

Response to Editor review comments

We would like to thank Topical Editor Dr. Jeffrey Neal for his careful work to the manuscript, and we especially appreciate his insightful comments that concerns presentation and language issues. We have carefully read all the comments and made modifications accordingly. Besides presentation modifications we have made, the revised manuscript is also sent out to a professional language service for polishing. For the code availability section, we have reconstructed it for a clearer description. The responds to the comments and the main modifications to the paper are as following:

(Review comments are reported in red.)

[Specific comments]

Abstract

1. “The topographic impacts on modifying Earth systems variability have been well recognised.” This would be simpler to understand as “The impact of topography on Earth systems variability is well recognised”

Response:

We have modified it as the suggestion:

“The impact of topography on Earth systems variability is well recognised.”

2. This sentence “Numerical schemes from Earth systems either use empirical parameterization as sub-grid scale and downscaling skills to express topographic endogenous processes, or rely on insecure point interpolation to induce topographic forcing, which create bias and input uncertainties” should perhaps read “Numerical schemes of Earth systems either use empirical parameterization at sub-grid scale with downscaling to express topographic endogenous processes, or rely on insecure point interpolation to induce topographic forcing, which create bias and input uncertainties”

Response:

We have modified it as the suggestion:

“Numerical schemes of Earth systems either use empirical parameterization at sub-grid scale with downscaling to express topographic endogenous processes, or rely on insecure point interpolation to induce topographic forcing, which create bias and input uncertainties.”

3. “This article proposes a novel high-fidelity multiresolution DEM model with high-quality grids to meet the challenges of scale transformation.” I don’t understand the “with high quality grids”. Do you mean for generalising high quality topographic data?

Response:

We are sorry for the improper presentation, the phrase “with high quality grids” should be written as “with guaranteed grid quality”. The complete context is modified as:

“DEM (digital elevation model) generalisation provides more sophisticated systematic topographic transformation, but existing methods are often difficult to be incorporated because of unwarranted grid quality. Meanwhile, approaches over discrete sets often employ heuristic approximating which are generally not best performed. Based on DEM generalisation, this article proposes a high-fidelity multiresolution DEM model with guaranteed grid quality for Earth systems.”

4. Change “The cCVT model is then evaluated on real LiDAR-derived DEM datasets compared to the classical heuristic method.” to “The cCVT model was evaluated on real LiDAR-derived DEM datasets and compared to the classical heuristic method.”

Response:

We have changed it as the suggestion:

“The cCVT model was evaluated on real LiDAR-derived DEM datasets and compared to the classical heuristic method.”

Introduction

5. “However, as numerical simulation systems evolved to incorporate broader scales and finer processes to produce more fidelity predictions” I would consider using the wording more exact predictions rather than more fidelity predictions.

Response:

The sentence is changed as the suggestion:

“However, as numerical simulation systems evolved to incorporate broader scales and finer processes to produce more exact predictions.”

6. “Earth and environmental simulations usually adopt sub-grid scheme to exert topographical heterogeneity and rely on downscaling the finer observations to surface variables” Do you mean upscaling rather than downscaling. e.g. upscaling fine topographic observations to model surface variables. Your meaning here has implication for the following sentences also.

Response:

We are sorry for the in-proper implications here. We do not mean to use upscaling instead of downscaling, however, we want to argue that topography via sophisticated transformation might be useful to reduce simulation bias and uncertainties. For not mislead/dissuade readers, we have rewritten this sentence and the following sentence as:

“Earth and environmental simulations usually adopt sub-grid schemes to express topography heterogeneous processes (Fiddes and Gruber, 2014; Kumar et al., 2012; Wilby and Wigley, 1997). The sub-grid schemes are designed for the empirical parameterisation rather than accurate topography representation, which often leads to mixed-up uncertainties and bias of endogenous variability (Jiménez and Dudhia, 2013; Nunalee et al., 2015). However, under-resolved representation could be improved by

variable-resolution enhancement, and bias of simulations can be justified by more fidelity topography transformation (Nunalee et al., 2015; Ringler et al., 2011)."

