Overview

First, we would like to thank the reviewers who made comments. We have carefully considered all the points and made changes and additions that make the study more complete. One of the main changes we have made is to change the order of sections 3 and 4, which makes the paper flow better, with consideration of the resolution tests. Also, the terminology was made to be consistent throughout the paper, so that this is a “program” that produces “ice sheet reconstructions”. This has required that we revise the title of the paper to “ICESHEET 1.0: A program to produce paleo-ice sheet reconstructions with minimal assumptions” (previously titled “ICESHEET 1.0: A program to produce paleo-ice sheet models with minimal assumptions”). Figure 1 has been completely changed, and we hope it provides a more clear illustration of how the program works.

Below are the reviewer comments, followed by our response in italics. The marked up manuscript with changes follows that. Note that latexdiff could not handle the switched around sections 3 and 4, so they are reversed relative to what will be in the final document.

Response to comments by Anonymous Referee #1

The advance in science and computer power have led to an enormous increase in modelling techniques, number of models in computations, and complexity of models. Ice (sheet) models are input to several of modellings when it comes to e.g. climate change or sea-level rise. As the authors note, they also turn out to be more complex recently. I welcome this study as it (1) provides an easy tool for generating your own ice model and (2) shows that for many studies on glacial isostatic adjustment (GIA) the ice model does not have to be so complex as often indicated. The manuscript is well written and concise, figures and tables provide all information needed to understand the tool. While reading I made a few notes only where some additional information would help make points finally clear.

Minor suggestions:

L109. Specify the "limited impact".

Response: *The final results change by less than 100 m near the margins if the initial sea level is adjusted. This is noted in the text.*

L111. What is "a component of GIA"?

Response: *The wording there was awkward. We changed this to say “In subsequent iterations, the topography is adjusted for glacial-isostatic adjustment...”*

L114. Add a few words on CALSEA such as underlying theory, resolution, dimension, e.g. is it a spheric harmonic viscoelastic Maxwell body description up to degree 256 using PREM? Does it include rotational feedback and moving shorelines? 1-2 sentences here or in section 4.3.

Response: *We added the following sentences to explain how CALSEA works:*

CALSEA computes glacial-isostatic adjustment using a spherically symmetric Earth, with a Maxwell rheology mantle and elastic lithosphere, using the PREM model (Dziewonski and Anderson, 1981) for other Earth model parameters. In includes time evolving shorelines and rotational feedback.

Sect. 3.2. Add maximum differences in the text. Aren’t the narrow ice streams at the edge and in the fjord areas?

Response: *We added a numerical value to the text (>400 m). Also added that indeed, the largest discrepancies are at the edge of the ice sheet sheet, where the ice stream locations are located.*

L231/2. This sentences is out of place here. Suggest to remove and add to L226 "...were performed, whose results can be found in Table 1." Also add "(Table 1)" after polygons in L234.

Response: *We rearranged the reference to Table 1 in the first paragraph of section 4.2 and added the reference to it in the first sentence of the second paragraph.*

Figure 1. What are the red and green lines (name it in caption!)?

Response: *Figure 1 has been completely revised. This is elaborated in response to Anonymous Referee #2.*

Figure 4. Add ", see text" after ice sheet.

Response: *This has been added to the caption.*

Figure 5. Specify spacing and contour interval of the reference ice sheet model here as well.

Response: *We added the information to the caption (1 km spacing and 10 m contour interval)*

Table 1. Highlight/mark the reference model, the recommended result and the one used in Fig 5b.

Response: Notes have been added to not the reference and recommended reconstructions.

Response to comments by Anonymous Referee #2

This paper describes an ice-sheet reconstruction technique, that estimates ice surface elevations given the 2-D margins of the ice sheet. It is based on two assumptions: steady state, and perfect plasticity, i.e., the basal shear stress is a given yield value. Both the bedrock topography and the basal yield stress (which can be uniform or spatially varying) need to be specified, but if they are not known, one or both can be adjusted iteratively to fit surface elevations (or ice sheet thicknesses) that may be known to some extent by independent means.

The reconstruction uses the method of characteristics, making direct use of two earlier studies: Reeh (1982) and Fisher et al. (1985). The authors have coded the method into a usable general-purpose program, which includes the capability to automatically correct for topographic barriers and contour crossovers. Although I have some reservations as described below, I think the study will be suitable for GMD, and the program may be an interesting and worthwhile tool for some glaciological situations.

General comments:

Several significant aspects are not well explained, leaving some questions hanging, as described in points 1-3. Much of this can probably be corrected by clarification and additional text to provide more information and context.

1. The overall purpose and outcomes of the study and the program are not clearly defined. Given a map of ice-sheet margins, basically there are 3 unknowns: surface elevation, basal shear (yield) stress, and bedrock elevations (all 2-D maps). Eqs. 5 to 9 and the method of characteristics relate the latter two fields to the first. But if only one of the 3 is known or is available from some external means, the other 2 cannot both be determined uniquely. 2 of them have to be known to determine the 3rd. In the examples given, Greenland's bed topography and surface elevations are known from independent data, and the method determines (iteratively) the basal shear stress map. For the Barents, bed topography is given from a modern dataset (which neglects depression under paleo grounded ice), and basal stress is "adjusted" to yield GIA-estimated paleo ice thicknesses or equivalently surface elevations.

These applications confuse what many readers may assume up front is the main goal, which is to reconstruct paleo-ice sheet elevations (or thicknesses) given the margins. It would help to put all this in perspective, and to clarify the main outputs and purpose of the program.

The basic problem of constraining both basal topography and basal stresses crops up in many related papers, mostly using process-based physical ice sheet models (e.g., van Pelt et al., The Cryo., 2013 and references therein). Some perspective discussing the connection to these types of studies could be given.

Response: *To address this, we have added a sentence to the third paragraph of the introduction:*

"The goal of this software is to provide an compromise between the GIA-only ice sheet reconstructions that have limited or no physics applied to their construction, and the full glacial systems models that demand considerable computational resources."

We also added at the end of that paragraph:

"Ultimately, the goal would be to reconstruct in a time-stepped fashion the entire history of an ice sheet complex. In this case, the basal topography is relatively well determined (since there is no existing ice), and the basal shear stress can be established to a certain extent by the surficial geology and geomorphology. The ice topography and basal shear stress are determined through time using external evidence, such as the nature of GIA. An example of this is presented for the western Laurentide Ice Sheet in Gowan et al (2016)."

As for the studies like the one by van Pelt et al (2013), they determine the basal parameters from direct observations of ice dynamics, something that is not really possible in a paleo-ice sheet (except perhaps from trying to match flowline patterns that happen to be preserved on the landscape from near the end of glaciation). The end goal of the Greenland example is basically to show that it is possible to reconstruct a modern ice sheet using our software, and applying GIA deformation is not necessary. That said, we now include reference and comparison to the results of *these dynamic studies in section 3.*

2. Perhaps related to point # 1: if basal topography and/or basal stresses are unknown, and have to be iteratively adjusted so that the simulated surface elevations match those from another data source, then what is learned from the exercise? In other words, if the surface elevations (or thicknesses) need to be known a priori, then why re-simulate them? There may be good reasons: perhaps to produce a higher-resolution surface elevation map, or to produce an iterated map of basal stresses which provides insights into bedrock geology or basal liquid/thermal regimes. But these issues are not clearly addressed and should be discussed clearly.

Response: *The entire goal of this study is to present a way to produce basic paleo-ice sheet models (for GIA modelling), and in most cases the ice sheet thickness is not known a priori, but basal topoography usually is. Although our software could be used to determine the basal characteristics of contemporary ice sheets, we have not and are not suggesting it be done (since higher order dynamic ice sheet modelling is possible in these cases). This was already stated explicitly in sections 1, 3 and 4, and are covered by the additional sentences added (as above).*

3. The basic sequence of using the method of characteristics to step elevation contours incrementally upwards from one contour to the next is clearly explained here,
as it is in the previous papers. However, the procedures used to handle topographic barriers (nunataks), and crossovers in the contours, are very opaque. Fig. 1 does not explain them well. For instance, in Fig. 1 and its caption: - it would help to say that a topographic barrier is the result of E becoming < B (where E is ice surface elevation along a flowline and B is bedrock elevation). - What are the triangles and the orange lines in Fig. 1? - What is "resampling"? This should be described in more detail.

