Response to the Referees' Comments

First of all, we would like to give our sincerest thanks to the three reviewers for the beneficial suggestions and comments, and we deeply appreciate your contributions, which help us for correcting and improving the manuscript. Our responses are listed as follows by using the red fonts. If there are still unclear or incorrect parts, the authors are very willing to make further corrections and improvements based on the reviewer's comments. Thanks again for your contributions.

Interactive comment on "A multilayer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea" by Yihao Wu et al.

Anonymous Referee #1

Authors present elegant and well-written numerical study for the SRBF gravimetric quasigeoid modelling using the multi-layer approach and compared results with a single-layer approach. This case study is very suitable for geodetic proceedings, but the modelling of quasigeoid surface is out of geophysical interest. This is main reason I recommend rejection of this article. Authors attempt to add some geophysical content (page 12/ line 14 to page 13/ line 5) is irrelevant. This is also evident from gravity signal decomposition in Fig. 2 that does not reflect any real geological features, rather than reflects the properties of kernel for different depths. There are additional major issues to be addressed by authors before considering further publication.

Response: The authors thank the reviewer for these beneficial comments. Before discussing the geophysical meaning of this study, the authors would like to introduce its motivation. With aspect to new modeling approach development, we develop a new parameterization of SRBFs' network for regional gravity field recovery. Based on the idea multi-resolution representation, we not only parameterize the multi-scale method in a mathematical way, but also linked the detailed signals to the anomaly sources at different depths beneath the topography, which are recovered by the different layers. To our knowledge, no existing researches studied this issue. From this point, we believe this study may be within the scope of "Geoscientific Model Development", since we notice that describing developments such as new parameterizations is one of scopes of this journal, please see the information in https://www.geoscientific-model-development.net/. Besides, to our knowledge, no direct comparisons have been made between the single-layer approach and multi-scale one regarding the performances in local gravity. In this study, we assess the performances of the multilayer approach and traditionally-used single-layer one, where the advantages and disadvantages of different methods are analyzed. According to the reviewer's comments, we enhance the relevant part the updated manuscript and make the motivation more clearly, please see pp 2-3 in the revised version. While, for the geophysical meanings of this study, the authors think there may have several aspects we can contribute. First, local gravity field is helpful for many applications in geodesy and geophysics, e.g., studying the structure of lithosphere and ocean circulation, and a new parameterization of local gravity field may be beneficial for this issue, which can be used as the inputs for geophysical applications. Moreover, we also compute the mean dynamic topography based on the gravimetric quasi-geoid modeled in this study, which can be used for studying the ocean circulation and mass transport in the North Sea. We also enhance this part based on the reviewer's comments, please refer to pp 27-29 in the updated version.

Yes, the authors believe the reviewer is right regarding this gravity signal decomposition in Figure 2 (in the original version) didn't include enough real geological features, and the statements in page 12/ line 14 to page 13/ line 5 didn't provide enough geophysical information for the patterns of these wavelet details in the original manuscript. However, the motivation of this study is to develop a new parameterization of gravity field based SRBFs in the framework of MRR, and the wavelet analysis is used to separate the contributions of different anomaly sources, which is finally used to design the parameterizations of multiply layers. And, the detailed investigation of the structure of lithosphere using the wavelet method is out the scope of this study. The author believe our work may contribute to study the geophysical features of bodies beneath the topography if we provide a better gravity field, however, this is not the main target for this study. However, according the reviewer's comments, we also provide the geophysical evidences for the demonstrated patterns of decomposed wavelet details and approximation (see Figure 1 and 2 in the updated version), and we believe these decomposed gravity anomalies can reveal the tectonic structure of study area at different depths. Please refer to the information in pp13-14 in the revised version.

1/ The values of variance factors for different types of observations are not given, so final accuracy and -most importantly - the claim that multi-layer approach provides better accuracy is not justified. This is especially evident from Table 5, where achieved accuracy in terms of gravity residuals is much too optimistic, because errors of gravity observations (especially for ship-borne data) are larger.

Response: Thanks the reviewer for the comments. Yes, we believe the reviewer is right, and the variance factors for different types of observations are important. According to the reviewer's comments, we add this information in the updated version, please see pp 17. For justifying the accuracies of different approaches, we actually consider several aspects. First, we check the data residuals after the least squares adjustment, and we agree with the reviewer's statement, we can't not confirm the multilayer approach works better even we derive a better fit of the data due to the noise level of gravity observations. Besides, since these data have been used for modeling, thus the comparison of SD values of data residuals can only be considered as the internal validation, not the external one. Thus, we introduce another high-quality independent data, i.e., GPS/leveling data, for validations in terms of quasi-geoid height. And, the associated validation results with GPS/leveling data, see Figure 6 and Table 6 in the updated version give us more confidence for the performances of different approaches. According to the reviewer's comments, we modify and enhance this part, please refer to pp 18-23 in the updated version.

2/ Another aspect related to validation of results is the ability of realistically extrapolating the gravity field. For this purpose sets of control point is chosen with given values that are not incorporated into gravimetric solution, but used to independently validate the result. Authors do not offer such validation.

Response: The authors thank the reviewer for these beneficial comments. We agree with the reviewer that the important aspect for the validation of results is extrapolating the gravity field, which is comparing the predicted values derived from the gravity model (e.g., model from the multilayer or single-layer approach) and ones derived from independent survey/measurements. For this aspect, we use independent GPS/leveling data for validating the result in terms of quasi-geoid heights, which is actually test the ability of the computed gravity field for realistically extrapolation. Let us explain it in more details, for modeling the regional gravity field using multilayer/single-layer approach, only the terrestrial and shipboard gravity data in terms of gravity anomalies are used, and no GPS/leveling data are combined. Then, after we solving the lease squares equation, i.e., eq.(8), we compute the unknown coefficients of SRBFs, and in this way, the regional gravity field model parameterized by SRBFs is known. Then, we use the independent GPS/leveling data for externally validate the regional SRBFs models. Since the GPS/leveling data are provided in terms of quasi-geoid heights, and their 3D coordinates don't coincide with the positions of gravity data, we need to reconstruct the SRBFs model based on the computed SRBFs' coefficients and coordinates of GPS/leveling data, e.g., see eq.(6), and compute the gravimetric quasi-geoid heights, which are actually ones derived from the gravity field model. In the meanwhile, we also have the measured geometric quasi-geoid heights from the high-quality GPS survey and leveling measurements, which are the observed values. Then, we compute the standard deviation (SD) of the point-wise difference between GPS/leveling data and the gravimetric quasi-geoid height from the regional approach, which is actually external validation. We have thousands of GPS/leveling points over the target region, and these statistics support the results for validation of different regional models. According to the reviewer's comments, we enhance this part in the updated manuscript, please refer to pp 20-22 in the updated version.

3/ Even if the geophysical application of this study is not substantiated, it is clear that the geodetic relevance is also not fully fulfilled. This is evident from Fig. 7, showing differences between the gravimetric and geometric (GPS/levelling) quasigeoid heights that are biased differently for each country. In gravimetic quasigeoid modelling, the final step is required to combine gravity and GPS/levelling data to remove such systematic bias. This step is missing and study is therefore not completed.

Response: Thanks the reviewer for these beneficial comments. We agree with the reviewer's comments that there are biases between the modeled purely gravimetric quasi-geoid and local GPS/leveling data, mainly due to the commission errors in the GGM and uncorrected systematic errors in the local gravity data and leveling system. These biases also show up when we compare the local GPS/leveling data and existing gravimetric solutions (e.g., EGG08, EGM2008, and EIGEN-6C4). Generally, corrector-surface (Fotopoulos 2005; Nahavandchi and Soltanpour 2006) or more complicated algorithms, e.g., least squares collocation (Tscherning 1978) and boundary-value methodology (Klees and Prutkin 2008; Prutkin and Klees 2008), can be applied to reduce systematic errors and properly combine GPS/leveling data and gravimetric solutions. Also, the authors proposed a direct approach to properly combine GPS/leveling data with the gravimetric quasi-geoid/geoid, where GPS/leveling data are treated as an additional observation group to form a new functional model, see Wu et al. (2017a). However, the target for this study is to develop a multilayer approach for gravimetric quasi-geoid modeling, which is served as a basic surface for geophysical applications, e.g., study the ocean circulation and structure of lithosphere. While, after implementing these methods for combining local GPS/leveling and gravimetric model, the derived quasi-geoid is not purely gravimetric, e.g., see the case in Wu et al. (2017a). Besides, we only have the well distributed GPS/leveling data in the limited region, i.e., in Netherlands, Belgium, and Germany; while, in other regions, no high-quality data are available. Thus, if we use the locally distributed GPS/leveling data for removing these systematic errors and computing the combined quasi-geoid, the final solution may be distorted in other regions, especially in the ocean parts, since no control data in these regions have been combined. And, this may be detrimental for geophysical applications in this area, e.g., investigating the ocean circulation in the North Sea. Over all, based on the reviewer's comments, we enhance the relevant part and add the necessary information, please refer to pp 21-22 in the revised version.

Overall, the application of multi-layer instead of single-layer approach cannot justified the publication in research-focused journals mainly due to a low scientific impact.

Response: Thanks for the reviewer's comments. First, we notice that the model development approach may coincide with the scope of "Geoscientific Model Development", and we also see describing developments such as new parameterizations is one of scopes of this journal. Moreover, we develop a new parameterization of SRBFs' networks for local gravity field modeling based on the idea of MRR, inspired by the power spectrum analysis of local gravity signals. Instead of constructing the multi-scale method in a purely mathematical way, we link the different detailed signals to the anomaly sources located at different depths, which are recovered by the various SRBFs' layers. To our knowledge, no existing literatures studied this issue. Besides, we directly compare the performances the multilayer approach and single-layer one, and this may also provide references for assessing the advantages and disadvantages of different methods. In addition, for justifying the performances of different approaches, four aspects are considered in this study. First, from the spectrums of different approaches, i.e., Figure 4 in the new version (Figure 5 in the original one), we notice that the single-layer approach is only sensitive to parts of the signals' spectrum; while, for the high-frequency band, this approach is less sensitive. However, the multilayer approach effectively covers the spectrum of the local gravity signals, which is both sensitive to the low- and high-frequency bands. This gives us the original insight for the performances of different approaches from a theoretical perspective of view. Then, we check the data residuals after the least squares adjustment, which show the multilayer approach fits the data better, especially in regions with strong topography variations, where the high-frequency signals correlated with local topography dominate the small-scale features of regional gravity field. And, this result also coincides with the analysis of spectrums of different approaches, where the multilayer approach is more sensitive to the high-frequency bands. However, based on the reviewer's comments, we admit that the analysis of data residuals can't be treated as the criteria for justifying the performances of different approaches, since these gravity data have been used for modeling purpose, and the SD values for the data residuals derived from different methods should be the internal agreement. Besides, due to the limitation of the accuracies of gravity data, we can't make conclusions too firmly only depends on the analysis of data residuals. Moreover, based on the comments of Referee #2, we implement a Akaike information criterion (AIC) test for different models. AIC rewards the goodness of fit of data, but also includes a penalty with the increasing of the number of estimated parameters. In other words, it deals with the trade-off between the goodness of fit of the model and the simplicity of the model. AIC value is an estimator of the relative quality of statistical models for a given set of data, providing a means for model selection, and the model that gives the minimum AIC value may be more preferable (Akaike, 1974; Burnham and Anderson, 2002). The associated results demonstrate that the multilayer model gives a smaller AIC value, which reaches a better balance between the goodness of fit of data and the simplicity of the model. This gives us the value information regarding the performances of different approaches in the view of statistical test, please see pp 19 for details in the revised manuscript. In addition, we test the test the ability of realistic extrapolation of different regional models recovered from various methods, where another independent data set, i.e., GPS/leveling measurements, is introduced for external validation. From these results, we see that the multilayer approach not only lead to a reduction for the data residuals in the least squares adjustment, but also derives a better solution assessed by the independent control data, compared to the single-layer approach. Based on these results, the authors believe this study may contribute to the literatures. Based on the reviewer's comments, we restructure the relevant parts and add the necessary information, please refer to the revised version.

Interactive comment on "A multilayer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea" by Yihao Wu et al.
Anonymous Referee #2
Received and published: 10 April 2018

I have read the interesting manuscript "A multi-layer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea" by Yihao Wu, Zhicai Luo, Bo Zhong, and Chuang Xu. The manuscript focuses on a multi-layer approach compared to a single layer approach in the computation of the local gravity geoid.

I have the following comments: 1. Muliti layer approach gives (according to Table 5 and Fig 7 and page 16-17) a better fit than single layer approach. The fit would naturally increase with incrasing level of parameters, but it is statistical significant. A statistical test such as AIC (Akaike information criterion) or BIC would give valuable information.

Response: The authors thank the reviewer for this beneficial comment. Yes, the authors totally agree with the reviewer's comment, and the fit with the data using the multilayer approach with more parameters naturally increase from the view of statistical analysis. We believe it is a very good suggestion for implementing the Akaike information criterion (AIC) or Bayesian information criterion (BIC) test of different models. In this study, we implement the AIC test, which may provide value information for model selection in another aspect. AIC rewards the goodness of fit of data, but also includes a penalty that is an increasing function of the number of estimated parameters. It deals with the trade-off between the goodness of fit of the model and the simplicity of the model. AIC test is an estimator of the relative quality of statistical models for a given set of data, providing a means for model selection, and the model that gives the minimum AIC value may be more preferable (Akaike, 1974). The AIC value of the model is defined as $AIC = 2k\text{-}2\ln(\hat{L})$, where k is the number of estimated parameters in the model, and $\hat{L}$ is the maximum value of the likelihood function for the model (Burnham and Anderson, 2002).
For gravity field modeling in this study, we work within the framework of least squares adjustment, i.e., the unknown coefficients of Poisson wavelets of different approaches (the multilayer and single-layer approach) are computed through the least squares method. We also assume that the data residuals derived from different approaches are distributed according to independent identical normal distributions with zero mean values, also see the information of data residuals in Table 5 in the revised manuscript. Then, the maximum likelihood estimate for the variance of a model's residuals distributions is $\hat{\theta}^2 = RSS / n$, where RSS is the residual sum of

squares (RSS), and n is the number of observations (Burnham and Anderson, 2002). Then, the AIC value of model is given as $AIC = 2k + n\ln(RSS/n) + C$, and C is a constant independent of the model (Burnham and Anderson, 2002). Since only differences in AIC are meaningful, the constant C can be ignored, and we can conveniently take $AIC = 2k + n\ln(RSS/n)$ for model comparisons. In this study, we compare the performances of the multilayer and single-layer model through the AIC test. In details, the number of gravity observations is $n = 894649$, and the numbers of estimated parameters k in the multilayer and single-layer model are 47504 and 19477, respectively. The RSS values for the multilayer and single-layer model are $8.8527 \times 10^5\ mGal^2$ and $1.3296 \times 10^6\ mGal^2$, respectively, based on the data residuals after the least squares adjustment. Then, the AIC values for the multilayer and single-layer model are estimated as 85581 and 393400, respectively. Based on these statistics, we notice that the multilayer model gives a smaller AIC value, which may be more preferable since it reaches a better balance between the goodness of fit of data and the simplicity of the model. According to the reviewer's comments, we add the information of AIC test in the revised manuscript, please refer to the abstract (pp 1) section 3.3 (pp 19), conclusion (pp 30), and the Appendix (pp 32) in the updated version.

2. For a better comparison with EGG08 the same or similar global geopotential model should be used.

Response: The authors thank the reviewer for this beneficial comment. For further validate the quality of QGNSea V1.0, we compare it with other existing models, where a regional model call EGG08 and other global geopotential models (GGMs) are introduced. EGG08 is a regional gravimetric quasi-geoid model covers most areas in Europe; this model was recovered by stokes integral based on locally distributed gravity data, which was provided in terms of gridded data instead of spherical harmonics like GGMs (e.g., EGM2008 and EIGEN-6C4), and the space resolution of which is 1′ in latitude and 1.5′ in longitude, see Denker (2013). We also use other global geopotential models for comparisons since the authors don't have access to other regional gravimetric quasi-geoid models; for example, a new Europe gravimetric quasi-geoid called EGG2015 has been implemented (Denker, 2015), however, this model is seems not publicly available. Thus, the two high-order GGMs, i.e., EGM2008 (d/o 2190) with the spatial resolution of 5′×5′, EIGEN-6C4 (d/o 2190) with the spatial resolution of 5′×5′are incorporated for further comparisons, since these two models have relatively higher spatial resolutions and better accuracies compared to most of other available GGMs, when compared with the globally distributed GPS/leveling data, see the information in http://icgem.gfz-potsdam.de/home. However, according to the reviewer's comments, we introduce another two recently published high-order GGMs (i.e., GECO (d/o 2190)

(Gilardoni et al. 2015), and SGG-UGM-1 (d/o 2159) (Liang et al. 2018)), which were developed by combining GOCE data into EGM2008, for further comparisons. We also restructure and modify the relevant parts in the updated manuscript based on the reviewer's comments, please see pp. 24-27 in the revised version.

