Response to the referee Almut Gassmann's comments

We thank Dr. Gassmann for very useful comments.

Answer to comment (1):

We can think of two major effects of (vertically) varying basic-state stability expressed by the Brunt-Vaisala frequency.

1- Abrupt change of stability

This is the case with the tropopause. The abrupt change of stability at the tropopause can act like a lid for the troposphere. As a result, the vertical extent of the waves is limited. From this point of view, we can estimate that the longest realizable vertical wavelength is limited to, say, n=9 for the domain used in our analyses. Recall that n=9 corresponds to roughly a 9-km half-wavelength that fits in to the troposphere. The discussion in section 7 of our manuscript indicates that nonhydrostatic effects are more important for the deep modes (see Fig. 15). The absence of very deep waves in realistic situations may be the reason that quasi-hydrostatic models, in practice, can give good solutions with horizontal grid spacing as small as 10 km

2-Gradual change of stability

Gradual changing static stability can cause internal refractions and reflections of waves, which also may limit their vertical extent.

Answer to comment (2):

Lindzen&Fox-Robinovitz (1989) suggest that the maximum vertical grid spacing used to resolve the quasi-geostrophic (and quasi-static) modes in the presence of critical layers should satisfy $(\delta z)_{max} = (f/N)\delta x$. In practice, this means that the vertical grid spacing should not exceed about 1% of the horizontal grid spacing. For nonhydrostatic models with small horizontal grid spacings, using such a small vertical grid spacing can immensely increase the computational cost. However, the nonhydrostatic models may not have to satisfy Lindzen&Fox-Robinovitz rule, because the resolutions that are needed to simulate nonhydrostatic motions may automatically resolve the QG modes.

We prepared the following table, showing the horizontal and vertical propagation speeds of the waves found in our analysis. The table can used in the selection of the horizontal and vertical grid spacings. In making the table, we assumed that the deepest and fastest wave has n=8.

$m = \pi n / z_T \quad z_T = 80$ km

Inertio-gravity waves

	n	$K^* = \sqrt{2}\,\pi/d$	$c_H = \nu/K^*$	$c_z = \nu/m$	c_z/c_H
$d = 2$ km	1	0.222144146908E-02	0.481904844357E+01	0.340750862011E+02	0.707106781187E+01
	3	0.222144146908E-02	[illegible]	[illegible]	[illegible]
	10	0.222144146908E-02	[illegible]	[illegible]	[illegible]
$d = 10$ km	1	0.444288293816E-03	[illegible]	[illegible]	[illegible]
	3	0.444288293816E-03	[illegible]	[illegible]	0.125707872210E+01
	10	0.444288293816E-03	0.182234773773E+02	0.200173113154E+02	0.183137084990E+01
$d = 25$ km	1	0.177715317526E-03	[illegible]	[illegible]	[illegible]
	3	0.177715317526E-03	[illegible]	[illegible]	[illegible]
	10	0.177715317526E-03	0.250599688908E+02	0.113400602291E+02	0.452548339959E+00
$d = 100$ km	1	0.444288293816E-04	[illegible]	[illegible]	[illegible]
	3	0.444288293816E-04	0.303885838749E+02	0.381946192673E+01	0.125707872210E+00
	10	0.444288293816E-04	0.274118268153E+02	0.318129418012E+01	0.183137084990E+00

Rossby waves

	n	$K^* = \sqrt{2}\,\pi/d$	$c_H = \nu/K^*$	$c_z = \nu/m$	c_z/c_H
$d = 2$ km	1	0.222144146908E-02	[illegible]	[illegible]	[illegible]
	3	0.222144146908E-02	-0.232123963774E-05	-0.143902187430E-04	0.628539861035E+01
	10	0.222144146908E-02	[illegible]	[illegible]	[illegible]
$d = 10$ km	1	0.444288293816E-03	[illegible]	[illegible]	[illegible]
	3	0.444288293816E-03	-0.580292138587E-04	-0.729472897449E-04	0.125707872210E+01
	10	0.444288293816E-03	[illegible]	[illegible]	0.183137084990E+01
$d = 25$ km	1	0.177715317526E-03	-0.362604902351E-03	-0.205120323640E-03	[illegible]
	3	0.177715317526E-03	[illegible]	[illegible]	[illegible]
	10	0.177715317526E-03	[illegible]	[illegible]	[illegible]
$d = 100$ km	1	0.444288293816E-04	[illegible]	[illegible]	[illegible]
	3	0.444288293816E-04	-0.577183383693E-02	[illegible]	[illegible]
	10	0.444288293816E-04	-0.570460370944E-02	-0.652190460596E-03	0.183137084990E+00

Answer to comment (3):

At the end of the C-grid subsection, we added a short paragraph discussing the C-grid discretization on the hexagonal and triangular grids (page 12, line 17, of revised Part I).

Response to the comments of the anonymous referee

We thank the referee for very instructive comments and going through the lengthy derivatives. We clarified in the text that Skamarock (2008) presents an analysis of the time continuous system.

<u>Response to the interactive comments of Harris and Chen</u>

<u>1-…"Unrepresentative of the discretizations used in modern dynamical cores"…:</u>

We do not agree that the linear analyses presented in this paper are irrelevant for today's dynamical cores. The methods that grid-point models use to simulate wave propagation have not changed over the years.

<u>2-…"Analyzing an oversimplifies second-order centered-difference system"…:</u>

Gravity waves propagate horizontally in all directions with the same speed, so the horizontal discretization of terms responsible for gravity wave propagation should be based on a centered treatment. Since our main concern is the behavior of the solutions near the smallest resolved horizontal scale (SRHS), we used a second-order scheme. Higher-order schemes produce more accurate solutions of the well-resolved features, but low-order schemes can actually be more accurate near the SRHS. We comment on the potential impact of higher-order schemes in section 7c below.

