

## ***Interactive comment on “The degree of freedom for signal assessment of measurement networks for joint chemical state and emission analysis” by Xueran Wu et al.***

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Dear Reviewer,

Thank you for your attention to our manuscript, and your valuable comments on the research, as well as the suggestions for improving the paper. We have tried to address your concerns. Details on the changes are below.

### General comments

- *This paper describes a methodology to determine the information content (in particular, the degree of freedom for signal) of joint chemical state and emission inversions.*

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*A detailed mathematical analysis is provided in the first part, followed by numerical illustrations. There are several issues with this article: 1) The method is known. Recent articles, such as, e.g., Bousserez and Henze (qjrms, 2017), Spantini et al. (2015), presented in detail this approach for information content analysis, with similar mathematical developments. Other related methods in the context of ensemble data assimilation are described in the literature, see for instance Anderson (2001) (Ensemble Adjustment Kalman Filter). This has to be acknowledged and discussed by the authors.*

**Thanks for reminding us about those references. We will definitely cite them in the following version of our paper.**

**There are indeed several similar equations. For example, we can find the same equation, such as Eq.(17) and (19), in our paper and the references by Spantini et al. (2015) and Bousserez and Henze (2017), which are the expression of the inverse of posterior covariance matrix. But those equations are well known and obtained from several previous papers, such as paper from Li and Navon, 2001 we cited. Besides, we use the singular value decomposition into the same matrix as the paper by Bousserez and Henze (2017). However, there are some points need to be emphasized about the similarity to the two references, differences and novel points of our paper.**

- **As to the similarity, we firstly have to clarify that the previous version of this paper has been published online in arXiv.org as a preprint with the title 'Efficiency and Sensitivity Analysis of Observation Networks for Atmospheric Inverse Modelling with Emissions' on Mar. 23, 2015. Then we reorganized the some part of the text and terminology in order to improve the preprint and got the current version submitted to GMD. Thus, the few same expression of the posterior covariance matrix as the one in paper by Spantini et al. (2015) is a coincidence. Besides, the similar equations from the paper by Bousserez and Henze (qjrms, 2017) are actually obtained by us earlier**

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(the preprint on Mar. 2015).

- Spantini et al. aimed at to approximate the posterior covariance in a subspace. It differs from our motivation to detect the capacity of a given observation network to optimize the state vector and emissions with the help of posterior covariance. It is worth noting that it is novel in our paper that we extended the state vector of the linearized atmospheric transport model so that the emissions were included in the model-state vector and the model turned to be homogeneous (Section 2). In section 3 and 4, based on the above extended model, our idea is to consider the normalization of the difference between the background forecast error covariance matrix and the analysis error covariance matrix as a criteria to investigate the capacity of observation networks to improve the estimation of both the concentrations and emissions. The normalization is crucial because it provides an uniform standard to any cases with diverse initial conditions and allows us to apply our approach into them directly and not considered in the paper by Spantini et al.. To be specific, compared with Eq. (3.6), (3.10), (4.4) and (4.13) in Spantini et al. (2015), the similar-looking equations (74), (72), the equation in line 13 on Page 9 and Eq. (54) in our paper are distinct.
- Bousserez and Henze discussed the theoretically equivalent approach to Bocquet et al (2011) they cited, yet different interpretation in terms of projection and aggregation framework. Both approaches, which can be considered, with some limits, as dual to each other, seek to optimize, in a controlled way, the complexity of the underlying problem space. The observational network is only involved by controlling the error of representatives. In contrast, our approach seeks to quantify the degree of freedom for signals for a given observation network and model/analysis grid with respect to its value for a heterogeneous parameter optimization (emission strengths and initial values in our case). In some sense, this can be considered as a test

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for prolongation of observational data to an extended parameter optimization space.

- Besides, in section 5, we originally derived the ensemble case of the approach in the section 3 and 4 without the non-singularity of the background covariance matrix, which differs from the derivation in the paper by Anderson (2001).

- 2) *The grammar needs to be thoroughly checked. In many places the text is unclear due to poor phrasing.*

**Thanks for your comments. We will go through the whole paper to correct the grammar mistakes and improve the phrasing and attempt to provide a satisfactory writing in the next version.**

- 3) *The main text contains too many equations, which is distracting and makes the reading difficult. Most of the mathematical developments should be moved to an Appendix.*

**Thanks for your valuable advise. Most of the mathematical developments will be moved to an Appendix and then the text will be reorganized.**

Detailed comments:

- 1) *Introduction: It is too long. Also, there are lots of redundancies. A number of references should be added (see general comments above for some of them) and discussed. In particular, the authors should clearly acknowledge previous works where similar analysis were conducted, and explain what their study adds to the current state of knowledge. If there is no real novelty in the approach, then the article should be*

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presented as a review paper focusing on methods for information content analysis, with a numerical application for the specific problem of joint chemical state and emission inversion.

**Thanks for the comments. The introduction is shortened and the novelties are emphasized.**

-2) P1, L17-18: Rephrase.

**We changed this to: Parameter mis-specifications in a model can only be identified within data assimilation intervals of space-time methods, if the the simulation is sufficiently sensitive and the error related observability of the measurement network is given.**

-3) P1, L19-24: Shorten. There are many repetitions.

**These sentences are now replaced by : “Otherwise, the forecast degrades beyond the observation controlled period.**

-4) P6, L13-23: Could be simplified (or should go to an Appendix).

**P6, L13-23 has been simplified and P7, L1-L7 has been put into Appendix A.**

-5) P7 L9-15: Redundancies. Poor phrasing.

**All sentences except the last are removed.**

-6) P7 L19: Good. You should do that simplification earlier. There is no need to split all

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the operators like in (11).

**Thanks for your comments. Though Eq. (15) and (16) are the general case of Eq. (11), Eq. (11) is novel in our paper.**

-7) P7, Eq(17): Please define the mathematical terms you use. For instance,  $t_{-1}$  is not defined here. Presumably  $P^{-1}(t_0|t_{-1})$  is the prior error covariance matrix, in which case it should be clearly stated.

**The required definitions are inserted.**

-8) P8, L1-2: I do not understand this sentence.

**It has been deleted.**

-9) P8. Eq(19): this equation is well-known and the previous developments are not needed.

**We arrive at a more comfortable notation.**

-10) P8 L13-14: Unclear. Please rephrase.

**We changed it into: “We aspire an expression, which allows for a direct and normalized comparison of sensitivities to initial values and emission rates. ”**

-11) P13, Eq (53): Notation V, S, U has already been used in (46) and (48), and the SVDs in (53) and (46) are not related. Please use another notation.

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**The notation has been changed accordingly.**

*-12) Section 6: Again, lots of mathematical developments that should be in an Appendix.*

**Most mathematical developments have been moved into Appendix B.**

Sincerely,

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Interactive comment on Geosci. Model Dev. Discuss., <https://doi.org/10.5194/gmd-2017-220>, 2017.