I suggest splitting Fig. 1 into two or even three separate figures: one for the basic stepping algorithm with no complications, one for a nunatak, and one for a crossover. This could greatly improve its potential to be understood.

Along with Fig. 1, the text describing some of the procedural steps in section 2.2 could be expanded and clarified, especially steps 6 and 7. Also, the connections between steps 1 to 14 in the text and the parts of the figure(s) could be specified more thoroughly.

Response: *We have redone figure 1 to specifically illustrate several of the steps explicitly, in particular steps 7 (stepping with no complications), 8 (hitting a nunatuk), 10, 11 (crossovers), 12 and 14 (resampling). The other steps basically relate to reading and writing files and the invocation of subroutines (i.e. steps 1-6).*

4. The authors are probably well aware of the limitations of the basic assumptions of steady state and perfect plasticity, and that there are some ice sheet regions and intervals where they may be seriously in error (e.g., Laurentide during deglaciation, West Antarctic ice streams). These should be mentioned as caveats.

Response: *Indeed we are aware of this (and it was mentioned in the abstract), we have added the following to the end of section 2.1:*

"It is important to note that assuming perfectly plastic, steady state conditions for the ice sheet is not accurate in areas where the ice sheet was highly dynamic. Due to this, the output basal shear

stress is unlikely to reflect the true basal shear stress in those areas".

5. I think the term "ice-sheet models" in the title and text is misleading. To many readers, this implies process-based time-stepping ice sheet models based on conservation equations of mass, momentum and heat. Instead I would suggest "reconstructions", as used for instance in Fisher et al.'s title ("...objective reconstructions").

Response: *We have changed the text to try and be consistent with the terminology throughout the paper in terms of calling these "reconstructions", including the title.*

Specific comments:

a. Text could be added to note how this study goes beyond Reeh (1982) and Fisher et al (1985), besides providing a program. For instance, bed topography can be iteratively adjusted here (pg. 4, line 110), rather than modifying the equations to represent isostatic equilibrium with the ice load.

Response: *We have added the following sentence to the end of section 2.1:*

"In the next subsection, we note some of the improvements to the original methodology, including adjustments to the base topography with realistic GIA, dealing with margins that are in marine environments, automatic determination of ice sheet saddles, and adjusting for the presence of nunataks."

b. It might be helpful to some readers to add hints for the derivation of Eq. 8 and 9, to give them a better chance of deriving them themselves if they are so inclined. The derivations of Eqs. 6 and 7 are straightforward, but Eq. 8 is more challenging. Just saying "As in the development of A6, one starts dq/dx = ..." in Fisher's Appendix would help a lot.

Response: *These equations were derived explicitly (in a general sense) in Differentialgleichungen: Losungsmethoden und Losungen by Kamke (1965). A reference to that has been added.*

Also, a couple of features of the equations could be stated which, although fairly obvious, might be helpful to readers on first perusal: (1) The flowline direction determined by Eq. 6 is the direction of local steepest ascent of the ice surface. (2) If x, y, E, p, q are known at a point on a flowline (and B and Hf are known everywhere), then Eqs. 6 to 8 (or 9) yield the next y, E, q along the flowline for a given increment in x. And then the next p is known from Eq. 5.

Response: *On point (1), the following text has been added immediately after the set of equations:*

"Equation 6 gives the direction of local maximum steepness."

As for point (2), although this is true, in general for a paleo-ice sheet we don't explicitly know what the values of E, p and q are beforehand, and if we did, there would be no point in going through this exercise.

c. In the Greenland example, it would be interesting to iterate on Greenland bed topography starting from a flat surface, instead of using modern bedrock data, and not iterate on basal stresses. That would be more analogous to a Laurentide application.

Response: *In the Laurentide example (Gowan et al, 2016), the basal topography is the only piece of information that we can be certain of. We did not start with a flat topography in that study, nor did the study of Fisher et al (1985). If you started from a flat surface for Greenland, you would need* *additional information (i.e. known values of basal shear stress) in order to uniquely determine the bedrock topography. We do not see the value of this kind of exercise since we are not aware of any studies where the basal shear stress has been determined for the entire ice sheet.*

d. Does Fig. 2c show the initial basal stresses, or the final iterated values? (see pg. 7, line 183).

Response: *This is the final iterated values. This is now indicated in the figure caption.*

e. In Fig. 4, it is unclear what is plotted. The caption says "changes" in basal topography and shear stress, but the plots are absolute fields.

Response: *This is refering to spatial changes in topography and basal shear stress. The word "spatial" has been added to the caption.*

Technical corrections:

pg. 2, line 26: "each flowline ray *is* allowed..."

Response: *The word "is" has been added to the text.*

pg. 2, Eq. 2: Note that this is only true for flat bedrock, B=0.

Response: *We included "neglecting basal topography" to the sentence after equation 2.*

pg. 4, line 105: What does "time interval" mean, given that everything is in steady state?

Response: *Yes, "interval" is an inaccurate term for this. We changed it to say "epoch".*

pg. 5, line 136: Instead of "too steep", "too high" would be more precise. And perhaps add "where E < B".

Response: *We changed that line to say "too high", and added E<B to the subsequent parenthesis.*

pg. 9, line 265: "dependence of ice volume on the Earth model..." (?)

Response: *We rephrased this sentence to say:*

"...a weak dependence on reconstructed ice volume and Earth model used to compute GIA"

Response to comments by Anonymous Referee #3

This manuscript describes a simple ice sheet model which can be used to simulate the first order surface elevation of an ice sheet given its extent. The model would be useful to many studies reconstructing past ice sheets from dating the chronology or retreat or using Glacial Isostatic Adjustment modelling. The manuscript is well, written, and concise and is well suited to this journal. I think the manuscript would need a moderate amount of corrections before publication. In particular, the terminology used in the manuscript needs adjusting and the applications to the Greenland and Eurasian ice sheets need some more detail. One important test is the sensitivity to the model resolution. This is done on the Eurasian ice sheet where there is no observational data on ice thickness. The resolution tests should be instead done for the Greenland ice sheet.

Details comments:

- In the manuscript, the model is referred to as “a program” (title, abstract) “a numerical program” (l29) “modelling software” (l41) and “program” and “software” in the conclusion. “model” is used here to describe an ice sheet “simulation” or “reconstruction” I think that this terminology is confusing. It should be described as a “numerical model”. You could also use the word “simulator” which some statisticians use to differentiate physical models from statistical models. If you have good reasons to stick to the terminology, please clarify the definitions you use.

Response: *Throughout the text, we now refer to 'models' as “reconstructions. We also refer to it as a “program” throughout the entire text. The usage of the word “simulation” is no longer made when refering to our program.*

- Similarly I would replace “modelling procedure” with “algorithm” or equivalent terminology

Response: *We have replaced the section header for Section 2.2 (where this was used) to:*

“Algorithm to reconstruct ice sheets”

- Please indicate how this model compares with other similar models, not only in terms of the equations, but also in the solving procedure.

Response: *The Barents Sea reconstruction was based off the results of the ANU model, the methodology which was described in section 2.1.*

- Replace “sample model” in titles 3 and 4 with “Example” or “application”

Response: *We have changed this to say “Sample reconstruction”*

- Section 3.1 how does the basal shear stress compare with other modelling studies of the Greenland ice sheet ? How much does it affect the results? This is important since the goal here is to comare the model results to observations.

Response: *The goal here was not really to attempt to fully model the basal shear stress of the modern-day Greenland ice sheet, but rather to show that we can reconstruct a modern ice sheet reasonably well with the program using a similar process as with paleo-ice sheet reconstructions. As far as we are aware, there have only been a couple of studies that have attempted to determine the basal shear stress of the Greenland ice sheet, and those have been focused on small parts of the ice sheet where there is streaming ice (i.e. where the basal shear stress can be determined through the inversion of surface velocity values). Reference to these studies has been added to the text:*

Direct inversions for basal shear stress have only been performed for some of the ice streams in (e.g. Sergienko et al. 2014 and Shapero et al. 2016}. In the study by Sergienko et al (2014), the basal shear stress exhibited a banded pattern, alternating between low (<50 kPa) to high (>150 kPa) values over spatial ranges of 5-20 km. Shapero et al (2016) found that the basal shear stress directly under fast flowing ice streams was almost negligible, but at the sides it could exceed 375 kPa. If averaged over a larger area, these values are consistent with the 100-200 kPa values in our reconstruction.