7. "The topography are also commonly treated as a static boundary layer in dynamics simulations" should read "Topography is commonly treated as a static boundary layer in dynamics simulations"

Response:

We have modified it as the suggestion:

"Topography is commonly treated as a static boundary layer in dynamics simulations"

8. "However, mesh from interpolated vertices does not necessarily comply with the terrain relief, and bed elevation errors are frequently reported as one input uncertainty" -> "However, a mesh from interpolated vertices does not necessarily comply with the terrain relief, and elevation errors are frequently reported as one input uncertainty"

Response:

The sentence is modified according to the suggestion:

"However, a mesh from interpolated vertices does not necessarily comply with the terrain relief, and elevation errors are frequently reported as one input uncertainty"

9. "While there are many situations where dynamic conditions are stressed for stronger impacts on modifying predictions" do you mean something like "Although there are many situations where dynamic conditions have stronger impacts on modifying predictions"

Response:

Yes, we mean "Although" instead of "While", we modified it as:

"Although there are many situations where dynamic conditions have stronger impacts on modifying predictions"

Section 1.2

10. "with the effects that the main relief features strongly stressed while non-structural details are massively suppressed" -> "with the effects that the main relief features are strongly stressed while non-structural details are massively suppressed"

Response:

We have modified the sentence as the suggestion:

"with the effects that the main relief features are strongly stressed while non-structural details are massively suppressed"

11. "map cognitive efforts are drawn to produce progressive data reduction" Sorry but it is not clear to me what is meant by map cognitive efforts are drawn

Response:

Here we would like to give some explanation. Some researchers, Ai and Li (2010), for example, had employed generalisation operations under map generalisation guidance to do DEM generalisation. One important map generalisation paradigm can be exemplified as the extraction of morphological lines (contours, ridges, streamlines, etc.), for these structures are most effective to “abstracting” a shape which has coherence to human (or map) cognition. Map generalisation tools have evolved a many of generalisation operators, such as smoothing, simplifying, deforming, etc.

We have modified this sentence as:

“Terrain generalisation emphasises geomorphology or landform depiction, where map generalisation measures (such as abstracting, smoothing) are drawn to produce progressive data reduction.”

12. “Since the static topographic layer are commonly composed directly by DEM datasets to diverse simulation interests, maintaining precise surface approximation for rigorous boundary conditions is more important than ‘sparse’ geomorphology representation.” - > “Since static topographic layers are commonly composed directly from DEM datasets for diverse simulation interests, maintaining precise surface approximation for rigorous boundary conditions is often more important than ‘sparse’ geomorphology representation.”

Response:

This sentence has been modified as the suggestion:

“Since static topographic layers are commonly composed directly from DEM datasets for diverse simulation interests, maintaining precise surface approximation for rigorous boundary conditions is often more important than ‘sparse’ geomorphology representation.”

13. “The first class of approaches is due to the computational bottleneck consideration, that determination which combination of vertices to a TIN surface approximates the original dense DEM surface best needs exponential time” This sentence needs to be rewritten, the meaning is unclear.

Response:

This sentence has been rewritten for a clearer statement as:

“The first class of approaches is due to the computational feasibility consideration, for selecting a TIN surface that best approximates the original DEM surface from exhaustively enumerating (of triangular combinations) requires exponential time (Chen and Li, 2012; Heckbert and Garland, 1997).”

14. Avoid the use of non-scientific terms like greedy, weeding

Response:

We removed the “greedy” term, for the involved sentence has contained enough information for the algorithm.

For the term “weeding” we replaced it with “rejection”. Here we explain it a little more: When selecting feature points (critical points, salient points, or significant points) from discrete sets, there may exist too much candidates due to numerical nature. Slight numeric difference would result in a feature point. DEM generalisation over discrete set often need some kind of “filtering” mechanism to get rid of the redundant feature points caused by this numeric sensitivity, Chen and Li (2012) described this process as “weeding”.

15. “The second class of approach is due to the consideration of TIN surface from feature points do not necessarily warrants best approximation to the original dense DEM, for feature points are commonly selected through some local metric.” I did not understand this sentence.