3. Figure 2: A comment related to the different patterns observed in Figure 2 would be of interest.

Response: The authors thank the reviewer for the comment. First of all, the authors believe the original wavelet details with stripe like patterns shown in Figure 2 are problematic (also see the interactive comments from the third referee), since we carefully check the source code for wavelet decomposition, and find bugs that may derive incorrect wavelet details. Based on the reviewer's comments, we redo the wavelet decomposition after the removal of bugs of source code, and compute the new wavelet details and approximation, please refer Figure 1 in the updated version, i.e., in pp 11, where no strange stripy patterns occur. Moreover, we provide the geophysical evidences for the patterns of different wavelet details. More specifically, D_1 and D_2 are seems dominated by the high-frequency signals correlate strongly with the local topography, which are mainly due to the uncorrected topographical signals in RTM corrections. D_3 and D_4 with respective average source depths of 4.5 km and 9.2 km primarily reflect the density distribution of the upper crust. The distribution of D_5 and D_6 is in agreement with the tectonic structure of the middle crust. D_7 is consistent with the Moho undulation. D_8 and A_8 represent density distribution of the upper mantle. Overall, these decomposed gravity anomalies can reveal the tectonic structure of study area at different depths. Based on the reviewer's comments, we add the detailed comments related to the different patterns of wavelet details in Figure 1 (Figure 2 in the original version) in the revised manuscript, please see the information in pp13-14. Moreover, we notice that the wavelet details and approximation change after we implement the wavelet decomposition with the errors corrected source code, and we redo the whole procedure for the multiply layers' network design, i.e., estimating the depths of different layers and the number of Poisson wavelets in each layer. Then, we recompute the solution based on the multilayer approach with the updated parameters (i.e., the depths of different layers and the number of Poisson wavelets in each layer), and redo the comparisons with existing models based on the updated solution. Following, the geodetic MDT (called MDTNS_QGNSea) based on the updated model derived from the multilayer approach is computed. Please refer to pp 13-29 in the revised manuscript.

Interactive comment on “A multilayer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea” by Yihao Wu et al.

With great interest, I read the article of Wu et al. "A multilayer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea". Unfortunately, the paper lacks many details which makes it hard to assess the results. My main concern is, however, that the authors are not very consistent compared to their previous study presented in Wu et al. 2017b. In that study, they used beside shipboard and terrestrial gravity anomalies also airborne gravity disturbances, multi-satellite altimetry measurements, and GOCE gravity gradients to compute a quasi-geoid model. To validate that model, the same GPS/leveling datasets are used as the ones used in this study. If we compare the statistics of solution A (obtained with the single-scale approach) in Wu et al. 2017b (solution computed without the use of GOCE gravity gradients) they obtained in terms of standard deviation 1.8, 1.8, and 1.6 cm for the Netherlands, Belgium, and Germany respectively. In this paper, they obtained using the single-scale approach 1.2, 2.8, and 2.9 cm for the Netherlands, Belgium, and Germany respectively. These differences are huge! Using their multiscale approach, they obtained 0.9, 2.2, and 2.1 cm for the Netherlands, Belgium, and Germany respectively. Hence, except for the Netherland this solution still has a lower quality compared to what the authors presented in Wu et al. 2017b. The differences become even larger in case I compare their solutions obtained including GOCE gravity gradients data. To me, this shows that apparently the use of different layers of SRBFs is not the main issue in obtaining a better quasi-geoid model. Below, I provide some other concerns.

Response: The authors thank the reviewer for these beneficial comments. To our knowledge, the solutions in this study are indeed inconsistent with ones shown in Wu et al. (2017b), and should not be made simply comparison with each other. There are several reasons that you find the accuracy of solution modeled with the single-layer approach in this study is different from the one displayed in Wu et al. (2017b). First, in this study we only use terrestrial and shipboard gravity data, no airborne or radar altimetry data are incorporated. While, for the solution A (without GOCE data) in Wu et al. (2017b), we used terrestrial, shipboard, and airborne gravity data, and radar altimetry data. Thus, even we use the same GPS/leveling data for validation, we observe the different statistics for accuracy assessment. Second, the target area in this study and the one in Wu et al. (2017b) are not consistent. The area in the study of Wu et al. (2017b) extends from 49.5 °N to 56 °N latitude and 0.25 °E to 8.25 °E longitude (see page 6 in Wu et al., 2017b); While, in this study we choose a much larger area, which covers an area of 49 °N-61 °N latitude and -6 °E-10 °E (see page 3 in the original manuscript). And, when we choose a larger region, more data in UK, Norway, and the North Sea are incorporated. However, we notice that the data in Norway are sparsely distributed, especially in the mountainous regions; and this situation also occurs in the north parts of the North Sea, see Fig.2 in Wu et al. (2017b). Consequently, the quality of the solution may be affected if different gravity data are introduced, even when we validate the solution only use the GPS/leveling data in the Netherlands, Belgium, and Germany. We should not directly compare these statistics if these solutions are modeled under different conditions. For the similar reasons, we can't simply compare the solutions computed in this study with the ones in Wu et al. (2017b).

pp 2: In the first paragraph the authors state (pp 2: 4-5): "However, one layer of SRBF's parameterization may be only sensitive to parts of signals' spectrum and reduce the quality of the solution." –> This may seems so if you look to the spectrum of the SRBFs being used. However, several authors (e.g., Slobbe 2013) have successfully computed quasi-geoid solutions using one or two layer(s) of SRBFs that have an accuracy comparable or even better than the authors present in this paper. The only prerequisite is that the energy in the data at the lowest and highest frequencies is reduced by using a reference GGM and a digital terrain model, respectively. (Slobbe, D. C. (2013), Roadmap to a mutually consistent set of offshore vertical reference frames, Ph.D. thesis, Delft University of Technology.).

Response: The authors thank the reviewer for these comments. Yes, we believe the reviewer's statement is right regarding this multilayers approach may work fine when only the residual gravity field is modeled from the ground-based data, i.e., the long- and short-wavelength parts have been removed. In this study, we also model the regional gravity field within the framework of remove-compute-restore method, and only the residual signals are parameterized, we emphasize this in the revised manuscript according to the reviewer's comments, see pp 2 in the updated manuscript. We also see the (one) two layers of SRBFs works fine, i.e., see Slobbe (2013) and Wittwer (2009). However, we should not compare the accuracies of the solutions if they are modeled under different solutions, see our detailed response to Q1. We also cite the contributions of the existing literatures regarding the modeling with single-layer approach, i.e., Wittwer (2009), Slobbe (2013). Moreover, we remove the "However, one layer of SRBF's parameterization may be only sensitive to parts of signals' spectrum and reduce the quality of the solution.", since we believe this is too absolute to some extent, which may lead to the wrong understanding. Based on the reviewer's comments, we modify and restructure the relevant contents, please see pp 2 in the updated version.

pp 2: I somehow have difficulties in understanding the main objective of this paper. The authors state without motivation (pp 2: 23-26): "However, differing from these methods mentioned above, we propose a multilayer approach, inspired by the power spectral analysis of local gravity observations, which indicates the gravity signals are the sum of the contributions generated from the anomaly sources that locate at different depths." In my opinion, a proper motivation is required. It should become clear what are the limitations in existing multi-resolution representation/multi-scale approaches and how the approach proposed by the authors is going to tackle these. Definitely, the authors are not the first ones that utilize a multi-scale approach as they mention themselves.

Response: The authors thank the reviewer for these beneficial comments. Yes, we think the reviewer's comments are right. In our opinion, there are two limitations for the existing studies. First, to our knowledge, no direct comparisons have been made between the single-layer approach and multi-scale one regarding the performances in local gravity field recovery. Besides, the existing multi-scale methods mainly construct the multi-scale framework in a mathematical way, where no explicit geophysical meanings are investigated. Thus, the main contributions of this study are twofold. First, to develop a new parameterization of SRBFs network in the framework of the MRR idea, i.e., the so-called multilayer approach; and the multiply layers are linked to the anomaly sources at different depths beneath the topography, which aim at recovering the signals at different levels. To our knowledge, no existing literatures studied this issue. Moreover, we assess the performances of the multilayer approach and traditionally-used single-layer one in this study, where the advantages and disadvantages of different methods are analyzed. According to the reviewer's comments, we modify the relevant part the updated manuscript and make the motivation more clearly, please see pp 2-3 in the revised version.

Section 2.1: It is not entirely clear to me whether or not the authors used GOCE gravity gradients as an additional datasets as they did in Wu et al. 2017b? The confusion is introduced by their sentence (pp 19: 6-8): "Moreover, the improvements in the frequency bands that GOCE data contribute may be also the reasons, since EGM2008/EGG08 was developed without GOCE data." This suggests that they used it. However, the dataset is not mentioned in Section 2.1. And what about the radar altimeter data and airborne gravity data the authors used in Wu et al. 2017b? If, indeed, these datasets are not used. What is the reason for that? In the abstract the authors mention that "A multilayer approach is set up for local gravity field modeling based on the idea of multi-resolution representation merging heterogeneous gravity data." What they do understand by "heterogeneous"? With their approach, can they not handle different data types?

Response: The authors thank the reviewer for these beneficial comments. We didn't directly use the along-track GOCE gradients as the additional groups as we did in Wu et al. (2017b). In fact, only the terrestrial and shipboard gravity data are introduced as the observation groups, Section 2.1 give the details regarding the data sets we use here. Although we didn't directly GOCE gradients, we used the GOCO05S as the reference model, which was computed with GOCE data. However, for the development of EGM2008/EGG08, no GOCE data were used. Thus, in the bandwidth that GOCE data contribute, i.e., in frequencies from 0.005 to 0.1 Hz, we believe our model may outperform EGM2008/EGG08. In this sense, we say "Moreover, the improvements in the frequency bands that GOCE data contribute may be also the reasons, since EGM2008/EGG08 was developed without GOCE data.", it doesn't not mean we directly combine the GOCE data as additional observation groups for modeling, but just use a more accurate reference model in the measurement bandwidth (MBW) of GOCE mission. The motivation of this study is to develop a new parameterization of SRBFs network in the framework of the MRR idea, i.e., the so-called multilayer approach, and compare it with the traditionally-used single-layer approach for the performances in regional gravity field recovery. For a case study, we only use the terrestrial and shipboard gravity data, and the results in case derive reasonable solutions, which can be used for supporting the conclusions of this study. The "heterogeneous" here not only means the different types of observations, but also refer to the data sets with different spatial resolutions/coverage, different noise levels, see Wu et al. (2017c) in the updated version regarding the details of heterogeneous data sets. The different types of observations groups can be combined through the multilayer approach just similar as the way the researchers did for in the single-layer approach, e.g., see Klees et al. (2008), and Slobbe (2013).

pp 6: From Figure 1, the authors conclude that "the gravity signals are the superstition (should be "superposition" I guess) of the contributions generated from the anomaly sources at different depths; and the signals originated from different anomaly sources have heterogeneous spectral contents". I have strong doubts. In Figure 1, I observe a quite smooth spectrum (no distinct peaks or whatsoever). The red lines are to me somewhat artificial.

Response: The authors thank the reviewer for this comment. First, we only model the residual gravity signals in this study, and the power spectrum showed in Figure 1 is based on the residual gravity data in Sect 2.1, the short- and long-wavelength signals are removed. Moreover, the local gravity signals are the sum of the contributions of different anomaly sources, i.e., the contributions from different anomaly sources have been separated, and the spectrum here shows the one for the mixed signals. After we separate the different signals with wavelet decomposition, and more distinguished spectrums occur, see Figure 3 in the revised manuscript. We also want to mention that Figure 1 is just an example support the statement that the gravity signals are the sum of the contributions of different sources, and red lines are also the illustrations show that slopes of the spectrum are different in different frequency bands, and please see our response to the question below regarding how we estimate the slopes (i.e., the red lines) of the spectrum. However, we also think this figure is confusing to some extent, and we remove this figure and restructure the relevant part based on the reviewer's comments, please see pp 6 in the updated version.

pp 6: It is not clear how the authors estimated/obtained A_W (first term of Eq. 4)? Given Eqs. 6-7, I suppose A_W is not estimated...?

Response: The authors thank the reviewer for this comment. Based on Eq.4, the gravity anomaly can be decomposed into a number of wavelet details and a wavelet approximation. Thus, the difference between the gravity anomaly and the sum of wavelet details is the wavelet approximation A_W, similar information can be found in Xu et al. (2017, 2018). The target for the wavelet decomposition is to design the parameterizations of multilayer approach, and for modeling purpose, the point-wise gravity data are combined just as we do in the single-layer approach.

pp 8: To compute their solutions, the authors applied variance component estimation and regularization. However, nowhere the regularization parameter is given, neither the estimated weights.

Response: The authors thank the reviewer for this beneficial comment. Yes, we believe the reviewer is right, and the variance factors for different types of observations are important, indicate their relative contributions, and play a key role in data combination. According to the reviewer's comments, we add the information of estimated variance factors of different observations groups and regularization parameter in the updated version, please see pp 17.

pp 8: It is not clear why the authors used 10 as the "preliminary maximum order for decomposition"? Why not 20 or 5?

Response: The authors thank the reviewer for this beneficial comment. This is a good question. To some extent, the original maximum order is arbitrarily chosen. However, wavelet analysis has a number of nice properties, for instance, the low-order details are invariant with the increase of decomposition order, and only the high-order details and wavelet approximation change. Thus, we can preliminarily choose a predefined order for decomposition, and analyze the derived details as we do in Section 3.1. If there are still details that are useful for constructing the multilayer model haven't been separated, we need to increase the decomposition order until all the useful details have been extracted; otherwise, we can truncated to a specific order as we do in this study, and compute the corresponding the necessary details and approximation for constructing the multiply layer's network. According to the reviewer's comment, we add and enhance this information in the updated version, please see pp 9.

pp 10: In the manuscript, the authors suggest that the wavelet details (D_W) have a kind of geophysical interpretation; for example D_W is explained as "the local anomaly originated from shallow and small-scale heterogeneous substances." If so, can the authors comment on the maps shown in Figure 2? To me, these are very peculiar. In particular D_5, D_6, and D_7 show strange stripy patterns...

Response: The authors thank the reviewer for these beneficial comments. We think the reviewer's concern is right regarding these strange stripe like signals, since we carefully check the source code for wavelet decomposition, and find bugs that may derive incorrect wavelet details. Based on the reviewer's comments, we redo the wavelet decomposition based on errors corrected source code, and compute the updated wavelet details and approximation, please refer to Figure 1 (in pp 11) in the updated version, and no strange stripy patterns occur. Moreover, we provide the geophysical evidences for the different patterns of various wavelet details. More specifically, D_1 and D_2 are seems dominated by the high-frequency signals correlate strongly with the local topography, which are mainly due to the uncorrected topographical signals in RTM corrections. D_3 and D_4 with respective average source depths 4.5 km and 9.2 km primarily reflect the density distribution of the upper crust. The distribution of D_5 and D_6 is in agreement with the tectonic structure of the middle crust. D_7 is consistent with the Moho undulation. D_8 and A_8 represent density distribution of the upper mantle. Overall, these decomposed gravity anomalies can reveal the tectonic structure of study area at different depths. Based on the reviewer's comments, we add the detailed comments related to the different patterns of Figure 1 in the revised manuscript, please see the information in pp13-14. We also notice that the wavelet details and approximation change after we implement the wavelet decomposition with the errors corrected source code, and we redo the whole procedure for the multiply layers' network design, i.e., estimating the depths of different layers and the number of Poisson wavelets in each layer. Then, we recompute the solution based on the multilayer approach with the updated parameters of multiply layers (i.e., the depths of different layers and the number of Poisson wavelets in each layer), and redo the comparisons with existing models based on the updated solution. Following, the geodetic MDT (called MDTNS_QGNSea) based on the updated model derived from the multilayer approach is computed. Please refer to pp 13-29 in the revised manuscript.

pp 13: With Figure 4, I have the same problem as I have with Figure 1. How they came up with the red lines?

Response: The authors thank the reviewer for this comment. The average depths for the power spectrum of wavelet details are estimated from the eq.(5). Actually, a number of literatures showed how to estimate the depths from these spectrums, e.g., see Figure 4 in Xu et al. (2018). More specifically, the red lines represent rates of change for logarithmic power relative to wave number, which are estimated by autoregressive method. The starting point and terminal point of the red lines are inflection points of the curves (green lines in Figure 3), recognized by us according to the trend of the curves. Based on the reviewer's comment, we also add this information in the revised manuscript, see pp 13 in the updated version.

pp 15: The authors mention without any motivation that "Point-wise terrestrial and shipboard gravity anomalies are merged for modeling." Why, these datasets usually have different accuracies...

Response: The authors thank the reviewer for this comment. For modeling purpose, point-wise terrestrial and shipboard data are combined. These data have different accuracies, and this is also one of the reasons why we need the MCVCE method for estimating the variance factors for different observation groups. The gridded gravity data is only used for wavelet decomposition, i.e., for designing the multiply layers' network, since this wavelet decomposition method needs the regularly distributed data. While, for modeling purpose, the point-wise data are directly used just the same as the single-layer approach. We also enhance this part for avoid confusing based on the reviewer's comment, please refer to pp 17 in the updated version.

pp 15-16: "These results demonstrate that the multilayer approach can more accurately recovers the local high-frequency signals than the single-layer one." –> Of course, the least-squares residuals are lower! In the multilayer approach you locate the SRBFs much shallower!