- Section 3.2: Please compare the difference in ice sheet volume modelled vs estimates from observations.

Response: *We have added a couple of sentences on this (the volume values from the reconstructions are in Table 1)*

The volume of the Greenland Ice Sheet, taken directly from the dataset by Morlighem et al (2014) is about 2.96x10^6 *km*^{3}*. From Table 1, the reconstructed volume is within 5% of this value, except in the lowest resolution tests.*

- Please include resolution tests for the Greenland ice sheet.

Response: *Resolution tests have been added to table one for the Greenland ice sheet. A paragraph has been added to elaborate on the:*

The resolution test was also performed with the Greenland simulation (Table 1). In this sample, the 5 km distance interval, 20 m contour interval does not perform quite as well as in the Barent Sea example. This is a result of having a larger area of mountainous terrain. Still, less than 3\% of the elements are greater than 100 m different from the reference reconstruction, using a smaller spacing value may not be worth the extra computation time. If the area of focus is predominantly mountainous, it may be prudent to decrease the distance interval.

- line 274 add “3D topographical” before “models of palaeo-ice sheets”

Response: *We have changed this sentence to say “reconstructions of paleo-ice sheets” to be consistent with previous terminology.*

- line 282: “those parameters” : rephrase to make it clear what parameters you are talking about.

Response: *We rephrased the final sentence to say:*

“A suite of ice sheet reconstructions through a glacial cycle could be used as independent inputs for climate and ice sheet dynamics modelling.”

- In your conclusion, please mention the main results from the sensitivity tests presented here.

Response: *We added the sentence:*

“It is also recommended (if a 5 km basal topography grid is used) to use a flowline spacing interval of 5 km and contour interval of 20 m for optimal calculation speed.”

Response to comments by Andy Wickert

Most recent scientific work in modeling past ice sheets has been aimed at a simple or a complex endmember. The former includes whole-ice-sheet simple flowline modeling and the "ice-cream scoop" approach of the ICE-nG models, in which ice volume is "scooped" from the ocean and placed on the map in a way that is semi-arbitrary but fits the GIA constraints. The latter includes all attempts to use time-evolving ice-dynamics models.

Gowan et al. present work that is sorely needed, that obeys the physics without over-fitting the geological constraints. I am very enthusiastic about this work, and see this as a necessary way forward. More specifically, I think this work embodies the null hypothesis: ice sheets in the past behave as physics dictates. Modeling them in an equilibrium state should be the zeroth- or first-order work that forms the basis for any more complex investigation, and has significant scientific value in and of itself.

A few minor comments follow.

Elevation: E should become z s (z-surface) or something with z in it – E to me is Young’s modulus, erosion, ... while z is a field vertical positions. Same goes with B, would be more intuitive to have z b . Thus the equations with E − B would become z s − z b , and this meaning would become immediately apparent to me.

Response: *The notation used in this paper is identical to that of the original studies by Reeh and Fisher et al. (as noted in the text). We have chosen to keep that notation for consistency.*

Line 66 – no comma needed

Response: *The comma at line 66 has been removed.*

p and q: once again, for readability, I would suggest avoiding variables like q that already mean something to glaciologists. Maybe some consecutive Greek letters or other ones from our standard alphabet would work. I don’t mean to be a stickler about this – it’s just that this makes the difference to me between being able to understand what you’re doing after a skim, and after a close reading, and I think that anything that you do to increase the at-a-glance readability will increase the paper’s impact.

Response: *As above, the notation is the same as what was used in the derivative studies, and we have kept this for consistency. The p and q notation is actually from the PDE solutions from Kamke (1965).*

Your steps in working through the model are good. How about a flowchart to accompany this? I find these very useful, and use a program called yEd, which is pretty quick.

Response: *The program mostly works in a serial order as noted in section 2.2 (aside from the main contour loop), so we don't feel a flowchart would improve readabilty enough to justify inclusion. The revised figure 1 should give a better indication of what is involved in each step.*

Nice examples, especially illustrative of the importance of basal shear stress inputs.

Software repository location: I would suggest that it could be useful to also provide the software on a non-personal website. This should increase its visibility and ensure its future availability. Some researchers like archives that have a doi, and GMD is in support of this. I personally use GitHub for everything, which is nice because it allows others to follow changes to one’s source code and/or check it out and modify it and suggest changes.

Response: *We are including the source code as a supplement to the paper. If future changes are required to the program, we will consider adding a Github or equivalent respoitory.*

Overall, this work is elegant in its simplicity, and I look forward to seeing additional applications.

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ICESHEET 1.0: A program to produce paleo-ice sheet ~~models~~ reconstructions with minimal assumptions

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Abstract. We describe a program that produces paleo-ice sheet ~~models~~ reconstructions using an assumption of steady state, perfectly plastic ice flow behaviour. It incorporates three input parameters: ice margin, basal shear stress and basal topography. Though it is unlikely that paleo-ice sheets were ever in complete steady-state conditions, this method can produce an ice sheet without relying on complicated and unconstrained parameters such as climate and ice dynamics. This makes it advantageous to use in glacial-isostatic adjustment ice sheet ~~models~~modelling, which are often used as input parameters in global climate modelling simulations. We test this program by applying it to the modern Greenland Ice Sheet and Last Glacial Maximum Barents Sea ice sheet and demonstrate the optimal parameters that balance computational time and accuracy.

1 Introduction

~~Modelling~~ Reconstructing past ice sheets is a complex task, due to the large number of parameters that can affect their growth and retreat. For example, ~~?~~ Tarasov et al. (2012) presented a glacial systems model that contained 39 parameters that could be tuned, which included climatology, Earth rheology, ice physics and margin chronology. Many of these parameters are poorly constrained by available observations. In particular, past climate is often parameterized based on ice core data from Greenland and Antarctica, or reconstructions from speleothems that are located far from where the ice sheets existed.

Since past climatic parameters are generally only well characterized in areas outside of where paleo-ice sheets existed, ice sheet ~~models~~ reconstructions that are independently determined using evidence of glacial-isostatic adjustment (GIA) are often used in paleo-climate simulations (*e.g.* Braconnot et al., 2007, 2012). One of the most commonly used GIA based ~~models~~ reconstructions of glaciation is the ICE-xG series (*e.g.* Peltier, 2004; Peltier et al., 2015). They produce configurations of ice sheets that minimize the misfits of geodetic and relative sea level data, with limited regard to the physical realism of the ice sheet itself. Another commonly used ~~model~~ reconstruction is the ANU model (*e.g.* Lambeck et al., 2010), which was developed using an assumed peak ice elevation at the center of ice sheets, and using a parabolic ice profile to the margins. In their formulation, each flowline ray is allowed to have different basal shear stress values, but is less flexible in regards to the direction of the flowline, and spatial variability in basal shear stress along it.

The ~~method~~ program presented in this paper ~~is a numerical program that~~ produces a physically realistic ice sheet ~~configuration~~ reconstructions while taking into account changes in basal shear stress and topography, while being simple enough that it does not depend on numerous parameters with large uncertainties. The ~~model is~~ goal of this program is to provide an compromise between the GIA-only ice sheet reconstructions that have limited or no physics applied to their construction, and the full glacial systems models that demand considerable computational resources. The reconstructions are based on the assumption of perfectly plastic, steady state ice conditions. It allows for the rapid determination of paleo-ice sheet configurations, which is desirable when matching observations of GIA. We present an example application of this ~~software~~ program to the Barents Sea Ice Sheet, a relatively short lived portion of the Eurasian Ice Sheet complex, by trying to match an existing GIA based model. We also apply the model to the contemporary Greenland ice sheet to provide an indication of how well the model is capable of reconstructing a known ice sheet geometry. ~~This software has also been used to produce a model of the full deglacial cycle of~~ Ultimately, the goal would be to reconstruct, in a timestepped fashion, the entire history of an ice sheet complex. In this case, the basal topography is relatively well determined (since there is no existing ice), and the basal shear stress can be established to a certain extent by the surficial geology and geomorphology. The ice topography and basal shear stress are determined through time using external evidence, such as the nature of GIA. An example of this is presented for the western Laurentide Ice Sheet ~~(?)~~ by Gowan et al. (2016) .