Response:

We have rewritten this sentence as:

“The second class of approaches recognised the TIN surface constructed from locally computed feature points as not-well approximation to the original DEM surface. Much research thus considered global approximation instead of relying on an elaborate feature point selection scheme.”

16. “ The purpose of this article is to devise a multiresolution DEM model that considers optimized surface approximation and guaranteed grid quality. The quality grid is demanded by the aforementioned easy incorporation with the simulation systems. Multiresolution is an effective paradigm to model scale diversity (Du et al., 2010; Guba et al., 2014; Ringler et al., 2011; Weller et al., 2016), among those promising plans, we are especially fascinated by the centroidal Voronoi tessellations 4 (CVTs) method for the intuitive way to redistribute samples with a designated function (Du et al., 1999; Du et al., 2010; Ringler et al., 2011), and develop CVT to an optimized surface transformation method to realize multiresolution terrain model” - > “The purpose of this article is to devise a multiresolution DEM model that optimises surface approximation and guaranteed grid quality, where the quality grid is demanded by the aforementioned easy incorporation with simulation systems. Multiresolution is an effective paradigm to model scale diversity (Du et al., 2010; Guba et al., 2014; Ringler et al., 2011; Weller et al., 2016). Amongst a number of promising approaches, we are especially fascinated by the centroidal Voronoi tessellations (CVTs) method as an intuitive way to redistribute samples with a designated function (Du et al., 1999; Du et al., 2010; Ringler et al., 2011) to develop an optimized surface transformation method to realize multiresolution terrain models.”

Response:

Thanks to the editor, we have modified it as the suggestion:

“The purpose of this article is to devise a multiresolution DEM model that optimises surface approximation and guarantees grid quality that can be easily incorporated into the simulation systems. Multiresolution is an effective paradigm to model scale diversity (Du et al., 2010; Guba et al., 2014; Ringler et al., 2011; Weller et al., 2016). Amongst a number of promising approaches, we are especially fascinated by the centroidal Voronoi tessellations (CVTs) method as an intuitive way to redistribute samples with a designated function (Du et al., 1999; Du et al., 2010; Ringler et al., 2011) to develop an optimised surface transformation method to realise multiresolution terrain models.”

17. “ The selection of feature points have important morphological structures embedded, computed (such as D8 flow algorithm) or auxiliary input morphological lines have been proved to have significant influence on the quality of transformed DEM surface.” I’m not sure what you mean here, should this read something like “The selection of feature points can have a significant influence on the quality of the transformed DEM surface, [meanwhile] by embedding morphological structures such as those from D8 flow algorithms and auxiliary morphological lines.”

Response:

Here we want to explain a little more for clarification. When building generalized DEM surface, it has been proven that selecting of feature points would be the most effect way. The approximation precision of the reconstructed DEM surface could be improved by embedding auxiliary or computed structural lines, this is because these structural information might be averaged and get lost when building the regular gridded raw DEM dataset.

Based on this recognition, we have modified the statement as:

“The selection of feature points have important morphological structures embedded (in the form of

serialised points sequences), for computed (such as D8 flow algorithm) or auxiliary input morphological lines has been proved to have significant influence on the quality of the transformed DEM surface (Zhou and Chen, 2011). The proposed method keeps the structural lines in the optimisation loop and makes it different to existing CVT implementations where stationary points are commonly not considered.”

18. “For the discrete TIN surface, we compute robust mean curvature as incident property. Upon this discrete spatial domain and property domain, the CVT loops and make discrete set equalized from both domains. Spatial equalization warrants a quasi-uniform quality TIN grid, while the intrinsic property domain equalization warrants distribution of the discrete facets conforms to inherent terrain variation. It thus a total different approach to DEM generalisation and we called it curvature CVT (cCVT)” I’m sorry but I don’t understand this sentence.

Response:

Here we would explain for clarification. Basically, CVT functions in a two-step manner optimization, spatial equalization from Voronoi tessellation and frequency equalization from barycenter adjustment. This processing might be somehow analogous to bi-lateral filtering. Based on the aforementioned observation (of DEM generalisation experience) that feature points would be most effective to construct a generalized DEM surface, and curvature has the capability describing the surface morphology, we compute the discrete curvature and take it as frequency distribution. Using this developed cCVT, we can hope of redistribution of selected samples conforming to terrain variation.