Response: The authors thank the reviewer for this enlightening comment. Yes, we believe the lower residuals may be attributed to the shallower SRBFs. Shallower SRBFs are more sensitive to the local high-frequency signals, and the corresponding spectrum also shifts to high-frequency bands, which may lead to a better fit to the data. However, there are still two aspects may be of concern. First, we parameterized the local gravity field by 7 layers with different depths, where the layer7 are still deeper than 40 km (where we locate the single-layer of SRBFs' grid), see Table 3 in the revised manuscript, thus not all the layers are shallower than 40 km. In addition, to our experience with the single-layer approach, the shallower SRBFs' grid may lead to a reduction of least square residuals, but not guarantee a better solution, i.e., the better fit to the independent control data for external validation, please refer to Figure 2, 3 in Wu et al. (2016), which clearly shows a shallower grid than 40 km may not derive a better solution. However, in this study, the multilayer approach not only derives a better fit to the data, but also obtains better solution validated by the control data. This can't acquire by solely putting the SRBFs' grid shallower. According to the reviewer's comments, and we restructure and enhance the relevant parts in the updated version, please refer to pp 18-19.

Table 6: The authors have used GNSS/leveling data to validate their quasi-geoid model. What is not clear to me at all is why the statistics presented in Table 6 for the single-layer approach are so different from the values they presented in Wu 2017b (solution A). In that paper, they obtained in terms of standard deviation 1.8, 1.8, and 1.6 cm for the Netherlands, Belgium, and Germany respectively. The parametrization they have used is the same. In this paper, they obtain 1.2, 2.8, and 2.9 cm for the Netherlands, Belgium, and Germany respectively. These differences are enormous! Can the authors explain what happened? Is that due to the fact that you did not use radar altimeter and airborne gravity data, and merged shipboard and terrestrial data sets. Anyway, it seems that compared to their work presented in Wu 2017b, their multi-scale approach performs still worser (except for the Netherlands)!

Response: The authors thank the reviewer for this comment. In our opinion, we should not directly compare these statistics if these solutions are modeled under different conditions. The solution derived from single-layer/multi-layer approach should be different from the solution A in Wu et al. (2017b), since the inputs for these solutions are inconsistent. Thus, even we use the same GPS/leveling data for validation, the derived statistics are heterogeneous. Please see our detailed response to the first question.

pp 19: "Apart from the application of different techniques for modeling, these differences are partly interpreted as the additional signals introduced by QGNSea V1.0, stemming from the incorporation of more high-quality gravimetry". This maybe applies to EGM2008 and EIGEN-6C4, but not to EGG2008.

Response: The authors thank the reviewer for the comment. Yes, we believe the reviewer is right. We also refer to EGM2008/EIGEN-6C4 when we say the additional signals introduced by QGNSea V1.0 are stemmed from the incorporation of more high-quality gravimetry. And, the sentence "Apart from the application of different techniques for modeling, these differences are partly interpreted as the additional signals introduced by QGNSea V1.0, stemming from the incorporation of more high-quality gravimetry" further explains "For EGM2008/EIGEN-6C4, remarkable differences show in south of Norway and northwest of Germany". However, according to the reviewer's comments, we modify this part slightly to eliminate misunderstanding, see pp 24 in the updated version.

Figure 8, the analysis is hampered by edge effects in QGNSea V1.0. The authors should exclude the edges of the area over which they computed QGNSea V1.0.

Response: The authors thank the reviewer for this comment. Yes, we believe the reviewer is right that the edge effects should be excluded. In fact, for plotting Figure 8 in the original manuscript (Figure 7 in the revised version), we have excluded the edge effects by contracted by 0.5 ° in all the directions. For modeling purpose, the boundary limits for the target area is chosen as 49 °N-61 °N latitude and -6 °E-10 °E longitude, see sect 2.1. While, for displaying the differences between different models, the signals only inside 49.5 °N-60.5 °N latitude and -5.5 °E-9.5 °E longitude have been extracted and compared. We also add this information in the updated version, please refer to pp 24.

The derived MDT models are not realistic. Please use DTU13MSS and EGG2008 to compute a MDT model and compare that to the one obtained using DTU13MSS and QGNSea V1.0. Prominent signals, like the Norwegian coastal current are not visible at all (e.g., Idžanovi ÌA¸c 2017)! (Idžanovi ÌA¸c, M., V. Ophaug, and O. B. Andersen (2017), The coastal mean dynamic topography in Norway observed by CryoSat-2 and GOCE, Geophys. Res. Lett., 44, 5609–5617, doi:10.1002/ 2017GL073777.)

Response: The authors thank the reviewer for these beneficial comments. Yes, we agree with the reviewer's comments, and the geodetic MDTs in the original manuscript are not realistic. The problem is seems due to the implementation of too strong filtering on the raw MDTs. In the original manuscript, we compared the MDT derived from QGNSea V1.0 with the existing global model called DTU13MDT. DTU13MDT was computed in a purely geodetic way, where the difference between DTU13MSS and the quasi-geoid derived from EGM2008 was used to estimate the raw MDT, and the derived MDT was further smoothed by a Guassian filter with a correlation length of 75 km to suppress the small-scale signals (Andersen et al., 2013). To make these comparisons consistently, in the original manuscript, the computed raw MDT (the difference between the DTUMSS13 and QGNSea V1.0) was also filtered by a Guassian filter with a correlation length of 75 km. However, based on the reviewer's comments, we believe this filter may be too strong since the prominent signals have been filtered out. According to the reviewer's comments, in the revised manuscript, we compute the raw MDT by computing the difference between DTUMSS13 and QGNSea V1.0/EGG08, and filter the raw MDT by a Gaussian filter to further smooth the derived MDT, which is called as MDTNS_QGNSea/MDTNS_EGG08. Considering the small-scale signals that have the wavelengths shorter than several kilometers can't be recovered from the local gravity data, since the mean distance between gravity data is approximately at 6~7 km level, the correlation length of Gaussian filter is chosen as 6 km instead of 75 km in the revised manuscript. This time, the derived MDTs show more realistic patterns, although MDTNS_QGNSea don't provide a full picture of Norwegian coastal currents due to the limited data coverage in Norway and its neighbouring ocean areas, please see Figure 9 in the updated version. According to the reviewer's comments, we restructure and modify the part for MDT comparison, please refer to pp 27-29 in the new version.

A multilayer approach and its application in modeling QGNSea V1.0: a local gravmetric quasi-geoid model over the North Sea

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Abstract: A multilayer approach is set up for local gravity field modeling recovery based on the ideain the framework of multi-resolution representation, where the gravity field is parameterized as the superposition of the multiply layers of Poisson wavelets located at the different depths beneath the topography merging heterogeneous gravity data. Different layers of Poisson wavelets' grids are formed designed to recover the signals at various levels, where the shallow and deep layers mainly capture the short- and long-wavelength signals, respectively. The depths of these layers beneath the topography are linked to the locations that the different anomaly sources locate, estimated by the wavelet decomposition and power spectrum analysis. For testing the performance of this approach, a gravimetric quasi-geoid over the North Sea in Europe called QGNSea V1.0 is computed and compared with other existing models. The results show that the multilayer approach outperforms fits the gravity data better than the traditionally used single-layer one in high-frequency bands, and the former fit the gravity data better, especially in regions with a tendency toward topographical variation. A Akaike information criterion (AIC) test demonstrates that the multilayer model gives a smaller AIC value, reaches a better balance between the goodness of fit of data and the simplicity of the model. Moreover, tThe evaluation with independent GPS/leveling data tests the ability of realistic extrapolation of regional models computed from different approaches, showing that the accuracies of QGNSea V1.0 modeled from the multilayer approach are improved by 0.4 cm, 0.9 cm and 1.1 cm in the Netherlands, Belgium and parts of Germany, respectively, compared to the original solution computed from the single-layer approach. Further validation with

existing models shows QGNSea V1.0 has the best quality, which may be beneficial for studying the ocean circulation between the North Sea and its neighbouring waters.

1. Introduction

Knowing of earth's gravity field at regional scales is crucial for a variety of applications in geodesy. It not only facilitates the use of Global Satellite Navigation System to determine orthometric/normal heights in geodesy and surveying engineering, but also plays a fundamental role in in oceanography and geophysics.

Regional gravity field determination is typically implemented within a framework of remove-compute-restore methodology (RCR) (Sjöberg, 2005), where the long-wavelength signals are often recovered by satellite-only global geopotential models (GGMs) derived from the dedicated satellite gravity missions, such as the GRACE (Gravity Field and Climate Experiment) (Tapley et al., 2004) and GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) (Rummel et al., 2002). Middle- and short-wavelength signals are extracted from the locally distributed gravity-related measurements (Wang et al., 2012; Wu et al., 2017c). Spherical radial basis functions (SRBFs) are of great interest for gravity field modeling at regional scales over years (Eicker et al., 2013; Naeimi et al., 2015). Typically, the widely-used SRBFs method is implemented by the so-called single-layer approach, i.e., the parameterization of gravity field is only based on a single-layer of SRBFs' grid (Wittwer, 2009; Bentel et al., 2013; Slobbe, 2013; Wu et al., 2017c). However, one layer of SRBF's parameterization may be only sensitive to parts of signals' spectrum and reduce the quality of the solution.

It has been suspected for long that if the single-layer approach can extract the full information of local gravity data, and the multi-resolution representation (MRR) method with SRBFs has been investigated over the recent years Contrary to the single-layer approach, SRBFs are also of special interest for multi-resolution representation (MRR) for merging different spectral contents of complementary observations techniques (Freeden et al., 1998; Fengler et al., 2004, 2007). The motivation behind this is the feasibility to compute the signals at different scales independently, and the ability to identify the certain geophysical features at the different spectral bands (Wittwer, 2009). Freeden and Schreiner (2006) proposed a multi-scale approach based on the locally supported wavelets for determining the regional geoid undulations from the deflections of the vertical. Freeden et al. (2009) demonstrated that the multi-scale approach using spherical wavelets provided local fine-structured features such as those caused by plumes, which allowed a scale- and space-dependent characterization of this geophysical phenomenon. Schmidt et al. (2005, 2006, 2007) developed a multi-representation method for static and spatiotemporal gravity field modeling through SRBFs, where the input gravity signals were decomposed into a certain number of frequency-dependent detail signals, and concluded that this approach could improve the spanning fixed time intervals with respect to the usual time-variable gravity fields. Chambodut et al. (2005) set up a multi-scale method for magnetic and gravity field recovery using Poisson wavelets, and created a set of hierarchical meshes associated with the wavelets at different scales, where a level of subdivision corresponded to a given wavelet scale. Panet et al. (2011) extended the approach developed by Chamboudt et al. (2005), and applied a domain decomposition approach to define the hierarchical subdomains of wavelets at different scales, which allowed to split a large problem into smaller ones. These results show the multi-scale approach with SRBFs has a good prospective in gravity field recovery~~modeling using heterogeneous data~~, however, to our knowledge, no direct comparisons have been made between the single-layer approach and multi-scale one regarding the performances in local gravity field recovery. Besides, the existing multi-scale methods mainly construct the multi-scale framework in a mathematical sense, where no explicit geophysical meanings are investigated.~~.~~ In this study, inspired by the power spectral analysis of local gravity signals, we develop a new parameterization of SRBFs network in the framework of the MRR idea, i.e., the so-called multilayer approach; and the multiply layers are linked to the anomaly sources at different depths beneath the topography, which aim at recovering the signals at different levels. In this way, the parameterization of multi-scale method can be linked to the different anomaly sources at different depths. Moreover, the performances of the multilayer approach and traditionally-used single-layer one are directly compared in this study, where the advantages and disadvantages of different methods are analyzed.

~~However, differing from these methods mentioned above, we propose a multilayer approach, inspired by the power spectral analysis of local gravity observations, which indicates the gravity signals are the sum of the contributions generated from the anomaly sources that locate at different depths.~~

The structure of the manuscript is as follows: the ~~heterogeneous~~ data in a study area in Europe are firstly described in Section 2. Then, the multilayer approach based on the MRR representation is introduced, and the wavelet decomposition and power spectrum analysis are applied for ~~estimating the depths of various layers beneath the topography~~constructing the networks of Poisson wavelets with the multilayer approach. In addition, ~~we set up~~ the function model based on the multilayer approach is set up, and the method for unknown coefficients of Poisson wavelets is introduced and combine the different types of gravity data. We construct the networks of multiply layersmultilayer model in section 3, and compare the performances of different approaches. Finally, the gravimetric quasi-geoid solution over the North Sea called QGNSea V1.0 is modeled by the multilayer approach and compared with other existing models for evaluating the additional values introduced by this approachcross validations. We summarize the main summaries and conclusions of this study in section 4.

2. Data and method

2.1. Study area and data

A local region in Europe is chosen as a case study, which covers an area of 49°N-61°N latitude and -6°E-10°E longitude, including the mainland of the Netherlands, Belgium, and parts of the North Sea, UK, Germany and France. Point-wise terrestrial and shipborne gravity anomalies are incorporated for testing the approach we developed in this study, which were provided by different institutions, see Slobbe et al. (2014). The details for data pre-processing procedures can be found inreferred to Wu et al. (2017b2017c), where crossover adjustment and low-pass filter were applied to remove systematic errors and reduce high-frequency noise, respectively, and datum transformations were performed on all the data. Moreover, the satellite-only reference model called GOCO05s with a full degree and order (d/o) of 280 (Mayer-Gürr et al., 2015) and RTM corrections were removed from the original observations to decrease the signal correlation length and smooth the data within the framework of remove-compute-restore (RCR) framework., and tThe details for the RTM reduction and residual gravity data could can be found in Wu et al. (2017b2017c).

2.2. Multilayer approach

According to Schmidt et al. (2006, 2007), the multi-resolution representation (MRR) of the Earth's potential $T(\boldsymbol{z})$ on position $\boldsymbol{z}$ is expressed as

$$T(\boldsymbol{z}) = \bar{T}(\boldsymbol{z}) + \sum_{i=1}^{I} t_i(\boldsymbol{z}) + \delta(\boldsymbol{z}) \tag{1}$$

where $T(\boldsymbol{z})$ is the disturbing potential in this study, $\bar{T}(\boldsymbol{z})$ means a reference model, e.g., a global geopotential model (GGM) computed from spherical harmonics; $\delta(\boldsymbol{z})$ represents the unmodeled signals; I is the number of levels (resolutions); $t_i(\boldsymbol{z})$ is the detailed signal of level i, and the higher the level value i is, the finer are the structures extractable from the input data; $t_i(\boldsymbol{z})$ is computed as the a linear combination of SRBFs (Schmidt et al., 2007)

$$t_i(\boldsymbol{z}) = \sum_{k=1}^{K_i} \beta_{i,k} \Psi_i(\boldsymbol{z}, \boldsymbol{y}_{i,k}) \tag{2}$$

where $\Psi(\boldsymbol{z}, \boldsymbol{y})$ is the SRBF, K_i and $\beta_{i,k}$ are the number and unknown coefficient of SRBF at level i, respectively, and $\boldsymbol{y}_{i,k}$ is the position of SRBF at this level.

We work with the RCR technique, and the reference GGM and RTM corrections are removed from the original data to decrease the signal correlation length and smooth the data (Omang and Forsberg, 2000). Then, only the residual gravity potential $T_{res}(\boldsymbol{z})$ is parameterized by SRBFs using the MRR approach. Neglecting the unmodeled signals, the residual potential is expressed as a series of the detailed signals at different levels when combining Eq.(1) and Eq.(2)

$$T_{res}(\boldsymbol{z}) = \sum_{i=1}^{I} \sum_{k=1}^{K_i} \beta_{i,k} \Psi_i(\boldsymbol{z}, \boldsymbol{y}_{i,k}) \tag{3}$$

where Ψ_i is computed as the difference of the spherical scaling functions with low-pass filter characteristics between the consecutive levels $i+1$ and i, but also can be expressed as the SRBF has the band-limited properties in the frequency domain (Schmidt et al., 2007). In this study, Ψ is chosen as the Poisson wavelet with band-limited properties ~~in the frequency domain~~ (Chambodut et al., 2005), and its full definition can be found in Holschneider and Iglewska-Nowak (2007).

Poisson wavelets can also be identified as the multipoles inside the Earth, and the scales of Poisson wavelets can be ~~related~~ linked to their depths, which are the key issues that determine their properties in space and frequency domain (Chambodut et al., 2005). The detailed signal at level i in Eq.(2) can be estimated by a linear combination of Poisson wavelets located at a specific depth. Poisson wavelets at depths demonstrate different properties in the frequency domain, as the depths going shallower, the scales decrease, and their spectrums shift towards the high degrees of the spherical harmonics (SH) and become more sensitive to the local features of signals with high-frequency properties, and vice versa (Chambodut et al., 2005). These properties are crucial for local gravity field modeling. First, the residual disturbing potential is typically the band-limited signal ~~under~~ within the RCR framework, and Poisson wavelets with band-pass filter characteristics are preferable for band-limited signal recovery (Bentel et al., 2013). Moreover, Poisson wavelets at different depths can be linked to the detailed signals at various levels, which are sensitive to different spectral contents of input signals, and ~~could~~ can be used for multi-resolution representation.