2 Methodology

2.1 Theory

The ~~ice sheet modelling software is~~ reconstructions produced by the ICESHEET program are based on the assumption that ice rheology adheres to perfectly plastic, steady-state conditions (*i.e.* ignoring lateral shear stresses, and assuming that the ice surface is not dynamically changing). The two-dimensional form of this theory was derived by Nye (1952), and neglects variability in topography and longitudinal changes in stress. In this equation, the ice surface gradient is directly related to the strength of the ice-bed interface, or basal shear stress. The basal shear stress is related to a number of factors, including basal geology, sediment thickness and strength, hydrology, temperature and bed roughness.

$$\frac{dE}{ds} = \frac{\tau_o}{\rho_i g H} \tag{1}$$

The ice surface elevation is E, s is the distance along ice flowline profile, τ_o is the shear stress at the base of the ice sheet, which balances the driving stress, ρ_i is the density of ice, g is the gravity at the Earth's surface, and H is the ice thickness. If the distance from the ice sheet margin to the centre of the ice sheet is known, then the thickness along the profile between the two points can be calculated using the following formula (Cuffey and Paterson, 2010).

$$H^2 = \frac{2\tau_o}{\rho_i g}[L - x] \tag{2}$$

In this equation, L is the distance between the margin and centre of the ice sheet, and x is the distance from the centre. Though this equation is simple, it can be used to make a rough estimate of the thickness of ice sheets, neglecting basal topography (Cuffey and Paterson, 2010). Eq. 2 was used to create the ANU ice sheet ~~model~~ reconstructions (*i.e.* Lambeck et al., 1998, 2006, 2010). The weakness of using this equation is that the center of the ice sheet has to be assumed a-priori. It also does not take into account changing basal shear stress conditions or changes in topography.

In order to overcome problems with spatial changes in basal topography and shear stress, in addition to the uncertainties in the location of the ice sheet center, Reeh (1982) and Fisher et al. (1985) presented expanded version of Eq. 1 that allows for changes in the direction of the flowline. The equation becomes the following partial differential equation.

$$\left(\frac{dE}{ds}\right)^2 = \left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 \tag{3}$$

The coordinate system is set up so that x points towards the center of the ice sheet, and y is parallel to the margin. Presented in the notation used by Reeh (1982), Eq. 1 is substituted into the left side of Eq. 3 ~~,~~ with the ice thickness represented in terms of ice surface elevation and basal topography elevation B, and substituting in a characteristic thickness, $H_f = \tau_o/\rho_i g$.

$$\left(\frac{H_f}{E - B}\right)^2 = \left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2 \tag{4}$$

The above equation describes the change in ice thickness over an arbitrary surface. This partial differential equation can be solved by the method of characteristics (Kamke, 1965). The x and y

partial derivatives in Equation 4 are substituted by $p = \partial E/\partial x$ and $q = \partial E/\partial y$, then rearranged in terms of p.

$$p = \sqrt{\left(\frac{H_f}{E-B}\right)^2 - q^2} \tag{5}$$

The solution to the partial differential equation then becomes three ordinary differential equations that are solved simultaneously, using the method of characteristics (Reeh, 1982).

$$\frac{dy}{dx} = \frac{q}{p} \tag{6}$$

$$\frac{dE}{dx} = \frac{p^2+q^2}{p} = \frac{H_f^2}{(E-B)^2 p} \tag{7}$$

$$\frac{dq}{dx} = \frac{(p^2+q^2)(\partial B/\partial y - q)}{p(E-B)} = \frac{H_f^2}{p(E-B)^3}\left(\frac{\partial B}{\partial y} - q\right) \tag{8}$$

Equation 6 gives the direction of local maximum steepness, while the other two equations describe how the elevation changes spatially in the x direction. Fisher et al. (1985) expanded Equation 8 to allow for changes in basal shear stress (in terms of the characteristic thickness, H_f).

$$\frac{dq}{dx} = \frac{H_f^2}{p(E-B)^3}\left(\frac{\partial B}{\partial y} - q\right) + \left(\frac{H_f}{p(E-B)^2}\right)\frac{\partial H_f}{\partial y} \tag{9}$$

These equations ~~can be~~ are solved by numerical integration to determine the course and gradient of an ice flowline. In the next subsection, we note some of the improvements to the original methodology, including adjustments to the base topography with realistic GIA, dealing with margins that are in marine environments, automatic determination of ice sheet saddles, and adjusting for the presence of nunataks.

It is important to note that assuming perfectly plastic, steady state conditions for the ice sheet is not accurate in areas where the ice sheet was highly dynamic, or where lateral shear stress was an important factor. Due to this, the output basal shear stress is unlikely to reflect the true basal shear stress in those areas.

2.2 ~~Modelling procedure~~Algorithm to reconstruct ice sheets

In order to solve the Eqs. 6-8, initial values for E, y and q are required. Starting ~~model~~ the calculation at the margin is convenient from the perspective of reducing a-priori assumptions on ice distribution, though it leads to a singularity because the ice thickness is zero ($E = B$). Consequently, the value of

E at the margin must be set to be a nominal value (in the sample problems presented in this study, 1 m). Although the actual thickness of ice near the margin may be as high as tens of metres, the choice of starting value will not have a large effect on the final model. For instance, the distance from the margin required in Eq. 2 to reach 10 m from a starting value of 1 m, and a low basal shear stress value (5 kPa) is 90 m, substantially smaller than the uncertainty in the margin location for paleo-ice sheets (Clark et al., 2012; Gowan, 2013; Hughes et al., 2016). For simplicity, the value of q is defined to be zero at the margin. This can be justified because near the margin the value of term $H_f/(E-B)$ will dominate Eq. 5 in the defined coordinate system.

The ice sheet ~~model~~ reconstruction is calculated in a piece-wise manner (see Fig. 1 for an illustration of the steps involved). The ice flowline calculation is initiated at intervals along the margin, which are user defined. The flowline calculation proceeds until it reaches a particular elevation (a user defined contour interval), at which point the program checks to see if any flowlines cross over, or if a saddle point in the ice sheet has been reached. A sequential list of the modelling steps is given below.

1. All parameters (ice sheet margin, shear stress map, topography map) are converted from geographical coordinates to a Cartesian coordinate system prior to the execution of the program.

2. Estimates of the basal shear stress for the area of interest are read into the program. The shear stress values must be adjusted for each time epoch to produce an appropriate ice sheet configuration.

3. The basal topography data for the area of interest are read in. For the first iteration of ice sheet model development, it uses modern topography or topography adjusted for changes in global mean sea level (in practice, it has limited impact on the final reconstruction, *i.e* < 100 m near the edge of the ice sheet and much less than that in the interior, even with predominantly marine based ice sheets). In subsequent iterations, the topography is adjusted for glacial-isostatic adjustment, to take into account the fact that the ice sheet will deform the Earth, and that the ice sheets will cause changes to sea level. The modified topography is calculated before running the ice sheet program. In the Barents Sea Ice Sheet sample problem, we use the CALSEA program to calculate GIA (Nakada and Lambeck, 1987; Lambeck et al., 2003) . CALSEA computes glacial-isostatic adjustment using a spherically symmetric Earth, with a Maxwell rheology mantle and elastic lithosphere, using the PREM model (Dziewonski and Anderson, 1981) for other Earth model parameters. In includes time evolving shorelines and rotational feedback.

4. The program reads in the margin, and defines locations along the perimeter where the flowline calculation initiates. The minimum distance along the margin between where flowline calculation is initiated is user-defined. The program defines the initial direction of flow to be perpendicular to the margin, away from the centre of the ice sheet.