Based on this recognition, we reconstruct the sentence for a clearer statement as:

“For the discrete TIN surface, we compute robust mean curvature on each facet. The attached curvature acts as a frequency distribution. In this discrete spatial domain and frequency domain, the CVT loops and makes sample facets equalised from both domains. Spatial equalisation warrants a quasi-uniform grid quality, while the curvature domain equalisation warrants adaptive distribution conforming to the terrain relief. It is thus a totally different DEM generalisation approach, and we called it curvature CVT (cCVT).”

19. “Existing CVT implementation often undertake clustering approach. However, clustering over discrete sets suffers from numeric issues such as zigzag boundaries, invalid cluster cells (Valette et al., 2008), and limited grid quality.” -> Existing CVT implementations often undertake a clustering approach. However, clustering over discrete sets suffers from numeric issues such as zigzag boundaries, invalid cluster cells (Valette et al., 2008) and limited grid quality.”

Response:

We have changed it as the suggestion:

“Existing CVT implementations often undertake a clustering approach. However, clustering over discrete sets suffers from numeric issues such as zigzag boundaries, invalid cluster cells (Valette et al., 2008) and limited grid quality.”

[Code Availability]

Furthermore, the description of the code availability is too vague. Specifically, what are the third party components of the code, in what software was the classic CVT implementation made. What are the novel aspects of the code and what comes from a third party? The paper must comply with the GMD data policy on the link below and this section needs to be more informative than simply asking the reader to email the authors:

http://www.geoscientific-model-development.net/about/code_and_data_policy.html

Response:

We implemented the cCVT and the comparing classical clustering algorithm using c++, with the open source Visualisation Toolkit (VTK, <https://vtk.org>). VTK provides wonderful flexible manipulation for polygonal datasets on vertex and cell level, but lacks easy control on edge structure. So, for the edge bi-sectioning based Voronoi tessellation approximation (approximated dual operation), we use the implementation of Valette et al. (2008). However, all other functions such as exact clipping and energy referring are implemented using VTK.

We have reconstructed this section as:

“We implement the cCVT and the comparing classical k-means clustering CVT algorithm using c++ on the open-source Visualisation Toolkit (The VTK, <https://www.vtk.org>). The approximated dual operation through edge bi-sectioning is based on the implementation of Valette et al. (2008). If anyone is interested in the technical details of the implementation, please contact the corresponding author for the source codes, demo datasets and necessary guide for compilation.”

Reference for Response to Editor Comments:

[1]. Ai, T. and Li, J.: A DEM generalization by minor valley branch detection and grid filling, *ISPRS Journal of Photogrammetry and Remote Sensing*, 65, 198-207, 2010, doi:10.1016/j.isprsjprs.2009.11.001.

[Other important changes]

Abstraction

20. P1L7 The sentence:

“As numerical simulations evolved to incorporate broader scales and finer processes, accurately embedding the underlying topography to simulate land – atmosphere – ocean interactions, or performing commensurate scale transformation to topography while considering high-fidelity retention have proven to be quite difficult.”

was reconstructed to simpler presentation:

“As numerical simulations evolved to incorporate broader scales and finer processes, accurately assimilating or transforming the topography to produce more exact land-atmosphere-ocean interactions, has proven to be quite challenging.”

21. P1L15, the sentence:

“The generalised DEM model is initially approximated as a triangulated irregular network (TIN) via selected terrain feature points, control points, and possible embedded terrain features.”

was simplified to a clearer presentation:

“The generalised DEM surface is initially approximated as a triangulated irregular network (TIN) via selected feature points and possible input features.”