Rather than using the name of MRR, we interpret Eq.(3) as the multilayer approach considering Poisson wavelets at different depths have various characteristics, and the different layers are corresponding to the Poisson wavelets' grids at various depths. We place Poisson wavelets ~~are placed~~ on the Fibonacci grids under the topography, and keep these grids ~~are also kept~~ parallel with the topography (Tenzer et al., 2012). Instead of associating the Poisson wavelets at different depths to the hierarchical meshes with various levels (Chambodut et al., 2005), we apply a wavelet analysis approach to estimate the depths of multiply layers, inspired by the power spectrum analysis of the residual gravity field. The power spectrum analysis of local gravity signals show ~~The green curve in Figure 1 shows the radially averaged power spectrum of the local gravity field using the data mentioned in sect. 2.1, the slopes of which change in different frequency bands (see the red straight lines), indicating~~ the gravity signals are the superposition ~~superstition~~ of the contributions generated from the anomaly sources at different depths; and the signals originated from different anomaly sources have heterogeneous spectral contents (Spector and Grant, 1970; Syberg, 1972; Xu et al, 2018). Since Poisson wavelets at different depths are sensitive to signals with heterogeneous frequency characteristics, ~~and~~ we put Poisson wavelets' grids at the locations where the anomaly sources situate. In this manner, the contributions from the anomaly sources at various depths can be estimated by different layers.

~~Figure 1. Power spectrum analysis of local gravity field. The green curve is the radially averaged power spectrum, and the red straight lines represent the slopes of the spectrum in different frequency bands~~

In order to separate the contributions stemmed from different anomaly sources, the wavelet multi-scale analysis, which is an excellent approach to extract the signals at different scales, is applied to decompose the gravity data Δg into wavelet approximation A_W and a number of wavelet details $D_w\ (w=1,2,3,\cdots,W)$ at different scales (Jiang et al., 2012; Audet, 2013; Xu et al., 2017)

$$\Delta g = A_W + \sum_{w=1}^{W} D_w \tag{4}$$

where W is the maximum order for decomposition, A_W is the regional anomaly caused by deep and large-scale geological bodies, and D_w is the local anomaly originated from shallow and small-scale heterogeneous substances. Wavelets analysis generates low-order wavelet details that are invariant with the decomposition order, and only the high-order wavelet details and corresponding wavelet approximation change with the decomposition order. Based on this property, we can choose the proper decomposition order to derive the desirable solutions.

The decomposed signals reveal the features of geological bodies, the average depths of which can be estimated from the power spectral analysis (Spector and Grant, 1970; Syberg, 1972; Cianciara and Marcak, 1976; Xu et al., 2018)

$$h_w = \frac{1}{4\pi} \frac{\Delta \ln P_k^{\ w}}{\Delta k_w} \quad w = 1, 2, \cdots, W \tag{5}$$

where h_w is the average depth of anomaly source corresponding to wavelet detail D_w; $\ln P_k^{\ w}$ is the logarithmic power spectrum of D_w; $\Delta \ln P_k^{\ w}$ and Δk_w are the change rates for $\ln P_k^{\ w}$ and radial wave number k_w, respectively.

In this study, terrestrial and shipborne gravity anomalies are merged for modeling. Gravity anomalies Δg and quasi-geoid height ζ are related to the disturbing potential based on the multilayer approach as follows:

$$\begin{aligned}\Delta g(\boldsymbol{z}) &\approx -\frac{2}{|\boldsymbol{z}|} T_{res}(\boldsymbol{z}) - \frac{\partial T_{res}(\boldsymbol{z})}{\partial |\boldsymbol{z}|} \\ &= \sum_{i=1}^{I} \sum_{k=1}^{K_i} \beta_{i,k} \left(-\frac{\partial}{\partial |\boldsymbol{z}|} \Psi_i(\boldsymbol{z}, \boldsymbol{y}_{i,k}) - \frac{2}{|\boldsymbol{z}|} \Psi_i(\boldsymbol{z}, \boldsymbol{y}_{i,k}) \right) \\ \zeta(\boldsymbol{z}) &= \frac{T_{res}(\boldsymbol{z})}{\gamma(\boldsymbol{z})} = \sum_{i=1}^{I} \sum_{k=1}^{K_i} \beta_{i,k} \frac{\Psi_i(\boldsymbol{z}, \boldsymbol{y}_{i,k})}{\gamma(\boldsymbol{z})}\end{aligned} \tag{6}$$

where γ is the normal gravity value.

We suppose the observational errors are white noises with zero mean, and the gravity field model using the multilayer approach is written as the standard Gauss-Markov model

$$\boldsymbol{l}_j - \boldsymbol{e}_j = \boldsymbol{A}_j \boldsymbol{x},\ E\{\boldsymbol{e}_j\} = 0,\ \ \mathrm{D}\{\boldsymbol{e}_j\} = \boldsymbol{C}_j = \sigma_j^2 \boldsymbol{Q}_j = \sigma_j^2 \boldsymbol{P}_j^{-1},\ \ j = 1, 2, \cdots, J \tag{7}$$

where $\boldsymbol{x}$ is the $K \times 1$ vector of unknown coefficients, including the unknown parameters of Poisson wavelets ~~from~~ of all the layers, i.e., $\boldsymbol{x} = \left[\beta_{1,1}, \beta_{1,2}, \cdots, \beta_{1,K_1}, \beta_{2,1}, \beta_{2,2}, \cdots, \beta_{2,K_2}, \cdots, \beta_{I,1}, \beta_{I,2}, \cdots, \beta_{I,K_I}\right]'$, and $K = K_1 + K_2 + \cdots + K_I$; $\boldsymbol{A}_j$ is the $m_j \times K$ design matrix of group j, $\boldsymbol{l}_j$ is the $m_j \times 1$ corresponding observation vector, $\boldsymbol{e}_j$ is the $m_j \times 1$ vector of corresponding stochastic errors, ~~and~~ m_j is the number of observations in group j, and J is the number of observation groups. $E\{\cdot\}$ and $D\{\cdot\}$ are the expectation and dispersion operators, respectively. $\boldsymbol{C}_j$ is the error variance-covariance matrix of group j, and σ_j^2, $\boldsymbol{Q}_j$ and $\boldsymbol{P}_j$ are the variance factor, cofactor matrix, and weight matrix of group j, respectively.

Data in different groups are assumed to be independent, and the weight matrix $\boldsymbol{P}_j$ is supposed to be the scaled diagonal matrix with white noise properties since it is usually difficult to acquire the realistic full error variance-covariance matrix in real-life measurements. Point-wise data can be directly combined for modeling through the functional described above. However, the heterogeneous characteristics ~~for~~ of the data, in terms of spatial coverages and noise properties, may result in an ill-conditioned normal matrix (Panet et al., 2011). We apply the first-order Tikhonov regularization for tackling the ill-conditioned problem (Kusche and Klees, 2002; Wu et al., 2017a). For a given α (regularization parameter) and κ (regularization matrix), the least-squares solution of Eq.(7) is (Klees et al., 2008):

$$\hat{\boldsymbol{x}} = \left(\sum_{j=1}^{J} \left(\frac{1}{\sigma_j^2} \boldsymbol{A}_j^T \boldsymbol{P}_j \boldsymbol{A}_j \right) + \alpha\kappa \right)^{-1} \left(\sum_{j=1}^{J} \left(\frac{1}{\sigma_j^2} \boldsymbol{A}_j^T \boldsymbol{P}_j \boldsymbol{l}_j \right) \right) \tag{8}$$

Moreover, we use the Monte-Carlo variance component estimation (MCVCE) to estimate the appropriate variance factors of various different observation groups and the regularization parameter (Koch and Kusche, 2002; Kusche, 2003; Wu et al., 2017c).

3. Numerical results and discussion

3.1. Wavelet analysis of local gravity signals

In order to determine the depths of different layers, the residual gravity data are decomposed into the signals at different scales based on wavelet analysis. The spline interpolation is used to compute the gridded data for wavelet decomposition., and Coif3 basis functions are chosen for wavelet decomposition (Xu et al., 2017). The preliminary maximum order for wavelet decomposition is arbitrarily chosen to some extent, however, since the low-order details are invariant with the increase of decomposition order, we can preliminarily choose a predefined order and implement the wavelet decomposition, and analyze the derived details. If there are still details that are useful for constructing the multilayer model haven't been separated, we need to increase the decomposition order until all the useful details have been extracted; otherwise, we truncate to a specific order, and compute the wavelet details and approximation to construct the multiply layer's parameterization. By trial and errors, the preliminary order for decomposition is chosen as nine, and Figure 1 shows the derived wavelet details (the corresponding statistics are provided in Table 1), where the maximum order for decomposition is preliminarily chosen as ten. With the increase of decomposition order, more long-wavelength features show upoccur. More specifically, the low-order details demonstrate the high-frequency signals stemmed from the shallow and small-scale substances;. wWhile, the high-order ones with long-wavelength patterns reflect the anomalies caused by deep and large-scale geological bodies. It is noticeable that the 1st- and 2nd-order details (i.e., D_1 and D_2) are seems dominated by the high-frequency signals correlate strongly with the local topography (the local digital terrain model (DTM) could can be found seen in Figure 1 in Wu et al., 2017b(2017c)). We mainly attribute this to the uncorrected topographical signals in RTM corrections, which is mainly due to the inaccuracy of the density parameters in RTM corrections and limitations of DTM both in terms of spatial resolution and precision. As a result, the small-scalehigh-frequency signals originated from local topography variation cannot thoroughly recovered from RTM reduction, and consequently, the uncorrected signals leak into the 1st- and 2nd-order details. However, these signals are of small magnitude (see Table 1)To avoid these high-frequency errors propagating into the final solution, and we neglect the first these two wavelet details for in designing the multiply layers' networks to avoid the adverse impacts introduced by these high-frequency noises. Moreover, with the order increasing to nine

~~and larger~~, we notice D_9 ~~and D_{10}~~ obviously reveal the large-scale signals with the wavelengths of hundreds of kilometers. Given that the mean distance between the data in this target area is approximately several kilometers and the spatial resolution of the applied GGM (i.e., GOCO05S) is roughly 72 km, the spectral contents of the residual signals need to be recovered is roughly between several kilometers and tens of kilometers within the RCR framework, i.e., approximately between degree 250 to 3000 in terms of spherical harmonics' representation. While, the spectral contents of the 9th~~- and 10th~~-order details exceed the frequency bands of the signals need to be ~~recovered~~modeled, and the maximum order for wavelet decomposition is truncated to eight. In this manner, the third- to eighth-order (D_3 - D_8) wavelet details and the final ~~wavelet~~ approximation (A_8) (see the information in Figure 2 and Table 2) are applied for constructing the multilayer model~~multiply layers' networks~~, ~~which are~~ consists of seven layers at various depths. ~~Different layers are sensitive to signals with heterogeneous frequency characteristics, and shallow and deep layers mainly capture the short- and long-wavelength signals, respectively.~~

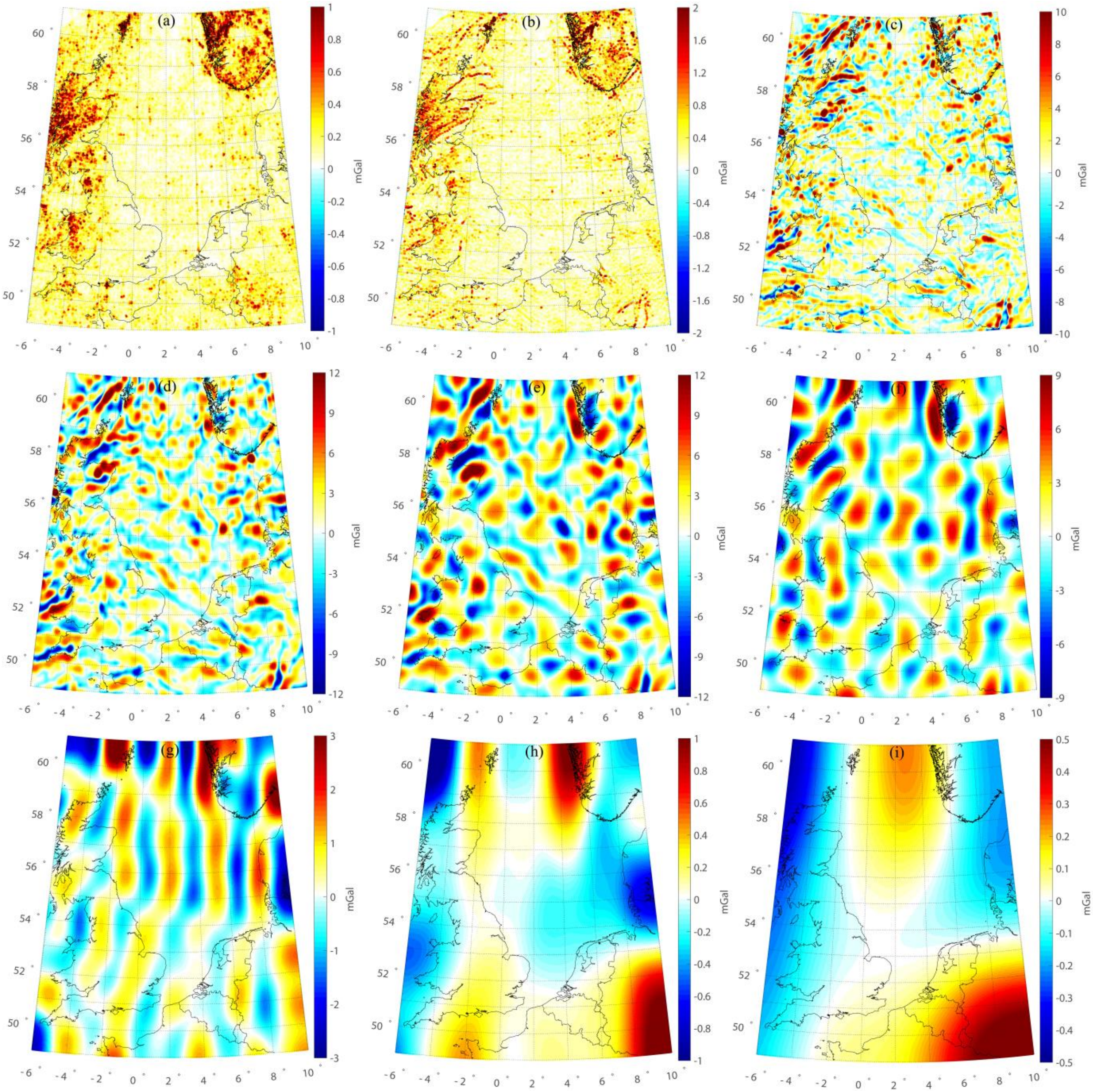

Figure 1. Wavelet details at various scales. (a) D_1, (b) D_2, (c) D_3, (d) D_4, (e) D_5, (f) D_6, (g) D_7, (h) D_8, and (i) D_9 .and (j) D_{10}

Table 1. Statistics of ~~various~~ different wavelet details (units: mGal)

	max	min	mean	sd
D_1	2.23	-2.78	0.00	0.20
D_2	4.52	-5.57	0.00	0.32
D_3	19.27	-16.26	0.00	2.30
D_4	21.71	-17.46	0.00	3.18
D_5	15.38	-16.47	0.00	3.80
D_6	10.60	-9.72	0.00	2.75
D_7	4.43	-3.33	0.00	0.95
D_8	1.23	-1.52	0.00	0.34
D_9	0.66	-0.45	0.00	0.18

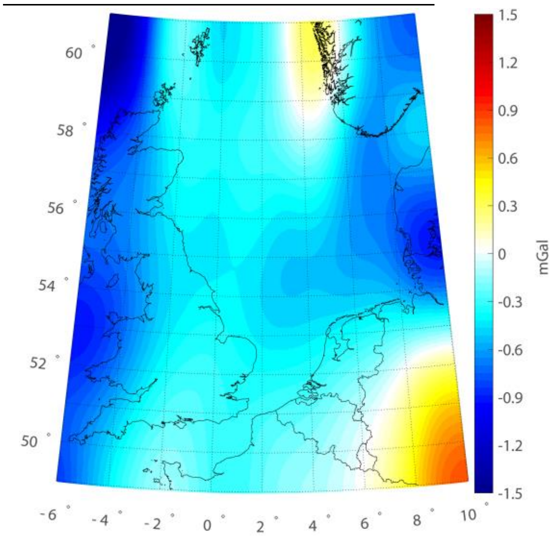

Figure 2. Wavelet approximation A_8 .