5. The margin is set to have an initial ice thickness of 1 m. If the margin is located where the topography is below sea level, it is assumed that the margin corresponds to the grounding line of the ice sheet. A conservative estimate of the thickness of ice at this point is set to $H = -B(1 - \rho_{seawater}/\rho_{ice})$, where $\rho_{seawater}$ is the density of sea water and ρ_{ice} is the density of ice, which is the thickness of ice corresponding to the equivalent mass of the water column at that point. There is a check to make sure that the ice surface slope between adjacent points on the boundary is not too steep for the given basal shear stress values. If it is, the ice thickness at the point with the lower elevation is increased. This check is only done where $B < 0$.

6. The calculation of ice elevation contours is a recursive process. If the contour crosses over itself (signifying a saddle on the surface of the ice sheet), the contour polygon is split, and the calculation is continued as separate polygons (see step 12).

7. The program searches for points on the contour that are below the next contour elevation. It then calculates the flowline by numerical integration of Eqs. 6-8, using the Runge-Kutta method (Press, 1992). When it reaches the next contour elevation, the calculation stops.

8. If the flowline calculation cannot reach the next contour elevation, which happens when the topography is too high ($H \to 0$, or $E < B$), the point is flagged and not included in the next contour (Fig. 1).

9. If the flowline direction changes sufficiently so that $q \geq H_f/(E - B)$ (*i.e.* p approaches zero), the local coordinate system is rotated so that p is in the direction of maximum flow.

10. If the calculated flowline goes outside the last calculated contour polygon, it is flagged and the point is not included in the next contour. This happens when the ice surface is near its peak height. This can also happen in areas where there is a sudden change in topography or basal shear stress, which causes a deflection in the flowline direction (Fig. 1).

11. After the flowlines are calculated for each applicable point along the polygon, the program checks to see if any of the calculated flowlines cross over. Offending crossovers are eliminated using a motorcycle algorithm (*e.g.* Vigneron and Yan, 2014) . The eliminated flowlines are flagged and not included in the next contour (Fig. 1).

12. At this point, an initial polygon of the next elevation contour can be constructed. This is checked to ensure that it is a simple polygon (*i.e.* a polygon that does not cross over itself). If it is not, then the program breaks it into several polygons, and determines whether they represent domes (ice gradient is increasing towards the centre of the polygon) or saddles (the ice gradient is decreasing towards the centre of the polygon). Where a saddle is identified, it is determined to have reached its peak elevation and is eliminated from subsequent calculations (Fig. 1).

13. The ice elevation and thickness for all points on a valid polygon (including flagged points) are written to file.

14. The polygon is resampled using the user-defined distance interval. There is also a check using Eq. 2 to estimate the distance to the next contour. If the difference in estimated distance between adjacent points is greater than the user defined distance threshold, additional points are included. This process excludes flagged points, and may incorporate basal topographic highs, where flowline calculation will not be initiated (Fig. 1).

This process is repeated for each time interval of interest. After calculation of the ice ~~model~~reconstruction, the calculated elevation values are averaged into a grid to be used as input for a GIA calculation program. The grid is created using a continuous smoothing algorithm, which is part of Generic Mapping Tools (Smith and Wessel, 1990).

3 Sample ~~model~~ reconstruction - Greenland Ice Sheet

3.1 Setup

The Greenland Ice Sheet serves as a good example of the capabilities of the ICESHEET program. The basal topography under the ice sheet is an observationally constrained, mass continuity based inversion of the contemporary ice thickness (Morlighem et al., 2014). Reeh (1982) ~~modelled~~ reconstructed the Greenland Ice Sheet reasonably well using the methodology explained earlier using a constant basal shear stress of 90 kPa. Since ICESHEET can have spatially variable basal shear stress and account for variable topography, it is possible to refine this. Advances in remote sensing over the last 30 years also allow a more accurate comparison to contemporary topography.

The goal of this example is to determine the misfit between the ICESHEET ~~modelled~~ reconstructed ice surface topography and the contemporary ice sheet using a methodology analogous to the reconstruction of a paleo-ice sheet. The input grounded ice margin and basal topography data come from the IceBridge BedMachine Greenland, Version 2 dataset (Morlighem et al., 2014, 2015). The basal shear stress value domains were designed the same way as a paleo-ice sheet would be constructed. The domains were constructed purely on the basis of basal topography (Fig. 2), ~~predominantly to divide~~ since information on basal geology is limited. They were predominantly divided into areas of rugged topography (*i.e.* mountainous regions), flat lying areas, and fjords. There intentionally was no attempt to divide it on the basis of modern ice flow patterns, given that it may not be possible to deduce them for a paleo-ice sheet. The shear stress values in the domains were adjusted iteratively in order to try to match the observed ice surface topography. In a paleo-ice sheet, it will not be possible to know what the ice surface topography was a-priori. In that case, other sources of data (*i.e.* GIA) must be used as the basis for the reconstruction.

3.2 Results

The resulting ~~model~~ reconstruction is shown in Fig. 2. For comparison purposes, the ice sheet is averaged into a 25 km grid. The ~~modelled~~ reconstructed ice sheet surface topography has an average difference of-37±2 m (within 200 m of the true topography for most of the ice sheet). The largest errors (>400 m) occur in places where there are narrow ice streams near the edge of the ice sheet, which could not be parameterized using the coarse resolution shear stress domains. In general, the shear stress values are highest in the mountainous regions in southeastern Greenland. The basal shear stress is lowest in the center of the ice sheet, likely reflecting the flat-lying basal topography. Direct inversions for basal shear stress have only been performed for some of the ice streams in (*e.g.* Sergienko et al., 2014; Shapero et al., 2016) . In the study by Sergienko et al. (2014) , the basal shear stress exhibited a banded pattern, alternating between low (<50 kPa) to high (>150 kPa) values over spatial ranges of 5-20 km. Shapero et al. (2016) found that the basal shear stress directly under fast flowing ice streams was almost negligible, but at the sides it could exceed 375 kPa. If averaged over a larger area, these values are consistent with the 100-200 kPa values in our reconstruction (Fig. 2)

The resolution test was also performed with the Greenland simulation (Table 1). In this sample, the 5 km distance interval, 20 m contour interval does not perform quite as well as in the Barents Sea example. This is a result of having a larger area of mountainous terrain. Still, less than 3% of the elements are greater than 100 m different from the reference reconstruction. If the area of focus is predominantly mountainous, it may be prudent to decrease the distance interval. The volume of the Greenland Ice Sheet, taken directly from the dataset by Morlighem et al. (2014) is about 2.96×10^6 km^3. From Table 1, the reconstructed volume is within 5% of this value, except in the lowest resolution tests.

4 Sample ~~model~~ reconstruction - Barents Sea Ice Sheet

4.1 Setup

The Barents Sea Ice Sheet was predominantly marine-based, and likely formed by the merging of isolated ice caps over Svalbard, Franz Josef Land, Novaya Zemlya and the Scandinavia Ice Sheet (Ingólfsson and Landvik, 2013). The hypothesis ~~to glaciate~~ explaining the glaciation of the entire Barents Sea is that GIA warped the ~~land upwards within~~ floor of the Barents Sea upwards, favouring the formation of grounded ice. At the Last Glacial Maximum (LGM) (about 20 ka), the ice sheet covered the entire continental shelf region west of Novaya Zemlya (Ingólfsson and Landvik, 2013). The extent was probably limited in the Kara Sea east of Novaya Zemlya, compared to the mid-Weichselian (45-55 ka) glaciation. At the LGM, the ice thickness was likely greatest to the east of Svalbard, on the basis of the pattern of paleo-sea level reconstructions (Lambeck, 1995).

In this sample problem, the ice sheet extent is taken as the “most likely" configuration at 20 ka from the DATED project (Hughes et al., 2016). Since the Barents Sea Ice sheet merged with the Scandinavian Ice Sheet at the LGM, the margin is cut off far enough south so that the northern part of the Scandinavian Ice Sheet is sufficiently represented. The basal topography used in this problem is from IBCAO (Jakobsson et al., 2012). The basal topography of Svalbard takes into account the thickness of modern ice cover. There is no published information on the thickness of ice on Novaya Zemlya, so we use contemporary ice surface topography. The basal shear stress was initially parameterized on the basis of topography and bedrock geology. The values were adjusted in order to produce an ice thickness distribution that is similar to the GIA based ANU model (Lambeck, 1995; Lambeck et al., 2006, 2010). Exact matching of ice thickness in ~~our~~ the sample problem to the ANU model was not attempted, since it is of low resolution, and has a different margin configuration to that of Hughes et al. (2016). Specifically, it is less extensive along the Bear Island Trough. In order to approximate the ice thickness from the ANU model, the basal shear stress was set to be high along the northern part of the ice sheet, and relatively low in the southern Barents Sea. Both the topography and basal shear stress values are sampled at 5 km (Fig. 3).