22. P1L17, then sentence:

“By devising a robust discrete curvature as a density function and exact geometry clipping as an energy reference, the developed curvature CVT (cCVT) converges, the generalised model evolves to a further approximation to the original DEM surface, and the points and their dual cells become equalized with the curvature distribution, exhibiting a quasi-uniform high-quality grid.”

was rewritten as:

“By devising a robust discrete curvature as density function and exact geometry clipping as energy reference, the developed curvature CVT (cCVT) converges, the generalised surface evolves to a further approximation to the original DEM surface, and the points with the dual triangles become spatially equalised with the curvature distribution, exhibiting a quasi-uniform high quality and an adaptive variable-resolution.”

Introduction

23. P3L4 The sentence:

“While DEM datasets are usually used interchangeably with topography or terrain in Earth systems, we will use DEMs and topography indiscriminately hereafter.”

was modified as:

“Since DEM datasets are usually used interchangeably with topography or terrain in Earth systems, we will use DEMs and topography indiscriminately hereafter.”

Section 2.2

24. P5L19 The sentence:

“The most classical energy minimization process of centroidal Voronoi tessellation is expressed by Lloyd’s Relaxation (Lloyd, 1982). The main idea of this algorithm is to first tessellate the surface; density integration over the area is then performed to find a ‘gravity’ barycentre for each tessellated cell, which is used as the new site for the iteration. The pseudo code of this procedure is shown below:”

was changed into:

“The most classical energy minimisation process of centroidal Voronoi tessellation is expressed by Lloyd’s Relaxation (Lloyd, 1982). The main idea of this algorithm is to first tessellate the surface, and then perform density integration over the area to find a ‘gravity’ barycentre for each tessellated cell, which is used as the new site for the iteration. The pseudo code of this procedure is shown below.”

Section 4, Discussion

25. P11L20 The sentence:

“Topography transformation of DEM surfaces has been a deeply studied topic in geoscience, simplification techniques and generalisation principles are widely realised and adopted. Extracting terrain feature points and using these points to construct a generalised surface is one successful approach,”

was modified as:

“Topography transformation of DEM surfaces has been a deeply studied topic in geoscience, simplification techniques and generalisation principles are widely realised and adopted. Extracting terrain feature points and using these points to construct a generalised surface has proven to be one valuable approach;”

26. P11L26 The sentence:

“This observation implies that, if global surface interpolation precision are more importantly demanded, robust approach like cCVT that incorporates surface approximation and terrain feature retention should be considered.”

was modified as:

“This observation implies that if global surface interpolation precision is of more importantly demanded, a robust approach that has overall considerations on surface approximation and terrain feature retention should be adopted.”

27. P13L2 The sentence:

“However, the impact of the feature point being-inserted on the feature points have-inserted is not considered because computational burden. As a result, feature points may cluster around relief with sharp variations,”

was modified as:

“However, the impact of the insertion of a new feature point on the inserted feature points is not considered due to computational burden. That is, modifications are only taken place on the triangle where the point with largest vertical error locates on. As a result, feature points may cluster around reliefs with sharp

variations,”

28. P12L10 The sentence:

“cCVT starts by constructing a terrain-adaptive multiresolution grid. The iterations use a robust mean curvature as a density function which is based on the curvature’s capability to characterise shapes and conduct shape evolution. CVT is essentially a two-stepped optimization loop (c.f. Algorithm1 in Section 2.2). The process of spatially equalising feature points has been seldom considered by classical approaches, which may explain why cCVT generally prevails over HFPRs (c.f. Table 1).”

was reconstructed for a consistent presentation as:

“cCVT starts by constructing a terrain-adaptive variable-resolution grid. The cCVT iteration uses a robust mean curvature as density function which is based on the curvature’s capability to characterise shapes and conduct shape evolution. Under the curvature guidance, the two-step optimisation (c.f. Algorithm1 in Section 2.2) loops both to spatial equalisation and frequency equalisation. The process of spatial equalising of feature points has been seldom considered by classical approaches, which may explain why cCVT generally prevails over HFPRs (c.f. Table 1). Notably, the triangles from the spatial equalisation exhibited a maximum aspect ratio that was less than 5.0, which implies that the constructed terrain grid satisfied the numerical stability requirement from classical finite element or finite volume computations.”