Table 2. Statistics of wavelet approximation (units: mGal).

max	min	mean	sd
0.83	-1.70	-0.41	0.32

3.2. Key parameters of Poisson wavelets

The order of Poisson wavelets is fixed at 3 to achieve a good compromise between the localization in space and frequency domain (Panet et al., 2011). In addition, the depth and number of Poisson wavelets are the crucial points affecting the solution quality (Klees et al., 2008). Poisson wavelets belong to different layers are placed on the Fibonacci grids at various depths beneath the topography, and the power spectrum analysis is applied to estimate the depths. As shown in Figure 3, the green curves show the radially averaged logarithm power spectrums ~~for~~ of ~~the~~ signals ~~of~~ at different scales, and the red straight lines represent the slopes of the spectrums, indicat~~ing~~e the depths of corresponding layers. The red lines represent rates of change for logarithmic power relative to wave number, estimated by a autoregressive method; and the starting point and terminal point of the red lines are the inflection points of the curves, recognized according to the trend of the curves (Xu et al., 2018). –The layers go deeper as the scales increase, and the shallow layers reflect the small-scale signals, while the deep ones recover the long-wavelength information. Table 3 provides the estimated depths ~~for~~ of different layers, ~~which are~~ limited between 4~~5 km~~ and ~~61~~ 60 km. The shallowest layer locates ~~5.7~~4.5 km underneath the topography, while the depth of the deepest one is approximately estimated as ~~60.2~~ 59.2 km. It is noticeable that the thickness of sediments in this ~~target~~ area is approximately 2~4 km, and the thickness of the upper-middle crust is roughly 15~20 km (Artemieva and Thybo, 2013). Thus, the first four layers (layer1, layer2, layer3 and layer4) locate between the sediments and upper-middle crust, and the corresponding wavelet details (D_3, D_4, D_5 and D_6) display as the small-scale patterns due to the highly heterogeneous structure of the crust. The distributions of D_3 and D_4 (with the average depths of 4.5 km and 9.2 km, respectively) on land are more dispersed than that in the ocean, demonstrate that the tectonic structure underneath the land is more complex than that beneath the ocean in the upper crust. Moreover, the gravity anomalies in the northern of North Sea are more dispersed than those in the central and southern of North Sea, which is consistent with that the Viking Graben and basin are located in the northern and southern of North Sea, respectively, e.g., see Fichler and Hospers (1990), and Blundell et al. (1991). The mean source depths of D_5 and D_6 are 13.7 km and 19.6 km, respectively, correspond to the depths of the middle crust. The gravity anomalies during these two layers present apparent positive-negative alternating patterns, which may be interpreted as the crustal shearing and extrusion (Blundell et al., 1991; Ziegler and Dèzes, 2006). While, the last three layers (layer5, layer6, and layer7) are supposed to be located between the Moho surface and upper mantle considering the Moho depth in this region is approximately 25~30 km (Grad and Tiira, 2009), and the corresponding details (D_7, D_8 and A_8) become smoother and more long-wavelength signals show upoccur.

D_7 with the mean source depth of 27.0 km primarily reflects the Moho undulation. The distribution of positive-negative alternating gravity anomalies in D_7 is nearly south-north oriented, which is in agreement with the features of the Moho relief in this area (Fichler and Hospers, 1990; Ziegler and Dèzes, 2006). The average source depths of D_8 and A_8 are 32.3 km and 59.0 km, respectively, correspond with the depth of the upper mantle, indicate that the density distribution of the upper mantle is relatively smooth. Overall, these decomposed gravity anomalies can reveal the tectonic structure of study area at different depths.

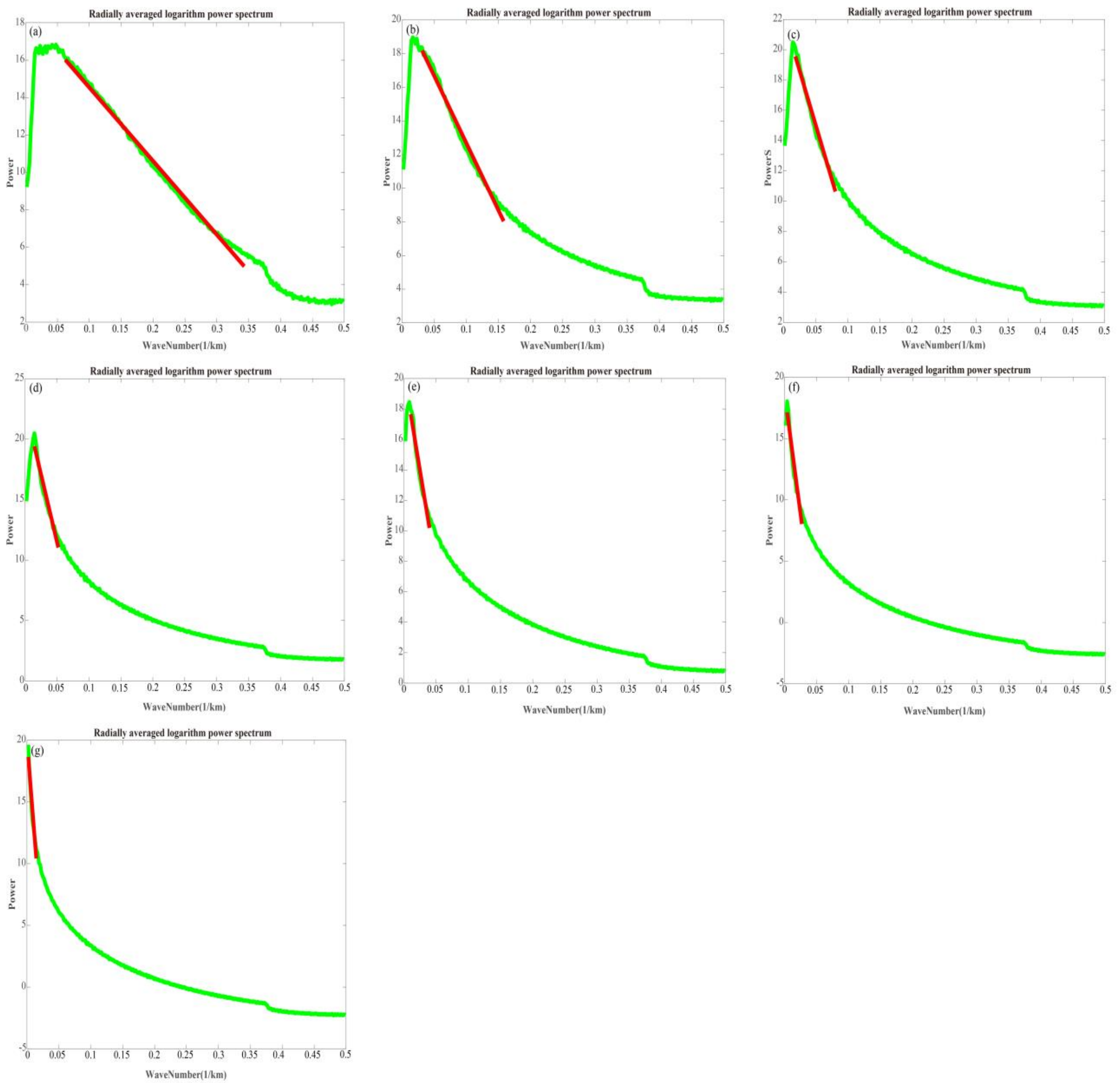

Figure 3. Power spectrum analysis of various wavelet signals. (a) D_3, (b) D_4, (c) D_5, (d) D_6, (e) D_7, (f) D_8, and (g) A_8.

The green curves are the radially averaged logarithm power spectrums, and the red straight lines– represent rates of change for logarithmic power relative to wave number.represent the slopes of the spectrums.

Table 3 Depths of multiply layers beneath the topography (Units: km).

layer1	4.5
layer2	9.2
layer3	13.7
layer4	19.6
layer5	27.0
layer6	32.3
layer7	59.2

As mentioned above, different layers are constructed designed to recover the wavelet details and approximation signals with various spectral contentsat different scales, and a trial-and-error approach is used to estimate the number of Poisson wavelets offor each layer (Wittwer, 2009). For a specific layer with the fixed depth, we predefine different number of Poisson wavelets to form a certain number of Fibonacci grids. Then, the signals reconstructed from these grids are compared with the true values, i.e., ones derived from wavelet decomposition, and the parameter that derives the smallest differences between the modeled and true signals is consider as the optimal one. By trail and errors, the spatial resolutions of Fibonacci grids (mean distance between Poisson wavelets) are changed from 20 to 14 km with a step of 1 km. Table 4 shows the accuracies of the solutions derived from different Fibonacci grids for of various multiply layers, and we take the situations of the first layer for instance. With more the increase of Poisson wavelets, the SD value of the differences between the reconstructed and true signals decreases gradually to 0.10 0.12 mGal when the spatial resolution of the grid increase to 16 km. Since then, no significant improvements show upoccur with incorporating more Poisson wavelets. Moreover, introducing more Poisson wavelets increases the overlapping between them, which may lead to the highly-conditioned normal matrices, and the associated heavy regularization may decrease the solution quality (Wu et al., 2017a2017b). The optimal mean distance between Poisson wavelets of the first layer is estimated as 16 km. Similarly, the spatial resolutions for the rest layers can be determined in this way, see Table 4.

Table 4 Accuracies of solutions derived from different various Fibonacci grids with various spatial resolutions for of different layers (Units: mGal).

	20 km	19 km	18 km	17 km	16 km	15 km	14 km
layer1	0.43	0.34	0.21	0.16	0.12	0.12	0.12
layer2	0.52	0.43	0.33	0.25	0.19	0.16	0.16
layer3	0.58	0.40	0.28	0.19	0.16	0.14	0.14
layer4	0.55	0.39	0.29	0.26	0.15	0.13	0.13
layer5	0.38	0.26	0.17	0.14	0.10	0.10	0.10
layer6	0.22	0.16	0.12	0.10	0.08	0.08	0.08
layer7	0.11	0.09	0.08	0.06	0.06	0.06	0.06

3.3. Regional solution and its validation

For regional gravity field recovery, pPoint-wise terrestrial and shipboard gravity anomalies are merged for modelingcombined. Since there are no accurate information for terrestrial and shipboard data, we assume the accuracies of 2 mGal for both of these two types of data, and the posterior variance factors of different observation groups are estimated from MCVCE method. The weights of different observation groups, indicate their relative contributions, and play a key role in data combination. The estimated variance factors for terrestrial and shipboard gravity data are approximately 1.45 mGal and 1.30 mGal through the MCVCE method, respectively, when we model the local gravity field based on the multilayer approach. For terrestrial data, the estimated accuracy is in good agreement with that derived by Klees et al. (2008), i.e., 1.48 mGal for parts of the Netherlands. However, it is difficult to judge whether this estimate is realistic in other regions because of a lack of accuracy information. While, for shipboard data, the computed value of 1.30 mGal is smaller than the results of crossover adjustments, where the standard deviation for the residuals at the crossovers was approximately estimated as 2.0 mGal (Slobbe, 2013). However, this value may be too optimistic considering much of the shipborne data were collected decades ago without GPS navigation. The first-order Tikhonov regularization is used to tackle the ill-conditioned problem (Kusche and Klees, 2002; Wu et al., 2017b), and the convergent regularization parameter is approximately 0.5×10^{-5} estimated from the MCVCE method; the details for regularization parameter estimation and comparisons with different methods can be referred to Wu et al. (2017b).

The performance of the traditionally-used single-layer is also investigated for comparison, and the parameterization of local gravity field based on the single-layer approach can be seen in, e.g., Klees et al. (2008) and Slobbe (2013). By trial and errors, the single layer of Poisson wavelets' grid is located 40 km beneath the topography, and the mean distance between Poisson wavelets is defined as 8.7 km (Wu et al., 2016). Figure 4 shows the normalized spectrums for different approaches. Considering the frequency range of the signals to be recovered in the target area is approximately between degree 250 to 3000 in spherical harmonics' representation, we note the single-layer approach is only sensitive to parts of the signals' spectrum, i.e., approximately between degree 300 to 1200 if we suppose half of the maximum value of the normalized spectrum is the criterion for determining whether it is sensitive or not within a specific frequency band. However, for the high-frequency band between degree 1200 to 3000, this approach is less sensitive. On the contrary, the multilayer approach effectively covers the spectrum of the local gravity signals, which is both sensitive to the low- and high-frequency bands. Figure 5 provides the residuals of data after least squares adjustment using different methods, shows~~ing~~ the residuals derived from the multilayer approach reduce significantly in the whole region compared with ones obtained from the single-layer approach, especially in western parts of UK, south of Norway, and southwest of Germany, where the high-frequency signals correlated with local topography dominate the features of regional gravity field. We also find the improvements occurring in the ocean parts, especially in waters around the English Channel, Irish Sea, northwest of North Sea, and Atlantic Ocean close to northwest UK. The statistics in Table 5 displays the standard deviation (SD) value for the residuals of terrestrial (shipborne) gravity anomalies decreases by 0.39 mGal (0.36 mGal) when the multilayer approach is used. These results are reasonable since the multilayer approach contains several layers shallower than 40 km, and the spectrums of these layers shift to the high-frequency bands. As a result, the spectrum of the multilayer approach is more sensitive to signals with high-frequency properties, and consequently, ~~demonstrate that the multilayer approach can more accurately recovers~~ the local high-frequency signals can be better fitted by the multilayer approach~~than the single-layer one~~. ~~The main reason is that the spectrum of the multilayer method covers the whole spectral contents of the regional gravity signals, which is more sensitive to the high-frequency signals. The statistics in displays the SD value for the residuals of terrestrial (shipborne) gravity anomalies decreases by 0.30 mGal (0.34 mGal) when the multilayer approach is used.~~ It is also worth to mention that the analysis of data residuals can't be treated as the only criteria for justifying the performances of different approaches, since these gravity data have been used for modeling purpose, and the SD values of data residuals should be regarded as the internal agreement. Besides, due to the limitation of the accuracies of gravity data, we can't make conclusions too firmly only depends on the analysis of data residuals. One may also argue that it may be possible to derive lower data residuals if we put the Poisson wavelets' grid shallower when the single-layer approach is used. However, we believe a shallower single grid may reduce the data residuals, but may not derive a better solution when validated against the independent control data, see the detailed discussions in Wu et al. (2016). In the following part, we introduce another high-quality independent data set, i.e., GPS/leveling data, for external validation, which give us more confidences with respect to the performances of different methods.

It is also of interest to implement a Akaike information criterion (AIC) test for different models. Although, the multilayer model fits the gravity observations better, but it also increases the level of estimated parameters. AIC rewards the goodness of fit of data, but also includes a penalty with the increasing of the number of estimated parameters. In other words, it deals with the trade-off between the goodness of fit of the model and the simplicity of the model. AIC value is an estimator of the relative quality of statistical models for a given set of data, providing a means for model selection, and the model that gives the minimum AIC value may be more preferable (Akaike, 1974; Burnham and Anderson, 2002). The definition for the AIC value can be seen in Eq.(A1) in the Appendix. Since we model the gravity field in the framework of least squares system, we can simply take $AIC = 2k + n\ln(RSS/n)$ for model comparision, where k is the number of estimated parameters in the model, n is the number of observations, and RSS is the residual sum of squares (RSS), see the details in the Appendix. In this study, the number of point-wise gravity observations used for modeling is 894649, and the numbers of estimated parameters in the multilayer and single-layer model are 47504 and 19477, respectively. The RSS values for the multilayer and single-layer model are computed as $8.8527\times10^{5}\ mGal^{2}$ and $1.3296\times10^{6}\ mGal^{2}$, respectively, based on the data residuals after the least squares adjustment. Then, the AIC values for the multilayer and single-layer model are estimated as 85581 and 393400, respectively. Based on these statistics, we notice that the multilayer model gives a smaller AIC value, which may be more preferable since it reaches a better balance between the goodness of fit of data and the simplicity of the model.