This purpose of this test is to demonstrate that GIA has an impact on the ice sheet ~~configuration~~reconstruction. This test only includes the Barents Sea Ice Sheet for the calculation of GIA. In a full ~~simulation~~glacial reconstruction (*e.g.* Gowan et al., 2016) , it is necessary to include the effects of far field ice sheets, and realistic ice sheet growth and decay.

4.2 Resolution test

In order to test the optimal parameters for producing ice sheet ~~configurations~~reconstructions, a series of tests with different distance and contour intervals were performed, the results can be found in Table 1. This test involved using modern topography minus the approximate 133 m reduction in global mean sea level at 20 ka (Fig. 3, Lambeck et al., 2014). The shear stress and basal topography values are shown in Fig. 3. Fig. 4 shows how changes in basal shear stress and basal topography affect the modelled ice sheet. The spacing between contours is greater in areas of low basal topography and shear stress, which replicates ice flow from areas of high to low basal topography, and around barriers that resist ice flow. ~~The results of this test are shown in Table. 1.~~

The program execution time largely depends on the chosen sampling interval along the contour polygons (Table. 1). The reference ice sheet configuration used a distance interval of 1 km, and a contour interval of 10 m (Fig. 5). Unsurprisingly, considering the 5 km resolution grid, all tests using distance intervals 5 km or less produced nearly identical ~~configurations~~reconstructions, as they captured the details of the grids. Using a contour interval of 20 m gives almost the same result as as 10 m, with diminishing accuracy when increased above this, without significant reductions in execution time. The optimal parameters for matching the reference configuration and fast execution time are a 5 km spacing and 20 m contour interval (Table. 1). Increasing the distance parameter decreases the execution time, but is unable to match the reference ~~model~~reconstruction, particularly in the mountainous regions of Svalbard and Scandinavia. There is a tendency towards overestimating the ice thickness when the initiation distance is larger than 5 km (Fig. 5). During the initial phases of GIA based ice model development, it may be prudent to decrease the resolution of the grids to quickly determine an estimate of basal shear stress, then increase the resolution when refinement is necessary.

4.3 GIA test

When an ice sheet grows, the basal topography is modified by GIA, which will significantly impact the Barents Sea Ice Sheet example. Therefore, in order to obtain an accurate characterization of the ice sheet surface topography and thickness, it is necessary to re-run the ~~simulation~~ program with the modified basal topography. The Earth model used in this sample problem is spherically symmetric and includes a 90 km thick elastic lithosphere, 4×10^{20} Pa s upper mantle viscosity and 10^{22} Pa s lower mantle, which is in the range of best fitting models for this region (Lambeck et al., 2010). The distance interval used is 5 km and contour interval is 20 m. Since there is a viscous component of the response, the ice sheet is allowed to grow linearly from 30 ka (when glaciation in the Barents Sea is presumed to be similar to present, Mangerud et al., 1998) to 20 ka, then linearly decrease back to present levels at 10 ka. After the first iteration of GIA, the ice sheet contribution to global mean sea level is subtracted to determine the Earth deformation. When combined with the actual global mean sea level at this time (-133 m), it should give a ~~good~~ reasonable estimate of local basal topography.

The results show that one iteration of GIA has a significant effect on ice sheet ~~configuration~~reconstruction, and in this case increases the total volume by about 5.8% (Fig. 6). In addition, since the basal topography becomes more depressed towards the center of the ice sheet relative to the initial ~~simulation, the modelled~~ reconstruction, the reconstructed ice surface topography is lower and has a more gentle gradient. A second iteration of GIA had only a minor effect on the ~~calculated~~ reconstructed ice sheet (0.4% increase in volume from the first iteration).

Additional tests by ~~(Gowan, 2014)~~ Gowan (2014) for the full deglacial Laurentide Ice Sheet showed that there is only a weak dependence on reconstructed ice volume and Earth model used to compute GIA. For three layer (lithosphere, upper mantle, lower mantle) Earth models, the ice volume varied most with changes in lower mantle viscosity at LGM extent, but the difference was less than 0.5% (though smaller ice sheets will have less dependence on the lower mantle). Towards the end of deglaciation, there was more dependence on upper mantle viscosity, but again, the volume difference was less than 0.5%. Though the volume was close to the same, there were slight differences in the distribution of ice, though not by more than 100 m in extreme cases. Therefore, the recommendation when creating an ice sheet model is to include at least one iteration of GIA, but the chosen Earth model is not as important.

5 Conclusions

ICESHEET 1.0 is a program that can quickly create ~~models~~ reconstructions of paleo-ice sheets, with a given margin configuration and estimated basal shear stress. We have provided two proof of concept examples showing ~~configurations~~ reconstructions of the modern Greenland Ice Sheet and the Barents Sea Ice Sheet at the LGM. It is recommended that at least one iteration of GIA is included to best characterize the thickness and ice surface topography. ~~This software~~ It is also recommended (if a 5 km basal topography grid is used) to use a flowline spacing interval of 5 km and contour interval of 20 m for optimal calculation speed. This program has been used to create a full late glacial GIA based ice sheet ~~model~~ reconstruction of the western Laurentide ice sheet ~~(?)~~ (Gowan et al., 2016) . It is ideal for producing ice sheet ~~models~~ reconstructions that have minimal input assumptions, but are glaciologically plausible. A suite of ice sheet ~~models~~ reconstructions through a glacial cycle could be used as independent inputs for climate and ice sheet dynamics modelling~~that are independent of those parameters~~.

5.1 Code availability

The source code, licensed under GPL version 3, and Greenland Ice Sheet example are available in the supplementary material. Software updates will be available on EJG's website (http://www.raisedbeaches.net).

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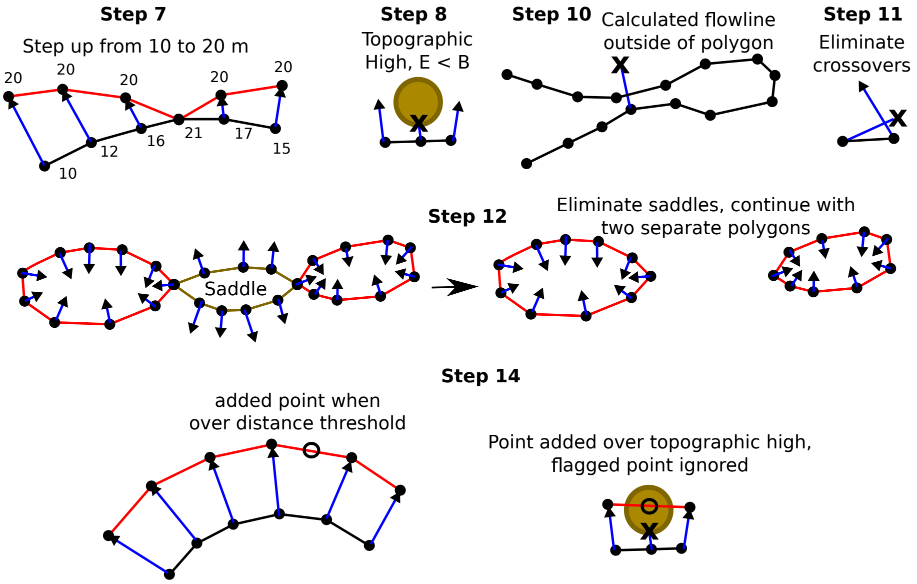

Figure 1. Schematic illustrating the steps in calculating the ice sheet, illustrating steps 7, 8, 10, 11, 12 and 14 in Section 2.2. The ~~blue~~ black lines ~~represent a contour of equal ice surface~~ indicate the initial elevation contour, ~~with increasing elevation towards the top. 1) Calculated~~ blue lines indicate calculated flowlines~~are flagged as they do not reach~~ , red lines indicate the next ~~contour~~ elevation ~~due to a topographic barrier (represented by the brown circle). 2) Flowline calculation to the next~~ contour~~is successful. 3) The points where~~ , black circles indicate flowline ~~calculation is initiated along~~ initiation points, unfilled circles indicate added initiation points for the next elevation contour~~polygon~~ , crosses indicate flagged points that are ~~resampled. Flowline calculation is~~ not ~~initiated for the point~~ included in the ~~brown circle, as its~~ next elevation ~~is too high. 4) Flowline calculation is successful for the points on either side of the topographic high. The point in the topographic high is not eliminated at this step. 5) After another flowline step, the distance along the flowline and the point within the topographic high is sufficient that the resampling puts points at this spot (black squares). 6) The flowline calculation causes the polygon to cross over itself. The polygon is isolated (magenta lines). Since the calculated direction of flow at these points is outside of this isolated polygon, it is eliminated. 7) Final~~ contour~~polygon includes the crossover point~~.