29. P12L17 The sentence:

“On the other hand, CVT is an approach within variational framework. The result from iteration relies on the boundary conditions and initial conditions. Hence, this article employed a feature point scheme (with additional input points considered) as a relatively stationary initial condition to maintain algorithm stability. The requirement of embedding feature points of interest, along with consideration to avoid the problematic k-means like clustering, prompt us to develop a non-clustering approach with an exact energy referring method. Experiments on ten million DEM points demonstrated that the exact clipping approach performed comparably to the elegant clustering approach. Notably, the triangles from the cCVT-generated TIN surface exhibited a maximum aspect ratio that was less than 5.0 (c.f. Table 1), which implies that the build-up terrain grid satisfied the numerical stability requirement from classical finite element or finite volume computations.”

was reconstructed as:

“On the other hand, CVT is an approach within variational framework. The result of the iteration depends on the boundary conditions and initial conditions. Hence, this article employed a feature point scheme (with additional input points considered) as a relatively stationary initial condition to maintain algorithm stability. The requirement of embedding feature points of interest, along with consideration of avoiding the problematic k-means like clustering, prompted us to develop a non-clustering approach with an exact energy referring method. Experiments on ten million DEM points demonstrated that the exact clipping approach performed comparably to the elegant clustering approach.”

Section 5, Conclusion

30. P12L24 The paragraph:

“In this article, a high-fidelity multiresolution DEM model was proposed for scale transformation. Multiresolution models are an essential tool to incorporate more scales. A high-fidelity generalised DEM model can build a concrete

topographic layer from which fine endogenous or exogenous processes can be assessed under proper scale conditions. These two aspects were achieved by our devised curvature-based CVT. cCVT optimization increases the precision of surface approximations compared to existing heuristic DEM generalisations, while the equalisation of feature points from both the spatial domain and curvature magnitude domain (i.e., frequency domain) facilitates multiresolution and high-fidelity approximations.”

was rearranged into a more concise presentation as:

“In this article, a high-fidelity multiresolution DEM model was proposed. The variable-resolution with high-fidelity was achieved by the developed curvature-based CVT. cCVT optimisation increases the precision of surface approximation compared to existing heuristic DEM generalisations, while the equalisation of feature points from the spatial domain guarantees a high-quality grid.”

31. P12L28 The paragraph:

“Evaluation of cCVT multiresolution DEM model on Earth and environmental systems in wide-ranged domains and scales is needed in further study. Considering the Earth system situation of global modeling tyranny (Ringler et al., 2011), this may imply a consideration of curvature of the Earth itself into the cCVT model.”

was reconstructed as:

“Multiresolution models are essential tools to incorporate more scales, while a high-fidelity generalised DEM model can be used to construct a concrete topographic layer from which fine endogenous or exogenous processes can be assessed under proper scale conditions. Evaluation of the cCVT multiresolution DEM model on Earth and environmental systems in wide-ranged domains and scales is a topic for future studies. Furthermore, considering the tyranny of global modelling of the Earth systems (Ringler et al., 2011), this may imply a consideration of curvature of the Earth itself into the cCVT model.”

[List of Changes]

1. P1L7. "The impact of topography on Earth systems variability is well recognised."

According to Comment #1

2. P1L7. "As numerical simulations evolved to incorporate broader scales and finer processes, accurately assimilating or transforming the topography to produce more exact land-atmosphere-ocean interactions, has proven to be quite challenging."

Correspond to Change #20

3. P1L9. "Numerical schemes of Earth systems often use empirical parameterisation at sub-grid scale with downscaling to express topographic endogenous processes, or rely on insecure point interpolation to induce topographic forcing, which creates bias and input uncertainties"

Response to Comment #2

4. P1L11. "DEM (digital elevation model) generalisation provides more sophisticated systematic topographic transformation, but existing methods are often difficult to be incorporated because of unwarranted grid quality. Meanwhile, approaches over discrete sets often employ heuristic approximating which are generally not best performed."

Response to Comment #3

5. P1L15. "The generalised DEM surface is initially approximated as a triangulated irregular network (TIN) via selected feature points and possible input features. The TIN surface is then optimised through an energy-minimised centroidal Voronoi tessellation (CVT)."