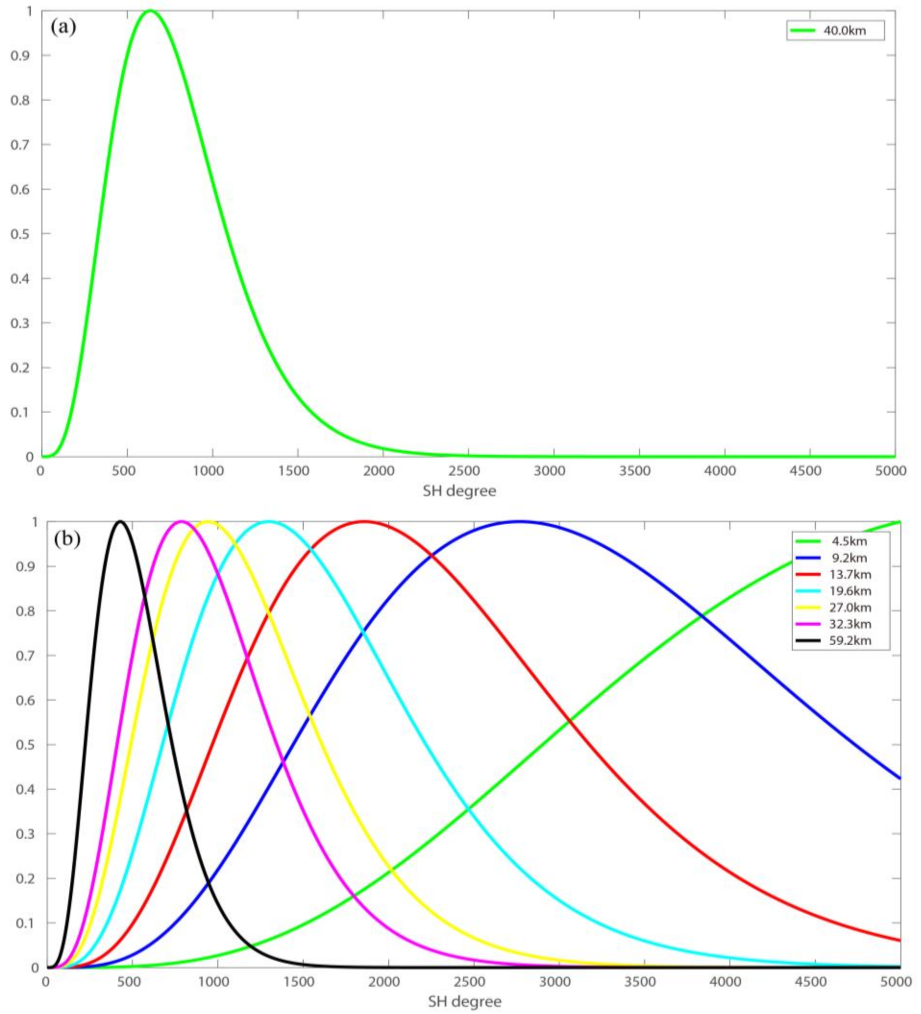

Figure 4. Normalized spectrums for (a) single-layer and (b) multilayer approach.

To test the ability of realistic extrapolation of different regional models recovered from various methods, which is actually comparing the predicted values derived from the regional model (e.g., model computed from the multilayer or single-layer approach) and ones derived from independent survey/measurements, we introduce GPS/leveling data in the Netherlands (534 points), Belgium (2707 points), and parts of Germany (213 points) ~~are used~~ as the independent validation data.~~,~~ These data are provided in terms of geometric quasi-geoid heights derived from the high-quality GPS measurements and leveling survey, and the overall estimated accuracy of these observed quasi-geoid heights is approximately at 1 cm level. It is worth to mention that these GPS/leveling data are not combined for modeling, and their three dimensional coordinates don't coincide with the positions of gravity data. For validating different models with GPS/leveling data, we need to reconstruct the regional model based on the computed Poisson wavelets' coefficients and coordinates of GPS/leveling points (see Eq.(6)), and compute the gravimetric quasi-geoid heights at these points, which are ones predicted from the regional model. Then, we compute the standard deviation (SD) of the point-wise difference between GPS/leveling data and the gravimetric quasi-geoid height derived from the regional approach, which is actually external validation. and tThe validation results demonstrate the discrepenciesy between the GPS/leveling points and quasi-geoid heights derived from the multilayer approach decrease substantially compared with ones computed from the single-layer approach, see Figure 6. The most prominent improvements occur in the northwest of Belgium, west of Germany, and eastern parts of Netherlands, which are in good agreement with the results for the gravity data residuals analysis demonstrated in Figure 5. As shown in Table 6, the accuracies of gravimetric quasi-geoid derived from the multilayer approach are improved by 0.4 cm, 0.9 cm and 1.1 cm in the Netherlands, Belgium and parts of Germany, respectively. Moreover, the mean values indicate that the solution with computed from the multilayer approach also further reduces the biases between gravimetric solution and local GPS/leveling data, with the magnitude of 0.8 cm, 0.7 cm, and 1.1 cm in these three regions, respectively, compared to the one modeled from the single-layer approach. From these results, we can see that the multilayer approach not only leads to a reduction for the data residuals, but also derives a better solution assessed by the independent control data, compared to the single-layer approach. For constructing the multilayer model, we consider that the gravity signals are the sum of the contributions generated from the anomaly sources, and different layers are designed for recovering these contributions with heterogeneous spectral contents. As a result, the spectrum of multilayer approach is sensitive to the frequency bands of local gravity signals, both in low- and high-frequency bands, and the local signals may be better recovered. Based on the evaluation results, we conclude the multilayer approach proposed in this study outperforms the traditionally-used single-layer method, which maybe more preferable in gravity field modeling using heterogeneous data. We also notice that there are still biases between the regional gravimetric solutions and local GPS/leveling data, see the mean values in Table 6, which are mainly due to the commission errors in the GGM and uncorrected systematic errors in the local gravity data and leveling systems (Fotopoulos, 2005). Generally, corrector-surface (Fotopoulos, 2005; Nahavandchi and Soltanpour, 2006) or more complicated algorithms, like least squares collocation (Tscherning, 1978), boundary-value methodology (Klees and Prutkin, 2008; Prutkin and Klees, 2008), and a direct approach (Wu et al., 2017a), can be applied to reduce the systematic errors and properly combine

GPS/leveling data and gravimetric solution. However, since the target for this study is to develop a multilayer approach for gravimetric quasi-geoid modeling, which is served as a basic surface for further geophysical applications, e.g., study the ocean circulation and structure of lithosphere; while, after implementing these methods for combining local GPS/leveling and gravimetric model, the derived quasi-geoid is not purely gravimetric. Besides, we only have the well distributed GPS/leveling data in the limited region, i.e., in Netherlands, Belgium, and parts of Germany, while in other regions, no high-quality control data are available. If we use the locally distributed GPS/leveling data for removing these systematic errors and computing the combined quasi-geoid, the final solution may be distorted in other regions, especially in the ocean parts, since no control data in these regions have been combined. Thus, we don't implement these methods mentioned above for computing the combined quasi-geoid. In following study, we use the gravimetric model derived from the multilayer approach, which is hereafter denoted as QGNSea V1.0 (quasi-geoid over the North Sea version 1.0).

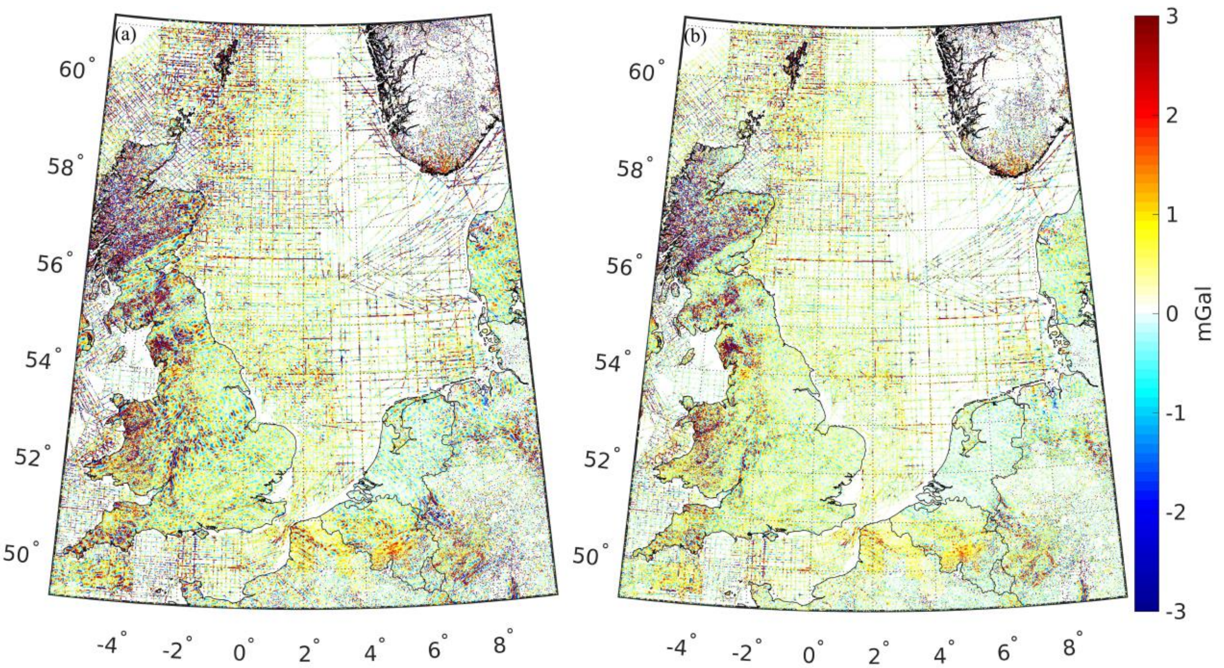

Figure 5. Residuals of gravity data derived from (a) single-layer and (b) multilayer approach.

Table 5 Statistics of the residuals of gravity data computed from different approaches (units: mGal).

		max	min	mean	sd
Single-layer approach	Terrestrial	19.58	-16.91	0.00	1.45
	Shipborne	11.91	-17.38	0.00	1.07
Multilayer approach	Terrestrial	16.96	-14.90	0.00	1.06
	Shipborne	9.25	-15.96	0.00	0.71

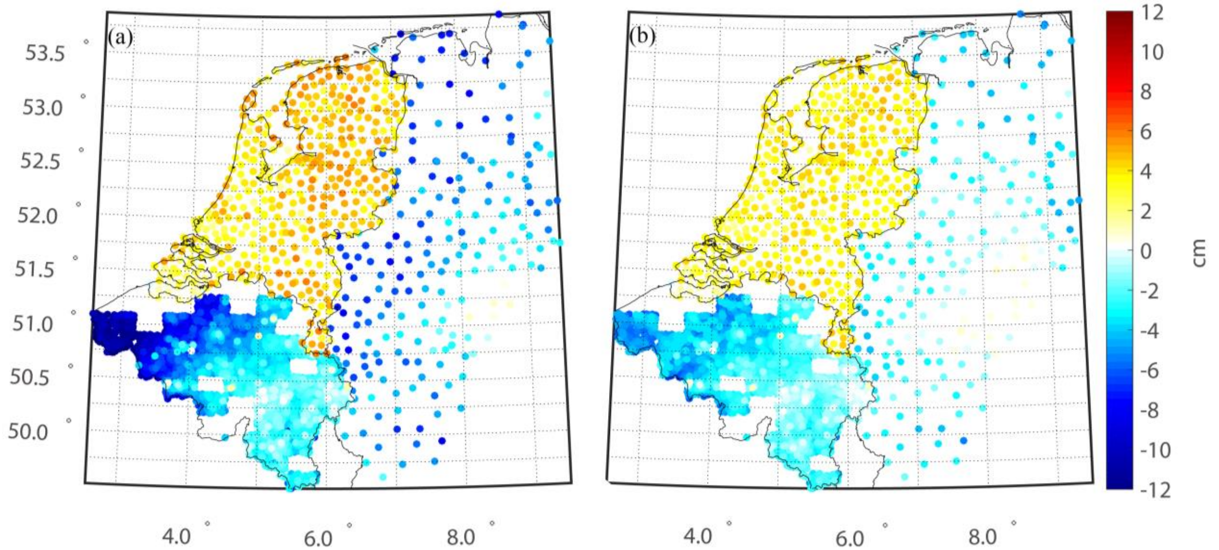

Figure 6. Differences between GPS/leveling data and gravimetric quasi-geoids computed from (a) single-layer and (b) multilayer approach.

Table 6 Evaluation of quasi-geoids modeled from different approaches (Units: cm).

		max	min	mean	sd
Single-layer approach	Netherlands	5.9	0.1	3.8	1.2
	Belgium	1.2	-13.1	-3.5	2.8
	Germany	1.2	-11.2	-3.6	2.9
Multilayer approach	Netherlands	4.8	0.0	3.0	0.8
	Belgium	1.2	-6.8	-2.8	1.9
	Germany	1.0	-6.7	-2.5	1.8

QGNSea V1.0 is compared with a regional model called EGG08 (Denker, 2013) and other ~~two~~ four recently published high-order GGMs, i.e., EGM2008 (d/o 2190) (Pavlis et al., 2012), ~~and~~ EIGEN-6C4 (d/o 2190) (Förste et al., 2014), GECO (d/o 2190) (Gilardoni et al., 2015), and SGG-UGM-1 (d/o 2159) (Liang et al., 2018), for ~~cross validation~~further comparisons. The reason for choosing these four GGMs for comparisons is that these models have relatively higher spatial resolutions and better accuracies compared to most of other available GGMs, see the information in http://icgem.gfz-potsdam.de/home. EGG08 is a regional gravimetric quasi-geoid model in Europe, which was recovered by stokes integral based on locally distributed gravity data. This model is provided in terms of gridded data instead of spherical harmonics, the space resolution of which is 1′ in latitude and 1.5′ in longitude, respectively (Denker, 2013). While, the rest four models are global geopotential models provided in terms of spherical harmonics, and EGM2008 was computed by merging GRACE measurements, terrestrial, altimetry-derived, and airborne gravity data. Since no GOCE data have been incorporated for developing EGM2008, and the recently published GGMs have been developed by combining GOCE data, which is supposed to improve the gravity field in the frequency bands approximately from degree 30 to 220 in spherical harmonics representation (Gruber et al., 2010). EIGEN-6C4 was computed by combining GRACE, GOCE, and terrestrial gravity data and other data sets; –GECO was computed by incorporating the GOCE-only TIM R5 (d/o 250) solution into EGM2008, and SGG-UGM-1 was computed by the combination of EGM2008 gravity anomalies and GOCE gravity gradients and satellite-to-satellite tracking data. Differences between QGNSea V1.0 and other models are shown in Figure 7 (the boundary limits for the area are contracted by 0.5° in all the directions to reduce edge effects),~~;~~ the magnitude of which reaches decimeter level. For EGG08, we note the most prominent differences appear in eastern parts of the Irish Sea and center of Germany. Different data pre-processing procedures and methods for parameterization partly account for these differences, e.g., QGNSea V1.0 is recovered from the multilayer approach using Poisson wavelets and proper weights for different observation groups are estimated through MCVCE; while the spectral combination technique and spectral weights were implemented in EGG08 for merging heterogeneous data (Denker, 2013). Larger differences are observed between QGNSea V1.0 and these four GGMs, and ~~For EGM2008/EIGEN-6C4,~~ remarkable differences show in southern –of Norway, northern of the North Sea, eastern of the Irish Sea, and northwest of Germany;~~.~~ besides ~~Apart~~ from the applications of different techniques for modeling, these differences are partly interpreted as the additional signals introduced by QGNSea V1.0, stemming from the incorporation of more high-quality ~~gravimetry~~ gravity data. The evaluation results with GPS/leveling data displayed in Figure 8 and Table 7 show the gravimetric quasi-geoid inversed from the multilayer approach has the best quality, especially in the north of the Netherlands and western and eastern parts of Belgium~~, and the accuracies for QGNSea V1.0, EGG08, EGM2008 and EIGEN-6C4 are 1.6 cm, 2.2 cm, 2.6 cm and 2.7 cm, respectively, when comparing with all the GPS/leveling data in the target area (see)~~. The SD value of the misfit between the GPS/leveling data and QGNSea V1.0 is 1.5 cm, while this value increases to 2.2 cm when EGG08 is validated. In contrast, the accuracies of these four GGMs are slightly worse than EGG08, which are approximately at 2.6 cm levels. Compared to these GGMs, t~~T~~he added values introduced by the local high-quality data lead to the primary improvements of QGNSea V1.0~~, which mainly contribute to the fine structures at short-wavelength bands~~. ~~Moreover, the improvements in the frequency bands that GOCE data contribute may be also the reasons, since EGM2008/EGG08 was developed without GOCE data.~~ We find that these four GGMs have the comparable accuracies, where the ones developed by combining GOCE data and EGM2008 (i.e., GECO and SGG-UGM-1) don't have better performances than EGM2008, and SGG-UGM-1 even has the slightly worse performance than EGM2008, which is especially prominent in the eastern parts of Belgium, however, the possible reasons need further investigation. We also notice that a new Europe gravimetric quasi-geoid called EGG2015 has been computed, where the GOCE-derived GGMs were used as the reference models (Denker, 2015). However, this model is not publicly available, and its performance can't be assessed in this local region.