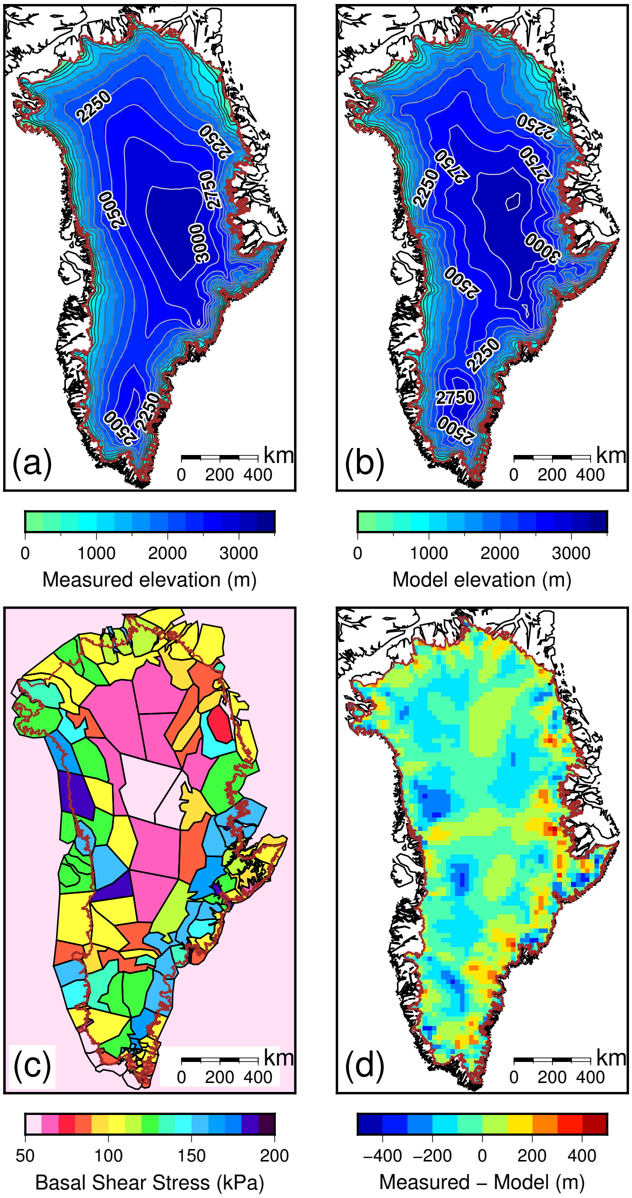

Figure 2. Sample ~~model~~ reconstruction of the contemporary Greenland Ice Sheet. (a) Modern topography. The brown line is the current grounded ice margin, the ~~green~~ black lines are the modern day coastlines. (b) ~~Modelled~~ Reconstructed topography. (c) ~~Basal~~ Final iterated basal shear stress domains and values used ~~to construct~~ for the ~~model~~reconstruction. (d) Difference between the observed topography and ~~modelled~~ reconstructed topography.

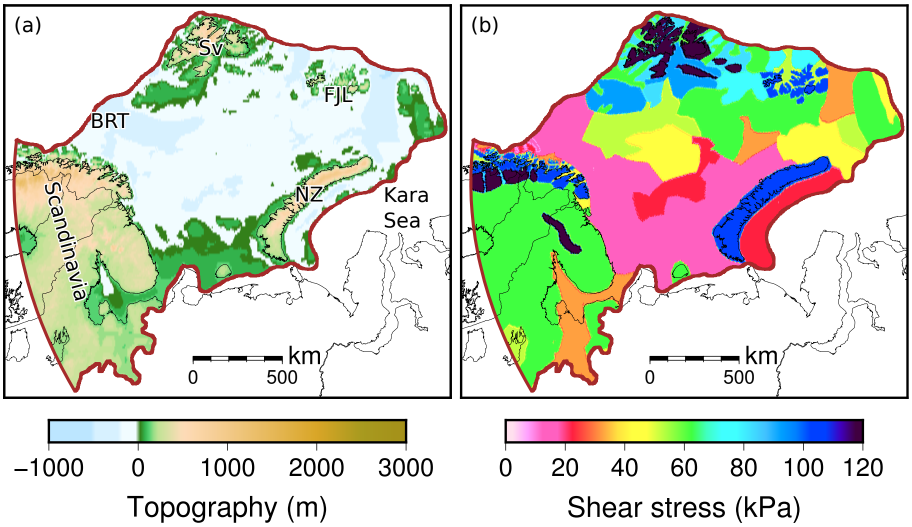

Figure 3. Basal topography used in the resolution test, which is modern topography minus the 133 m drop in global mean sea level at 20 ka. Also shown in brown is the 20 ka ice margin (Hughes et al., 2016) and the location of places described in the text. Sv - Svalbard. FJL - Franz Josef Land. NZ - Novaya Zemlya. BRT - Bear Island Trough. (b) Basal shear stress values used in the example in this paper.

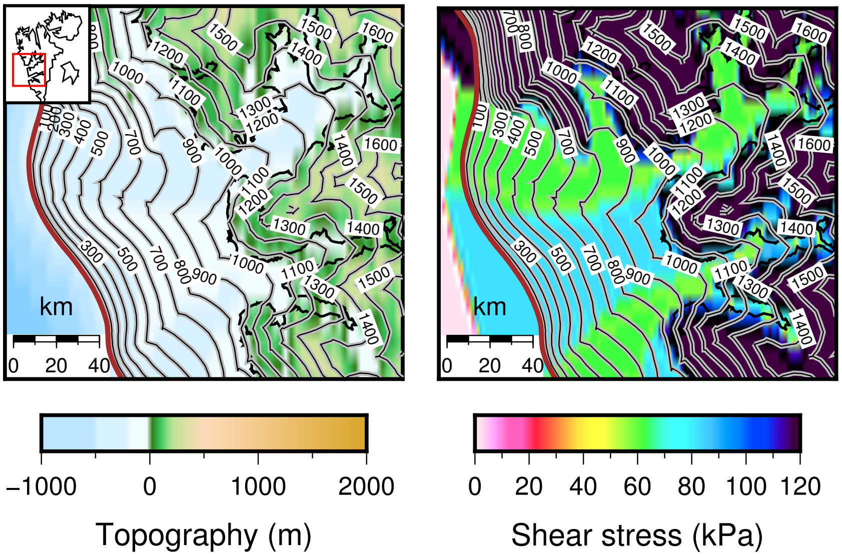

Figure 4. Example from central-western Svalbard of how spatial changes in basal topography and basal shear stress affect the ~~modelled~~ reconstructed ice surface topography of the ice sheet (see text). Contour interval is 100 m in the figure, though this sample was calculated with a 5 km spacing and 20 m contour interval. The dark black lines are the modern day coastlines.