Correspond to Change #21

6. P1L17. "By devising a robust discrete curvature as density function and exact geometry clipping as energy reference, the developed curvature CVT (cCVT) converges, the generalised surface evolves to a further approximation to the original DEM surface, and the points with the dual triangles become spatially equalised with the curvature distribution, exhibiting a quasi-uniform high quality and an adaptive variable-resolution."

Correspond to Change #22

7. P1L21. "The cCVT model was then evaluated on real LiDAR-derived DEM datasets and compared to the classical heuristic model."

Response to Comment #4

8. P1L28. "exist", Modified from "exists"

9. P1L28. "to", Modified from "into"

10. P2L3. "However, as numerical simulation systems evolved to incorporate broader scales and finer processes to produce more exact predictions (Ringler et al., 2011; Weller et al., 2016; Wilson, 2012; Zarzycki et al., 2014)"

Response to Comment #5

11. P2L4. "how to accurately assimilate or transform the fine resolution topography has proven to be a quite difficult

task (Bilskie et al., 2015; Chen et al., 2015; Tarolli, 2014).”
Correspond to Change #23

12. P2L7. “Earth and environmental simulations usually adopt sub-grid schemes to express topography heterogeneous processes (Fiddes and Gruber, 2014; Kumar et al., 2012; Wilby and Wigley, 1997). The sub-grid schemes are designed for the empirical parameterisation rather than accurate topography representation, which often leads to mixed-up uncertainties and bias of endogenous variability (Jiménez and Dudhia, 2013; Nunalee et al., 2015). However, under-resolved representation could be improved by variable-resolution enhancement, and bias of simulations can be justified by more fidelity topography transformation (Nunalee et al., 2015; Ringler et al., 2011).”
Response to Comment #6

13. P2L12. “Topography is also commonly treated as a static boundary layer in dynamics simulations”
Response to Comment #7

14. P2L14. “But a mesh constructed from interpolated vertices does not necessarily comply with the terrain relief, and elevation errors are frequently reported as an input uncertainty”
Response to Comment #8

15. P2L17. “Although there are many situations where dynamic conditions are stressed for stronger impacts on modifying predictions”
Response to Comment #9

16. P2L18. “due to”, Modified from “for”

17. P2L25. “and terrain feature points (critical points or salient points from some significance metric) are selected for constructing the network”.
Modified for clear statement from “and critical points or salient points with terrain significance are selected for the network feature points”.

18. P2L26. “shcemes”, Modified from “shceme”

19. P2L31. “Terrain generalisation emphasises geomorphology or landform depiction, where map generalisation measures (such as abstracting, smoothing) are drawn to produce progressive data reduction”
Response to Comment #11

20. P2L33. “with the effect that the main relief features are strongly stressed while non-structural details are massively suppressed”
Response to Comment #10

21. P3L1. “Since the static topographic layers are commonly composed directly from DEM datasets for diverse simulation interests, maintaining precise surface approximation for rigorous boundary conditions is often more important than ‘sparse’ geomorphology representation.”
Response to Comment #12

22. P3L6. “two broad classes”, Modified from “two broad groups”

23. P3L6. “heuristic refinements and smooth-fitting”, Modified from “heuristic refinements and smooth-fittings”

24. P3L7. “The first class of approaches is due to the computational feasibility consideration, for selecting a TIN surface that best approximates the original DEM surface from exhaustively enumerating (of triangular combinations) requires exponential time (Chen and Li, 2012; Heckbert and Garland, 1997).”

Response to Comment #13

25. P3L10. “in which insertion (or deletion) refinements on feature points are adopted”, Modified from “which insertion (or deletion) refinements on feature points”

26. P3L12. “prohibitive”, Modified from “inhibitive”

27. P3L17. “The second class of approaches recognised the TIN surface constructed from locally computed feature points as not-well approximation to the original DEM surface. Much research thus considered global approximation instead of relying on an elaborate feature point selection scheme,”

Response to Comment #15

28. P3L23. “rejection”, Response to Comment #14

29. P3L25. “well performing with respect to loyalty to the original terrain surface nor easily incorporated by the numerical schemes” , Modified from “ best performed with loyalty to the original terrain surface, nor easily incorporated by the numerical schemes”