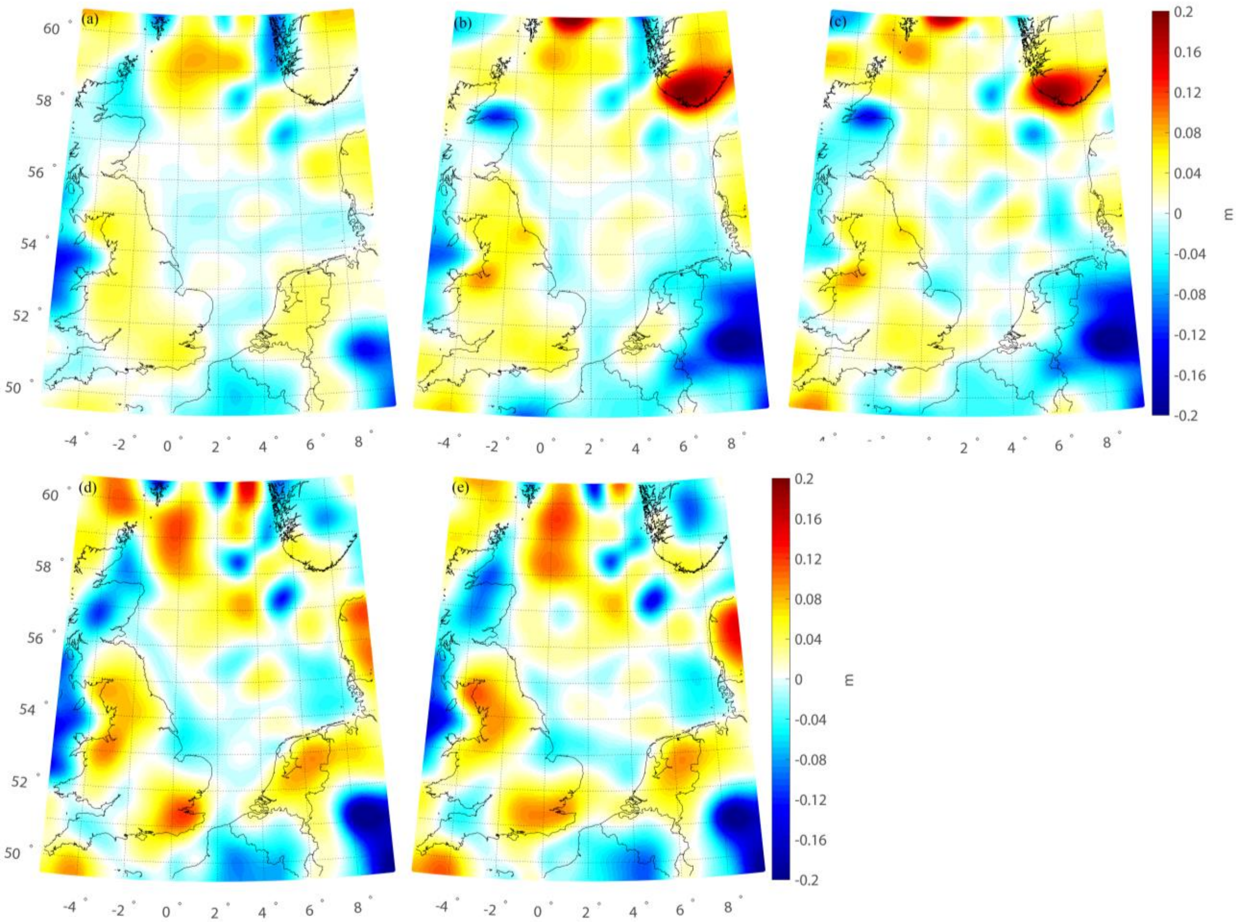

Figure 7. Difference between QGNSea V1.0 and (a) EGG08, (b) EGM2008, (c) EIGEN-6C4, (d) GECO, (e) SGG-UGM-1. Note that the mean differences are removed.

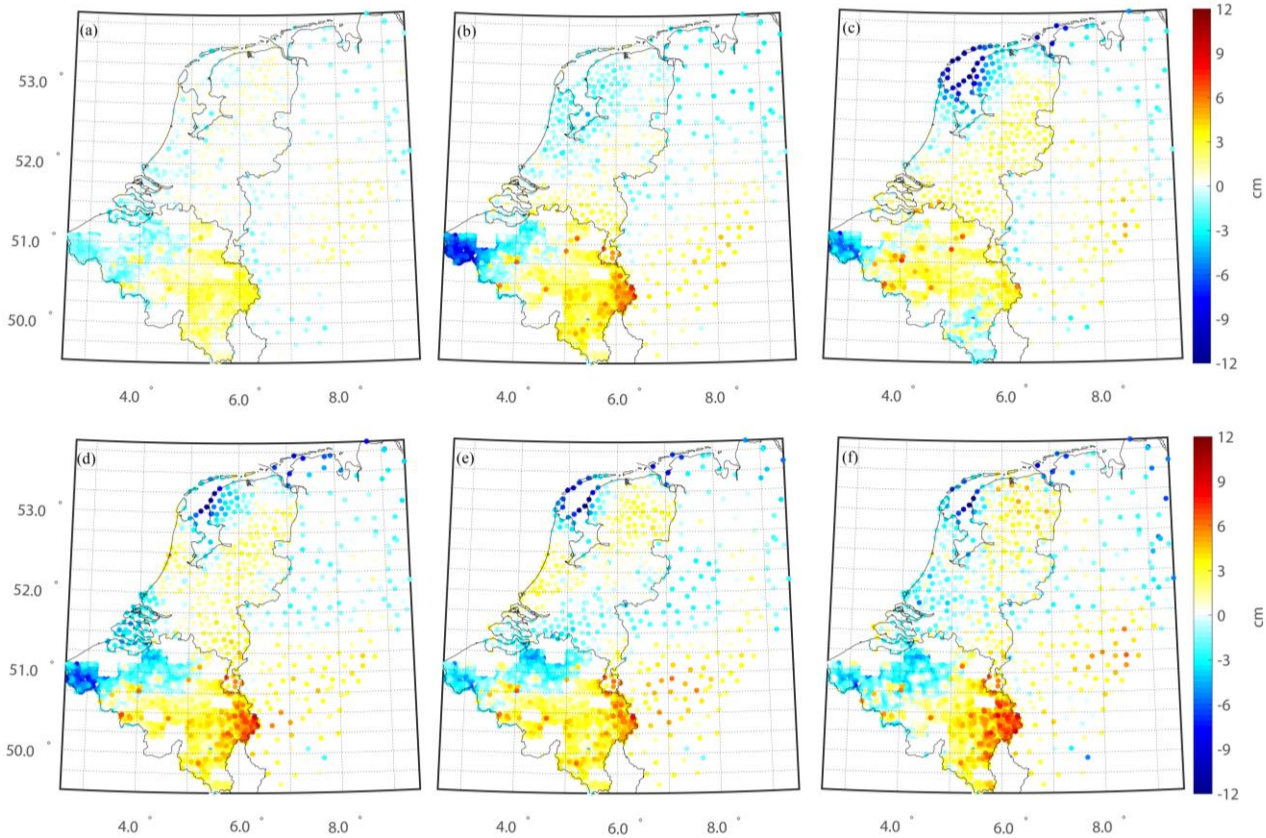

Figure 8. Evaluation of the various quasi-geoids. (a) QGNSea V1.0, (b) EGG08, (c) EGM2008, (d) EIGEN-6C4, (e) GECO, (d) EIGEN-6C4and (f) SGG-UGM-1. Note that the mean differences are removed.

Table 7. Statistics of accuracy of various quasi-geoids. (units: cm). Note that the mean differences are removed.

	max	min	sd
QGNSea V1.0	5.2	-3.9	1.5
EGG08	7.8	-9.4	2.2
EGM2008	8.4	-10.0	2.6
EIGEN-6C4	9.0	-11.9	2.7
GECO	8.3	-12.8	2.6
SGG-UGM-1	8.8	-12.7	2.7

For further comparisons, a high-resolutionwe compute the local mean dynamic topography (MDT) over the target region is derived using QGNSea V1.0, which illustrates the departure of the mean sea surface (MSS) from the quasi-geoid/geoid (Becker et al., 2014; Bingham et al., 2014). is compared with an existing model called DTU13MDT with the spatial resolution of 1′×1′ (Figure 10 (b))We compute the MDTs in a geodetic way, and the raw MDTs are computed as the differences between MSS and local geoid/quasi-geoid models, and the derived MDTs are further smoothed with a Guassian filter to suppress the small-scale signals that can't be resolved from the MSS or local geoid/quasi-geoid (Andersen et al., 2013). DTU13MSS from 1993-2012 is chosen as the MSS, and this model is provided as the gridded data, with the spatial resolution of 1′×1′ (Andersen et al., 2013). Considering QGNSea V1.0 and EGG08 have better performances than other models compared with local GPS/leveling data, we only compute the local MDTs based on these two gravimetric quasi-geoids. Similar as the methods for computing DTU13MDT (Andersen et al., 2013), the local MDT is computed in a purely geodetic way, where DTU13MSS and QGNSea V1.0/EGG08 are directly combined to obtain the raw MDT. Then,, and a Gaussian filter with a correlation length of 675 km is further applied to smooth the derived MDT, considering the small-scale signals that have the wavelengths shorter than several kilometers can't be recovered from the local gravity data, since the mean distance between gravity data is approximately at 6~7 km level. The modeled MDTs based on QGNSea V1.0 and EGG08 are denoted as called MDTNS_QGNSeaNSeaMDT and MDTNS_EGG08, respectively (Mean dynamic topography over the North Sea), is displayed insee Figure 9 (a), showinging in good agreement with DTU13MDT each other in most areas over the North Sea. Although the misfit between QGNSea V1.0 and EGM2008 reaches several centimeters in the North Sea (see (c)), the applied Gaussian filter seems attenuates these differences and consequently, these two MDTs demonstrate similar structures in the spatial domain. Prominent signals like the Norwegian coastal currents can be seen in these two MDTs, also see e.g., Idžanović et al. (2017), although the signals observed in MDTNS_QGNSea don't provide a full picture of Norwegian coastal currents due to the limited data coverage in Norway and its neighbouring ocean areas. While, in other areas of the North Sea, the MDTs show quite smooth patterns, indicate the small change in sea surface topography, which is consistent with Hipkin et al. (2004). It is also worth noting that observable differences appear between these MDTs, especially in the northern parts of the North Sea and east parts of the Irish Sea. The geostrophic velocities in Figure 11 indicate the geostrophic surface currents are rather smooth in the North Sea, where the SD values for the zonal (meridian) components are approximately 1.96 cm/s (1.86 cm/s) and the absolute values for both the zonal and meridian components are within 8 cm/s in the open sea areas. However, extreme values are observed surrounding most offshore areas, e.g., see the features over the offshore regions closed to The Wash (around 0.5 °W and 53 °N) and Thames estuary (around 1 °W and 51.5 °N) in England, and along the coastal areas of France, Netherlands, and Germany, which are typically identified as errors (Hipkin et al., 2004). The problems for computing geodetic MDTs in offshore regions are twofold. First, the quasi-geoid/geoid is poorly modeled in coastal areas due to the unfavorable data coverage, and data inconsistencies are usually observed when combining land and marine gravity surveys. Moreover, the quality of altimetry data is dramatically reduced near the offshore areas, and associated errors in the derived MSS propagate into the final MDT (Andersen et al., 2013). However, airborne gravity measurements provide a seamless way for gravity measurements over land and seas, which may allay this situation (Andersen and Knudsen, 2000). ~~Similar results can also be found in Hipkin et al. (2004).~~

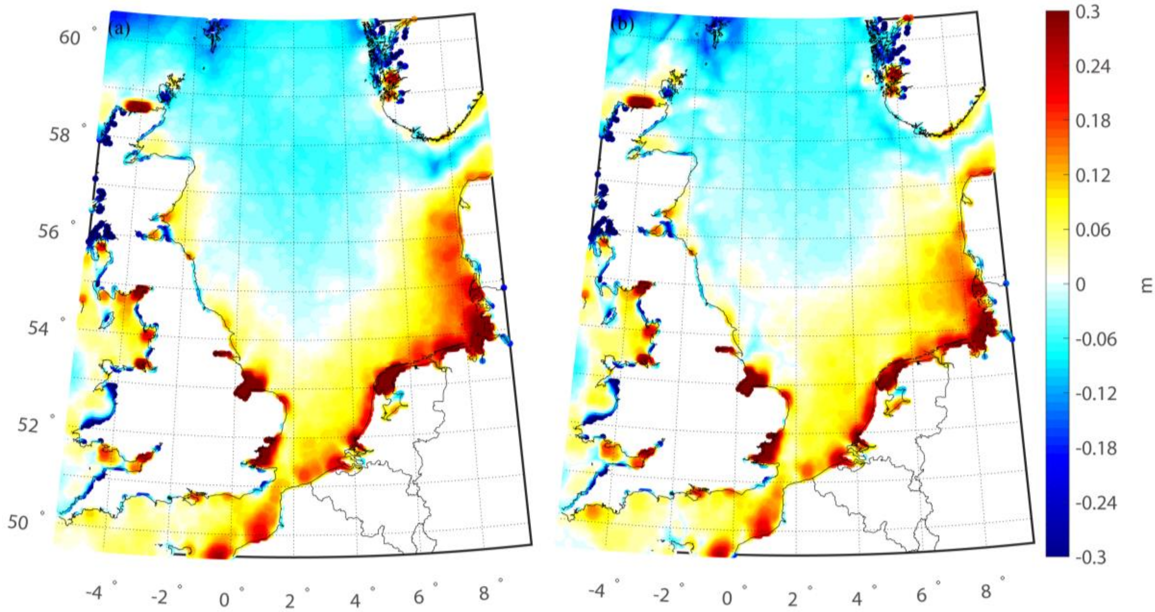

Figure 9. Different geodetic MDTs in North Sea. (a) MDTNS_QGNSea; (b) MDTNS_EGG08. For all profiles the mean value has been removed.

4. Conclusions

A multilayer approach is developed for gravity field recovery at regional scales ~~from heterogeneous data based on the idea~~in the framework of multi-resolution representation, where the residual gravity field is parameterized as the superposition of the multiply layers of Poisson wavelets~~' grids~~ located at the different depths beneath the topography. Since the gravity signals is the sum of the contributions generated from the anomaly sources at different depths, we put the multiply layers at the locations where different anomaly sources situate. Further, wavelet decomposition and power spectrum analysis are applied for estimating the depths of different layers.

For testing the performance of this multilayer approach, a local gravimetric quasi-geoid called QGNSea V1.0 over the North Sea in Europe is modeled and compared with other models~~, where a dense coverage of high-quality measurements extending continuously from land to ocean are available~~. Based on wavelet ~~analysis~~decomposition and power spectrum analysis, multiply layers that situate between 4.5 km and 59.2 km underneath the topography are built to capture the signals ~~with different spectral contents~~at different scales. The numerical results show that the multilayer approach is sensitive to the spectrum of signals, both in the low- and high-frequency bands; while, the traditionally-used single-layer approach is only sensitive for parts of signals' spectrum. The comparisons with the single-layer approach show that ~~the residuals of data derived from~~ the multilayer~~s~~ approach ~~reduce significantly in the target area~~fits the gravity observations better, especially in the regions where the gravity signals show strong correlations with the variation of local topography. Moreover, we introduce a Akaike information criterion (AIC) test for different models, which is an estimator of the relative quality of statistical models for a given set of data, providing a means for model selection in the view of statistical test. The associated results demonstrate that the multilayer model gives a smaller AIC value, which reaches a better balance between the goodness of fit of data and the simplicity of the model. The evaluation with independent GPS/leveling data tests the ability of realistic extrapolation of regional models recovered from different methods, reveals the model called QGNSea V1.0 computed by multilayer approach~~–~~ fits the local GPS/leveling data better~~deriving a more accurate quasi-geoid~~, ~~where QGNSea V1.0 outperforms the solution obtained by the single-layer approach,~~ by the magnitudes of 0.4 cm, 0.9 cm and 1.1 cm in the Netherlands, Belgium and parts of Germany, respectively, compared to the one recovered from the single-layer approach. Further comparisons with the existing models ~~indicates~~ show that QGNSea V1.0 has the best performance, which ~~could~~ may be ~~used~~ beneficial for investigating the ocean circulation in the North Sea and surrounding oceanic areas~~local areas~~.

Future work is needed for further improving the QGNSea V1.0. First, the satellite data (e.g., K-band Range Rate data and gravity gradients) from GRACE and GOCE missions can be combined with the ground-based gravity data~~ for further improving the solution quality~~. However, deeper ~~Poisson wavelet's grids~~layers than ones we use to combine surface data may be implemented to incorporate satellite observations, since these data ~~are more sensitive~~mainly contribute to low-frequency ~~gravity signals~~bands of gravity field. In addition, the stochastic model may need to be refined. For instance, the effects on the solutions caused by the GGM's errors may be quantified if ~~we incorporate~~ the full error variance-covariance matrix of the spherical coefficients is incorporated into the stochastic model. ~~Consequently~~In this way, the different data may be more properly weighted, and the solution ~~can~~ may be further improved.

Author contributions. All authors have contributed to designing the approach and writing the manuscript.

Code and data availability. The source code is included as the Supplement. Gravity data were provided by the British Geological Service; the Geological Survey of Northern Ireland; the Nordic Geodetic Commission; Bundesamt für Kartographie und Geodäsie (Germany); Institut für Erdmessung (Germany); the Bureau Gravimétrique International IAG service (France); the Banque de données Gravimétriques de la France; and the Bureau de Recherches Géologiques et Minières (France). GPS/leveling data were provided by the Geo-information and ICT of Rijkswaterstaat (RWS-AGI) and the GPS Kernnet of the Kadaster, National Geographic Institute (NGI) and the Royal Observatory (ROB), and Bundesamt für Kartographie und Geodäsie.

Competing interests. The authors declare that they have no conflict of interest.