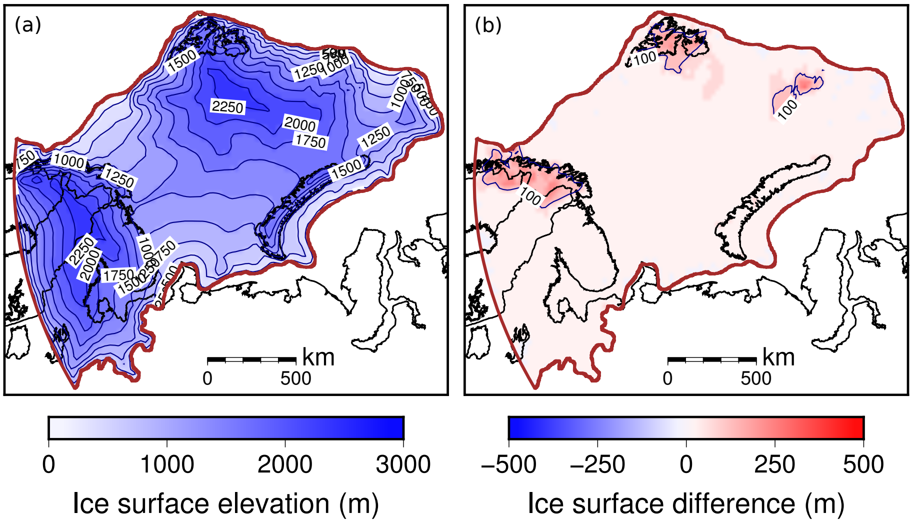

Figure 5. (a) Reference ice sheet ~~model~~ reconstruction with a 1 km spacing and 10 m contour interval using the topography and basal shear stress in Fig. 3. (b) The difference between a model calculated with a 20 km spacing and 20 m contour interval and the reference ~~model~~ reconstruction shown in (a). This demonstrates that the lower resolution tends to overestimate the ice surface elevation in mountainous regions.

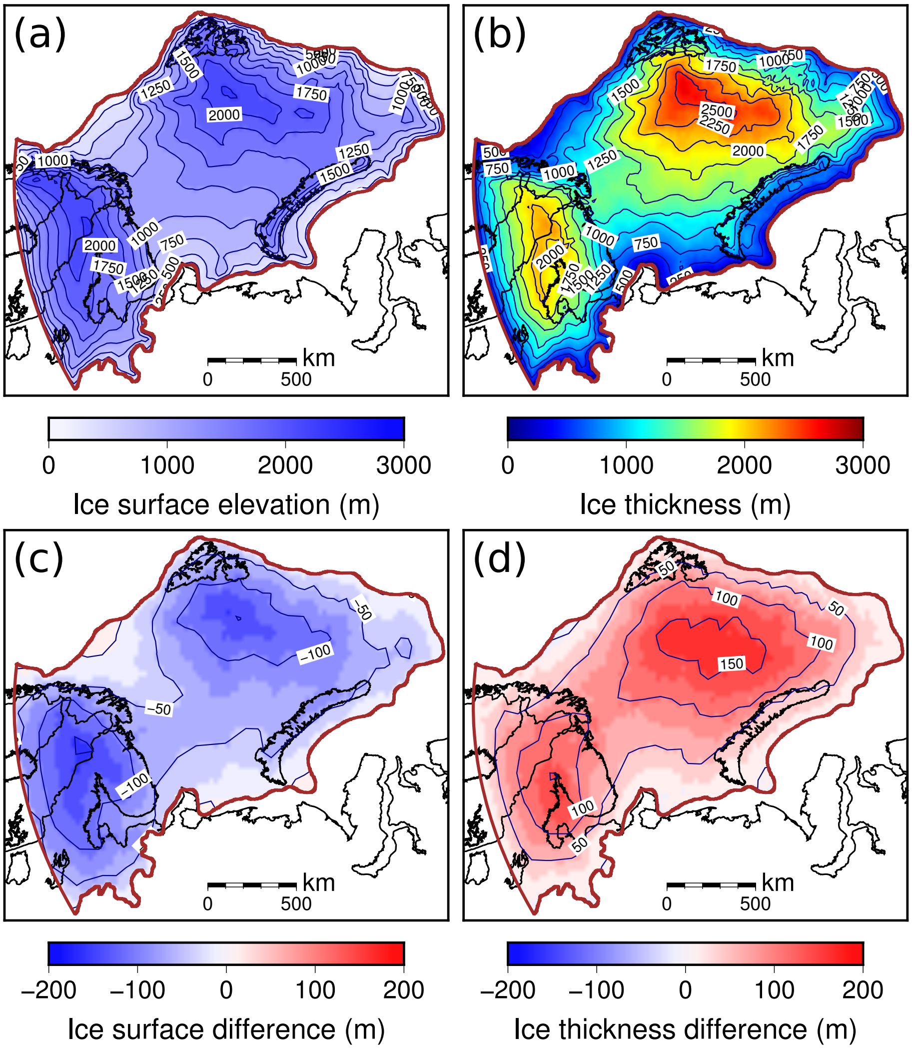

Figure 6. Ice sheet ~~model~~ reconstruction after one iteration of GIA. (a) Ice surface elevation. (b) Ice thickness (c) Difference in elevation between (a) and the initial model without GIA deformed topography. (d) same as (c) but for ice thickness.

Table 1. Results of the resolution test

		Barents Sea			Greenland		
Spacing (km)	Contour interval (m)	CPU Execution time (min)[1]	Ice Volume (10^6 km^3)	Element difference (%)[2]	CPU Execution time (min)[1]	Ice Volume (10^6 km^3)	Element difference (%)[2]
1	10[3]	21.3	3.635	0.00	53.3	2.815	0.00
1	20	14.6	3.640	0.00	29.8	2.816	0.00
1	30	12.2	3.647	0.00	21.6	2.817	0.00
1	40	10.9	3.657	0.12	17.9	2.820	0.01
1	50	10.2	3.675	0.66	15.3	2.824	0.05
3	10	6.3	3.651	0.00	11.0	2.865	0.33
3	20	4.4	3.655	0.02	6.5	2.867	0.46
3	30	3.6	3.661	0.07	5.0	2.870	0.53
3	40	3.3	3.668	0.23	4.1	2.875	0.94
3	50	3.1	3.676	0.55	3.8	2.876	0.98
5	10	3.9	3.667	0.25	6.1	2.915	2.53
5	20[4]	2.6	3.671	0.47	3.7	2.918	2.79
5	30	2.2	3.675	0.59	2.8	2.921	2.89
5	40	2.1	3.683	0.68	2.4	2.921	3.05
5	50	1.9	3.691	1.05	2.2	2.924	3.68
10	10	1.9	3.703	1.69	2.9	2.998	7.53
10	20	1.3	3.704	1.71	1.8	3.005	7.47
10	30	1.1	3.714	1.93	1.4	3.008	7.51
10	40	1.0	3.722	2.16	1.2	3.012	7.94
10	50	0.9	3.726	2.14	1.1	3.010	8.02
15	10	1.3	3.743	2.71	2.0	3.060	10.07
15	20	0.9	3.742	2.85	1.2	3.055	9.44
15	30	0.8	3.748	2.84	0.9	3.061	10.45
15	40	0.7	3.753	2.87	0.8	3.057	10.04
15	50	0.6	3.766	3.20	0.7	3.061	10.18
20	10	1.0	3.769	3.76	1.5	3.099	11.99
20	20	0.7	3.771	3.81	0.9	3.102	11.95
20	30	0.6	3.767	3.83	0.7	3.095	11.84
20	40	0.5	3.779	4.12	0.7	3.108	13.29
20	50	0.5	3.779	4.21	0.6	3.103	12.19
30	10	0.7	3.819	6.33	1.1	3.130	13.55
30	20	0.5	3.817	6.37	0.7	3.141	14.04
30	30	0.4	3.816	6.40	0.6	3.129	14.01
30	40	0.3	3.826	7.16	0.5	3.135	13.92
30	50	0.3	3.840	7.54	0.4	3.141	14.24

[1] Execution time on Terrawulf III (Sambridge et al., 2009), Dual Intel Xeon X5650 at 2.66 GHz running OpenSuse 13.2. Compiled with ifort 15 with -O2 flag.

[2] Percent of 0.5° longitude by 0.25° elements that are $>$ 100 m different from the reference model (out of 23205 total elements for Barents Sea, and 21901 for Greenland)

[3] Reference reconstructions [4] Recommended reconstructions