<u>3-…"Analysis method neglects nonlinear vorticity advection"…:</u>

Nonlinear processes are important to control the spurious cascades of (potential) enstrophy and kinetic energy to small scales, and they limit the accumulation of noise near the SRHS. They act slowly, however, and should not be expected to "cure" the rapid adverse-effects of poorly simulated linear wave dispersion.

When parameterized physical processes and topography are included, noise can be generated even without a spurious cascade. In such a case, wave propagation on the smallest resolved scales can disperse the energy and thereby reduce the noise. Of course, diffusion can also help to dissipate the noise, but a poor scheme may require excessive diffusion that also damps some of the better-resolved scales, thus effectively reducing the resolution of the model.

<u>4-…"The analysis is entirely inviscid"…:</u>

Yes, our analysis is entirely inviscid, and it should be. Viscous effects do not affect the dispersion of waves, but simply reduce their amplitudes. Avoiding the need for excessive diffusion is an important example of "good practices in dynamical core development."

<u>5-…" The CD grid analyzed has little resemblance to that used in the FV3"…:</u>

We gave the reviewers every chance to tell us what their scheme is. We personally asked them twice. In response, they gave some references, which describe the scheme in vague terms, without equations. The Editor asked the reviewers to specify the changes that would be needed to make equations (32)-(42) consistent with the formulation of FV3. The reviewers have not responded.

We have analyzed more than a dozen possible CD schemes, some of which have not produced closed systems of equations. (We will explain what we mean by closed systems in item 7a below.) All of the schemes that produced closed systems have been analyzed and presented in our manuscript and the supplementary material.

6-…" The C grid vorticity temperature and mass discarded"…:

Schemes III and V (pages 57 and 70 of the supplementary material, respectively) discard the vorticity obtained on the C grid, and pass the divergence to the D grid. At least in the shallow water sense, these schemes mimic what the reviewers describe as their scheme. The frequencies for scheme III and V are shown in figures on page 62 and page 75 of the supplementary material, respectively. The frequencies shown in these figures are virtually identical to the frequencies for scheme I that is discussed in the manuscript and all other schemes discretized on the CD grid.

All of the CD schemes presented in our manuscript and in the supplementary material correspond to closed systems. In all cases, the CD grid behaves like the D grid as far as the propagation of the gravity waves is considered.

In short, we stand behind our linear analysis of the CD grid. Our conclusions are consistent with those of Skamarock (2008).

7-Further comments :

7a-In what sense are some schemes not closed?:

The CD grid discretization combines the C and D grid solutions in a time-split predictor-corrector sequence. The linearized system solves for the unknowns that are the values of predicted variables (divergence, vorticity, potential temperature etc.) in the next time step using the values of the known quantities that are provided for present time step. The number of unknowns must be equal to the number of equations to have a closed system.

7b-What are the consequences if the system is not closed? :

With a scheme that is not closed, the solutions can include physically decoupled computational modes.

Here is a simple example: A large majority of the CD schemes that do not produce closed systems fail to produce a quadratic equation for the frequency, or an equivalent form given by Eq. (46a), which yields the same solution for positive or negative real-frequencies. Since the gravity waves horizontally propagate with the same speed every direction, this condition has to be satisfied by every "consistent" scheme.

Another example is the C-grid staggering on the triangular grid, which does not yield a closed system (Gassmann, 2011). An early version of ICON was based on a triangular grid and suffered from a checkerboard pattern its the divergence field as reported by its developers. Diffusion can of course render such noise invisible.

7c-How are these solutions affected by the use of high-order schemes? :

The use of high-order schemes has only a minor impact on our analysis. To see this, consider the discrete dispersion relation for the C-grid given by

$$\nu^2 = \frac{N^2\left(\xi^2 k^2 + \eta^2 \ell^2\right) + \mu^2 f^2 \left(m^2 + \frac{1}{4H^2}\right)}{\left(\xi^2 k^2 + \eta^2 \ell^2\right) + \left(m^2 + \frac{1}{4H^2}\right)},$$

which is Eq. (17) of Part I. This form will be the same for high-order schemes, but with different definitions of ξ and η. What is gained with the use of higher-order schemes is that the errors will be more confined near the SRHS. The errors at the SRHS are not improved by the use of higher-order schemes.

7d-Comments on the importance and use of linear analysis:

Linear analysis is an optimal tool to examine the behavior of waves near the SRHS. Obviously, we expect significant errors near the SRHS, but our manuscript demonstrates that *the nature and size of the errors depend on the grid used*. In the development of a dynamical core, it is useful to choose the grid staggering that behaves as well as possible near the SRHS, all other factors being equal. If there are compelling reasons to select a different grid, then the advantages of that grid should be demonstrated through precise quantitative tests.

Linear tests also allow us to determine whether all of the primary dynamical processes properly interact with each other in the discrete system. In particular, the unknowns and the number of equations should be balanced for the system to be closed. Without such closure, uncontrolled modes may appear. The C-grid discretization of the momentum equations on the hexagonal system has been achieved in this way (Gassmann, 2011).

References

Skamarock, W. C. A linear analysis of the NCAR CCSM finite-volume dynamical core. *Mon. Weather Rev.*, **136**, 2112–2119, 2008:.

Gassmann, A.: Inspection of hexagonal and triangular C-grid discretizations of the shallow water equations. *J. Comput. Phys.*, **230**, 2706-2721, 2011.