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Appendix A: Akaike information criterion

Suppose that we have a statistical model of some data, and the Akaike information criterion (AIC) value of the model is (Burnham and Anderson, 2002)

$$AIC = 2k - 2\ln(\hat{L}) \quad \text{(A1)}$$

where k is the number of estimated parameters in the model, and $\hat{L}$ is the maximum value of the likelihood function for the model (Akaike, 1974; Burnham and Anderson, 2002).

For least squares fitting, the maximum likelihood estimate for the variance of a model's residuals distributions is

$$\hat{\theta}^2 = RSS / n \qquad \text{(A2)}$$

where RSS is the residual sum of squares (RSS), and n is the number of observations.

Then, the maximum value of a log-likelihood function of least square model is (Burnham and Anderson, 2002)

$$-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\hat{\theta}^2) - \frac{1}{2\hat{\theta}^2} RSS = -\frac{n}{2}\ln(RSS / n) + C \qquad \text{(A3)}$$

where C is a constant independent of the model.

Combining Eq.(A1) and Eq.(A3), for least square model, the AIC value is expressed as

$$AIC = 2k + n\ln(RSS / n) + C \qquad \text{(A4)}$$

Since only differences in AIC are meaningful, the constant C can be ignored, and we can conveniently take $AIC = 2k + n\ln(RSS / n)$ for model comparisons.

References

Akaike, H.: A new look at the statistical model identification, IEEE Transactions on Automatic Control, 19(6), 716-723, doi:10.1109/TAC.1974.1100705, MR 0423716, 1974.

Andersen, O. B., and Knudsen, P.: The role of satellite altimetry in gravity field modeling in coastal areas, Phys. Chem. Earth., 25(1), 17-24, doi:10.1016/S1464-1895(00)00004-1, 2000.

Andersen, O. B., Knudsen, P., and Stenseng, L.: The DTU13 global mean sea surface from 20 years of satellite altimetry, In: OSTST Meeting, Boulder, Colo, 2013.

Artemieva, I.M. and Thybo, H.: EUNAseis: A seismic model for Moho and crustal structure in Europe, Greenland, and the North Atlantic region, Tectonophysics, 609, 97-153, doi:10.1016/j.tecto.2013.08.004, 2013.

Audet, P.: Toward mapping the effective elastic thickness of planetary lithospheres from a spherical wavelet analysis of gravity and topography, Phys. Earth. Planet. In., 226(1), 48-82, doi: 10.1016/j.pepi.2013.09.011, 2014.

Becker, S., Brockmann, J. M., and Schuh, W. D.: Mean dynamic topography estimates purely based on GOCE gravity field models and altimetry, Geophys. Res. Lett., 41, 2063-2069, doi:10.1002/2014GL059510, 2014.

Bentel, K., Schmidt, M. and Rolstad, D.C.: Artifacts in regional gravity representations with spherical radial basis functions, Journal of Geodetic Science, 3(3), 173-187, doi:10.2478/jogs-2013-0029, 2013.

Bingham, R. J., Haines, K., and Lea, D. J.: How well can we measure the ocean's mean dynamic topography from space? J. Geophys. Res. Oceans, 119, 3336-3356, doi: 10.1002/2013JC009354, 2014.

Blundell, D.J., Hobbs, R.W., Klemperer, S.L., Scott-Robinson, R., Long, R.E., West, T.E. and Duin, E.: Crustal structure of the central and southern North Sea from BIRPS deep seismic reflection profiling, Journal of the Geological Society, 148, 445-457, 1991.

Burnham, K. P., Anderson, D. R.: Model Selection and Multimodel Inference: A practical information-theoretic approach (2nd ed.), Springer-Verlag, ISBN 0-387-95364-7, 2002.

Chambodut, A., Panet, I., Mandea, M., Diament, M., Holschneider, M. and Jamet, O.: Wavelet frames: an alternative to spherical harmonic representation of potential fields, Geophys. J. Int., 163(3), 875-899, doi:10.1111/j.1365-246X.2005.02754.x, 2005.

Cianciara, B. and Marcak, H.: Interpretation of gravity anomalies by means of local power spectra, Geophys. Prospect., 24 (2), 273-286, doi: 10.1111/j.1365-2478.1976.tb00925.x, 1976.

Denker, H.: Regional Gravity Field Modeling: Theory and Practical Results, In: G Xu (ed)Sciences of 641 Geodesy - II, 185-291, Springer, Berlin, Heidelberg, 2013.

Eicker, A., Schall, J. and Kusche, J.: Regional gravity modeling from spaceborne data: case studies with GOCE, Geophys. J. Int., 196(3), 1431-1440, doi:10.1093/gji/ggt485, 2013.

Fengler, M.J., Freeden, W., Kohlhaas, A., Michel, V. and Peters, T.: Wavelet Modeling of Regional and Temporal Variations of the Earth's Gravitational Potential Observed by GRACE, J. Geod., 81(1), 5-15, doi: 10.1007/s00190-006-0040-1, 2007.

Fengler, M.J., Freeden, W. and Michel, V.: The Kaiserslautern multiscale geopotential model SWITCH-03 from orbit perturbations of the satellite CHAMP and its comparison to the models EGM96, UCPH2002_02_0.5, EIGEN-1s and EIGEN-2, Geophys. J. Int., 157(2), 499-514, doi:10.1111/j.1365-246X.2004.02209.x, 2004.

Fichler, C. and Hospers, J.: Deep crustal structure of the northern North Sea Viking Graben: results from deep reflection seismic and gravity data, Tectonophysics, 178, 241-254, 1990.

Forsberg, R. and Tscherning, C.C.: An overview manual for the GRAVSOFT geodetic gravity field modeling programs, 2nd edn. National Space Institute, Denmark and Niels Bohr Institute, University of Copenhagen, 2008.

Förste, C., Bruinsma, S.L., Abrikosov, O., Lemoine, J.M., Schaller, T., Götze, H.J., Ebbing, J., Marty, J.C., Flechtner, F., Balmino, G. and Biancale, R.: EIGEN-6C4 The latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse. The 5th GOCE User Workshop, Paris, France, 2014.

Fotopoulos, G.: Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data. J. Geod., 79(1),111-123, doi:10.1007/s00190-005-0449-y, 2005.

Freeden, W., Gervens, T. and Schreiner, M.: Constructive Approximation on the Sphere (With Applications to Geomathematics). Oxford Sci. Publ., Clarendon Press, Oxford, 1998.

Freeden, W., Fehlinger, T., Klug, M., Mathar, D. and Wolf, K.: Classical globally reflected gravity field determination in modern locally oriented multiscale framework. J. Geod., 83, 1171-1191, doi:10.1007/s00190-009-0335-0, 2009.

Freeden, W. and Schreiner, M.: Local multiscale modelling of geoid undulations from deflections of the vertical, J. Geod., 79, 641-651, doi: 10.1007/s00190-005-0017-5, 2006.

Gilardoni, M., Reguzzoni, M., Sampietro, D.: GECO: a global gravity model by locally combining GOCE data and EGM2008, Studia Geophysica et Geodaetica, 60 (2), 228-247, doi:10.1007/s11200-015-1114-14, 2015.

Grad, M. and Tiira, T.: TheMoho depthmap of the European Plate. Geophys. J. Int., 176, 279-292, doi:10.1111/j.1365-246X.2008.03919.x, 2009.

Gruber, T., Rummel, R., Abrikosov, O., and van Hees, R.: GOCE Level 2 Product Data Handbook. GO-MA-HPF-GS-0110, Issue 4.2. (http://earth.esa.int/pub/ESA_DOC/GOCE/GOMA-HPF-GS-0110_4.2-ProductDataHandbook.pdf), 2010.

Hipkin, R.G., Haines, K., Beggan, C., Bingley, R., Hernandez, F., Holt, J. and Baker, T.: The geoid EDIN2000 and mean sea surface topography around the British Isles, Geophys. J. Int., 157(2), 565-577, doi:10.1111/j.1365-246X.2004.01989.x, 2004.

Holschneider, M. and Iglewska-Nowak, I.: Poisson wavelets on the sphere. J. Fourier. Anal. Appl., 13(4), 405-419, doi:10.1007/s00041-006-6909-9, 2007.

Idžanović, M., Ophaug, V., and Andersen, O. B.: The coastal mean dynamic topography in Norway observed by CryoSat-2 and GOCE, Geophys. Res. Lett., 44, 5609-5617, doi:10.1002/ 2017GL073777, 2017.

Jiang, W., Zhang, J., Tian, T. and Wang, X.: Crustal structure of Chuan-Dian region derived from gravity data and its tectonic implications, Phys. Earth. Planet. Interiors., 212-213, 76-87, doi: 10.1016/j.pepi.2012.07.001, 2012.

Klees, R., Prutkin, I.: The combination of GNSS-levelling data and gravimetric (quasi-) geoid heights in the presence of noise. J. Geod., 84(12), 731-749, doi:10.1007/s00190-010-0406-2, 2008.

Klees, R., Tenzer, R., Prutkin, I. and Wittwer, T.: A data-driven approach to local gravity field modelling using spherical radial basis functions, J. Geod., 82(8), 457-471, doi:10.1007/s00190-007-0196-3, 2008.

Koch, K.R. and Kusche, J.: Regularization of geopotential determination from satellite data by variance components, J. Geod., 76, 259-268, doi: 10.1007/s00190-002-0245-x, 2002.

Kusche, J.: A Monte-Carlo technique for weight estimation in satellite geodesy, J. Geod., 76(11), 641-652, doi:10.1007/s00190-002-0302-5, 2003.

Liang, W., Xu, X., Li, J. and Zhu, G.: The determination of an ultra high gravity field model SGG-UGM-1 by combining EGM2008 gravity anomaly and GOCE observation data, Acta Geodaeticaet Cartographica Sinica, 47(4), 425-434, doi:10.11947/j.AGCS.2018.20170269, 2018.

Lieb, V., Schmidt, M., Dettmering, D. and Börger, K.: Combination of various observation techniques for regional modeling of the gravity field, J. Geophys. Res. Solid Earth, 121, 3825-3845, doi:10.1002/2015JB012586, 2016.

Mayer-Gürr, T., Pail, R., Gruber, T., Fecher, T., Rexer, M., Schuh, W.-D., Kusche, J., Brockmann, J.-M., Rieser, D., Zehentner, N., Kvas, A., Klinger, B., Baur, O., Höck, E., Krauss, S., and Jäggi, A.: The combined satellite gravity field model GOCO05s, Geophys Res Abs 17:EGU2015-12364, 2015.

Naeimi, M., Flury, J. and Brieden, P.: On the regularization of regional gravity field solutions in spherical radial base functions, Geophys. J. Int., 202, 1041-1053, doi:10.1093/gji/ggv210, 2015.

Nahavandchi, N., Soltanpour, A.: Improved determination of heights using a conversion surface by combining gravimetric quasi-geoid/geoid and GNSS-levelling height differences. Stud. Geophys. Geod., 50(2), 165-180, doi:10.1007/s11200-006-0010-3, 2006.

Omang, O.C.D. and Forsberg, R.: How to handle topography in practical geoid determination: three examples, J. Geod., 74(6), 458-466, doi:10.1007/s001900000107, 2000.

Panet, I., Kuroishi, Y. and Holschneider, M.: Wavelet modelling of the gravity field by domain decomposition methods: an example over Japan, Geophys. J. Int., 184(1), 203-219, doi:10.1111/j.1365-246X.2010.04840.x, 2011.

Pavlis, N.K., Holmes, S.A., Kenyon, S.C. and Factor, J.F.: The development and evaluation of Earth Gravitational Model (EGM2008), J. Geophys. Res., 117, B04406, doi:10.1029/2011JB008916, 2012.

Prutkin, I., Klees, R.: On the non-uniqueness of local quasi-geoids computed from terrestrial gravity anomalies. J. Geod., 82(3), 147-156, doi:10.1007/s00190-007-0161-1, 2008.

Rummel, R., Balmino, G., Johannessen, J., Visser, P., and Woodworth, P.: Dedicated gravity field missions-Principle and aims, J. Geodyn., 33(1), 3-20, doi:10.1016/S0264-3707(01)00050-3, 2002.

Schmidt, M., Fengler, M., Mayer-Gürr, T., Eicker, A., Kusche, J., Sánchez, L. and Han, S.C.: Regional gravity modeling in terms of sphericalbase functions, J. Geod., 81(1), 17-38, doi:10.1007/s00190-006-0101-5, 2007.

Schmidt, M., Fabert, O., Shum, C.K.: On the estimation of a multi-resolution representation of the gravity field based on spherical harmonics and wavelets, J. Geodyn., 39(1), 512-526, doi: 10.1016/j.jog.2005.04.007, 2005.

Schmidt, M., Han, S.C., Kusche, J., Sanchez, L. and Shum, C.K.: Regional high- resolution spatiotemporal gravity modeling from GRACE data using spherical wavelets, Geophys. Res. Lett., 33, L08403, doi:10.1029/2005GL025509, 2006.

Sjöberg, L. E.: A discussion on the approximations made in the practical implementation of the remove-compute-restore technique in regional geoid modelling, J. Geod., 78, 645-653, doi:10.1007/s00190-004-0430-1, 2005.

Slobbe, D. C.: Roadmap to a mutually consistent set of offshore vertical reference frames, PhD thesis, The Netherland, 2013.

Slobbe, D.C., Klees, R. and Gunter, B.C.: Realization of a consistent set of vertical reference surfaces in coastal areas, J. Geod., 88(6), 601-615, doi:10.1007/s00190-014-0709-9, 2014.

Spector, A. and Grant, F.S.: Statistical models for interpreting aeromagnetic data, Geophysics, 35(2), 293-302, doi:10.1190/1.1440092, 1970.

Syberg, F.J.R.: A Fourier method for the regional-residual problem of potential fields, Geophys. Prospect., 20 (1), 47-75, doi:10.1111/j.1365-2478.1972.tb00619.x, 1972.

Tapley, B. D., Bettadpur, S., Watkins, M. and Reigber, C.: The gravity recovery and climate experiment: Mission overview and early results, Geophys. Res. Lett., 31, L09607, doi:10.1029/2004GL019920, 2004.

Tenzer, R. and Klees, R.: The choice of the spherical radial basis functions in local gravity field modeling. Stud. Geophys. Geod., 52(3), 287-304, doi:10.1007/s11200-008-0022-2, 2008.

Tenzer, R., Klees, R. and Wittwer, T.: Local Gravity Field Modelling in Rugged Terrain Using Spherical Radial Basis Functions: Case Study for the Canadian Rocky Mountains. In S. Kenyon, Geodesy for Planet Earth, International Association of Geodesy Symposia 136, Springer-Verlag Berlin Heidelberg, pp 401-409, 2012.

Tscherning, C.C.: Introduction to functional analysis with a view to its application in approximation theory. In: Moritz H, Sünkel H(eds)Approximation methods in geodesy, Karlsruhe, Germany, 1978.

Wang, Y., Saleh, J., Li, X., and Roman, D. R.: The US Gravimetric Geoid of 2009 (USGG2009): model development and evaluation, J. Geod., 86(3), 165-180, doi:10.1007/s00190-011-0506-7, 2012.

Wittwer, T.: Regional gravity field modelling with radial basis functions, Dissertation, Delft University of Technology, Delft, The Netherlands, 2009.

Wu, Y., Luo, Z., Chen, W. and Chen, Y.: High-resolution regional gravity field recovery from Poisson wavelets using heterogeneous observational techniques, Earth Planets Space, 69:34, doi:10.1186/s40623-017-0618-2, 2017a.

Wu, Y., Luo, Z. and Zhou, B.: Regional gravity modelling based on heterogeneous data sets by using Poisson wavelets radial basis functions, Chin. J. Geophys (in Chinese), 59(3):852-864, doi:10.6038/cjg20160308, 2016.

Wu, Y., Zhong, B., Luo, Z.: Investigation of the Tikhonov regularization method in regional gravity field modeling by Poisson wavelets radial basis functions, Journal of Earth Science,9,1-10, doi: 10.1007/s12583-017-0771-32016, 2017b.

Wu, Y., Zhou, H., Zhong, B., and Luo, Z.: Regional gravity field recovery using the GOCE gravity gradient tensor and heterogeneous gravimetry and altimetry data, J. Geophys. Res. Solid Earth, 122(8), 6928-6952, doi: 10.1002/2017JB014196, 2017c~~b~~.

Xu, C., Liu, Z., Luo, Z., Wu, Y. and Wang, H.: Moho topography of the Tibetan Plateau using multi-scale gravity analysis and its tectonic implications, J. Asian. Earth. Sci., 138, 378-386, doi: 10.1016/j.jseaes.2017.02.028, 2017.

Xu, C., Luo, Z., Sun, R., Zhou, H., Wu, Y.: Multilayer densities using a wavelet-based gravity method and their tectonic implications beneath the Tibetan Plateau, Geophys. J. Int., 213, 2085-2095, doi: 10.1093/gji/ggy110, 2018.

Ziegler, P.A. and Dèzes, P.: Crustal evolution of western and central Europe, European Lithosphere Dynamics, In: Gee, D., Stephenson, R. (Eds.), Geol. Soc. London Sp. Publ., 32, 43-56, 2006.