{-3} \text{ s}^{-1})$, the offline model has more grid points with very high values (not shown).

Page 6, line 15, and Figures 10 and 11: "Polar filtering" is blamed for the mismatch at high latitudes. Seems likely that this is not the cause. The mismatch is not really in the "polar" regions - is this filtering really applied at 50-60oS in the ACC? Also, the nutrients are too high in the surface S. Ocean and too low in the deep S. Ocean âA T this seems to implicate the biological pump (e.g. particle formation/sinking) as the culprit. A more careful discussion of these differences and their possible causes is warranted.

Figures 10 and 11 are now Figures 5 and 6. The identical biogeochemical code is used in the online and offline models- particle formation and sinking is the same. The only way the biological pump could vary between models is by differences in the biomass at the surface. Difference plots have been added to Figure 2 which show generally higher biomass in the offline model, which would increase particle export, lowering surface nutrients and raising them in the deep ocean. This is consistent with the surface difference plot of phosphate in the Southern Ocean, which shows the online model has higher surface phosphate (lower primary production). The deep Pacific and deep South Atlantic also shows the offline model has higher phosphate which is also consistent with higher primary production in the offline model Southern Ocean.

The drivers of the differences in primary production must be due either to differences in the application of external forcing or to physical differences arising from the method of integration. Surface processes in the polar regions are likely to have downstream affects, which is why we suspect the absence of polar filtering may be contributing to differences at 50-60 degrees. Of course it could also be slightly different application of external forcing has an impact on the biology equations. However, in the Arctic, differences in deep alkalinity exist even at 60N even though biomass does not show a clear bias in the offline model. Polar filtering is the only plausible explanation for this.

The phosphate and nitrate paragraph in Section 3.1 has been amended to reflect the possibility seasonal forcing may also be driving differences in phytoplankton biomass at the high latitudes:

The absence of polar filtering in the offline model is contributing to these differences, as is slightly higher primary production in the offline Southern Ocean which enhances the biological pump (Fig. 2). This higher primary production may be due to slight differences in the application of external forcing, which is expected to introduce some bias to regions with strong seasonality. All of the above biases are small relative to discrepancies between the models and observations (see below).

Section 3.2.2 "Seasonal cycle". More discussion of how the seasonal cycle is handled is needed. Equations are needed. Is there a separate TM for each month? And then Euler forward (or backward) is applied to time-step the model? I'm assuming this is the case, but this should be made explicit. In regards to the difference between the TMM and online model, for example as seen in the Indian Ocean for phosphate (Fig. 14), how much is due to the time-averaging of the TMM, and how much to the time- stepping scheme? Would some of these differences be reduced with a more robust time-stepping scheme? e.g. Adams-Bashforth or Crank-Nicholson.

In response to this and comments by the other reviewers we have greatly expanded Sec. 2.1 and added a paragraph to Sec. 2.2, which we hope provides the additional details that the referee is asking for.

As for the differences between online and offline runs, they could arise from a number of factors, including time averaging of circulation (TMs) and other forcing fields (see last paragraph of Sec. 2.2) and differences in the time-stepping scheme. Unfortunately, short of implementing the same scheme offline as used by UVic, it is difficult to pinpoint the exact reason. As stated on Page 4, line 5, one of the key advantages of the TMM, as we see it, is that the same underlying framework can be used to "mix and match" circulations and biogeochemical models. This would be lost – or at least become quite complicated – if we had to implement time-stepping schemes for every ocean model from which TMs have been extracted (not only UVic, but others such as MITgcm and NEMO). Obviously this means that even without time averaging the offline run may not always exactly replicate the online one.