Geoscientific
Model Development
Discussions

Interactive comment on "Impacts of the Horizontal and Vertical Grids on the Numerical Solutions of the Dynamical Equations. Part I: Nonhydrostatic Inertia-Gravity Modes" *by* Celal S. Konor and David A. Randall

Anonymous Referee #2

Referee comment on "Impacts of the Horizontal and Vertical Grids on the Numerical Solutions of the Dynamical Equations. Part I: Nonhydrostatic Inertia-Gravity Modes" by C.S. Konor and D.A. Randall

The manuscript derives and discusses numerical dispersion relationships for finite difference approximations of the Lipps & Hemler anelastic equations, for various staggered arrangements of the variables. In part I, the analysis is done on a mid-latitude f−plane and focuses on inertia-gravity modes ; in part II the analysis is done on a mid-latitude β−plane and focuses on Rossby mode.

Such numerical dispersion analysis are useful to understand the behavior of numerical schemes near the grid scale. Although one can not expect any numerical scheme to be accurate at such scales, one hopes to avoid pathological behavior such as physically propagating modes being numerically stationary or vice-versa. The alternative is to apply a sufficiently large diffusion that damps the small-scale degrees of freedom. This waste of degrees of freedom can be deemed acceptable in exchange of a gain in simplicity (e.g. collocated A-grid) in certain cases, e.g. spectral methods with scale-selective high-order hyperviscosity, but would be much more disputable for staggered lowish-order finite-difference / finite-volume methods. That such linear analyses are useful is demonstrated by the long history of similar work, and the operational popularity of staggered-mesh methods or, more recently, compatible finite element methods.

Compared to previous work, the work presented here is novel in 3 ways : (a) it deals with three-dimensional, non-hydrostatic equations while previous work two-dimensional shallow-water equations or hydrostatic equations that are equivalent to the latter after separation of variables (e.g. Bell, Peixoto, Thuburn QJRMS 2017) (b) it adresses the C-D staggering, in addition to better understood staggerings (A-E, Z) (c) the analysis of the C-D staggering is done with discrete time, while the only existing analysis (Skamarock, MWR 2008) is for continuous time.

Anelastic equations rather than the fully compressible equations are analyzed. As discussed by the authors, it is plausible that despite some inaccuracies of anelastic equations this is not a serious limitation. Similarly, lowest-order centered finite-difference schemes are considered rather than higher-order, upwind biased schemes. This is not a serious limitation either, since (a) when linearizing about a state of rest, upwinding becomes irrelevant (b) increasing the order of a scheme with problematic dispersion properties only makes the problem worse as far as I am aware.

I was particularly interested by the analysis of the C-D grid. So far I have checked the math mostly of that part. As far as I understand Harris & Lin, 2013, the linearization (32-42) of the predictor-corrector time scheme is correct (notice that the horizontal curl

and div have been applied to the linearized momentum equations).

The final results refine as a function of horizontal scale previous wisdom accumulated on the pros and cons of the various staggerings. Especially it confirms that, with respect to numerical dispersion, the C-D grid is extremely similar to the D-grid, whose shortcomings with respect to the propagation of gravity waves are well-known. No breakthrough here, but a valuable addition of a missing piece to the puzzle.

Overall, this is a clearly a good paper, with a sound methodology and useful purposes, that deserves publication after possibly correcting minor issues (see separate comment).

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