30. P3L28. “The purpose of this article is to devise a multiresolution DEM model that optimises surface approximation and guarantees grid quality that can be easily incorporated into the simulation systems. Multiresolution is an effective paradigm to model scale diversity (Du et al., 2010; Guba et al., 2014; Ringler et al., 2011; Weller et al., 2016). Amongst a number of promising approaches, we are especially fascinated by the centroidal Voronoi tessellations (CVTs) method as an intuitive way to redistribute samples with a designated function (Du et al., 1999; Du et al., 2010; Ringler et al., 2011) to develop an optimised surface transformation method to realise multiresolution terrain models.”

Response to Comment #16

31. P4L2. “this general optimisation method”, Modified from “this general purposed optimization method”

32. P4L3. “we made the following contributions:”, Modified from “we made a few contributions as follow”

33. P4L4. “The generalised DEM surface is initially approximated by a triangular grid constructed from selected feature points. The selection of feature points have important morphological structures embedded (in the form of serialised points sequences), for computed (such as D8 flow algorithm) or auxiliary input morphological lines has been proved to have significant influence on the quality of the transformed DEM surface (Zhou and Chen, 2011). ”

Response to Comment #17

34. P4L10. “For the discrete TIN surface, we compute robust mean curvature on each facet. The attached curvature acts as a frequency distribution. In this discrete spatial domain and frequency domain, the CVT loops and makes sample facets equalised from both domains. Spatial equalisation warrants a quasi-uniform grid quality, while the curvature

domain equalisation warrants adaptive distribution conforming to the terrain relief. It is thus a totally different DEM generalisation approach, and we called it curvature CVT (cCVT)”

Response to Comment #18

35. P4L15. “Existing CVT implementations often undertake a clustering approach. However, clustering over discrete sets suffers from numerical issues such as zigzag boundaries, invalid cluster cells (Valette et al., 2008) and limited grid quality.”

Response to Comment #19

36. P4L19. “has a global optimisation mechanism”, Modified from “but has global optimization mechanism”

37. P4L20. “which can be used to build”, Added “can be used to”

38. P4L28. “Section 5 briefly presents a short conclusion and outlook.”, Edited from “Section 5 presents a short conclusion and outlooks briefly.”

39. P5L10. “by summing up the potential energies of all cells V_i .”, Modified from “by summing up every cell V_i ’s potential energy”

40. P5L12. “The energy minimiser is”, Added “The”

41. P5L14. “since”, Modified from “for”

42. P5L19. “The most classical energy minimisation process of centroidal Voronoi tessellation is expressed by Lloyd’s Relaxation (Lloyd, 1982). The main idea of this algorithm is to first tessellate the surface, and then perform density integration over the area to find a ‘gravity’ barycentre for each tessellated cell, which is used as the new site for the iteration. The pseudo code of this procedure is shown below.”

Correspond to Change #24

43. P6L8. “projecting”, Modified from “project”

44. P6L8. “using instead the constrained projection point for the new site”, Modified from “the constrained projection point is used instead”

45. P6L10. “for more accurate site calculations”, Modified from “for further accurate site calculation”

46. P6L11. “Fast convergence to DEM equilibria”, Modified from “Fast converge to DEM equilibrium”

47. P6L22. “zigzag cluster cell boundaries”, Edited from “zigzag cluster boundaries”

48. P6L24. “Furthermore”, Added for smoother statement

49. P6L29. “derivatives”, Modified from “derivative”

50. P6L29. “will describe a more fundamental terrain geometry shape”, Edited from “will be used to describe a more

detailed set of terrain surface parameters”

51. P7L1. “considering”, Modified from “consider”

52. P7L5. “Curvature’s ability to flexibly describe terrain morphology”, Edited from “Curvature’s flexible ability on depicting terrain morphology”

53. P7L6. “noted”, Modified from “pointed out”

54. P7L6. “compared to the results of the”, Modified from “compared to the result of”

55. P7L7. “is not limited to”, Modified from “has not limit to”

56. P7L11. “an edge bi-sectioning based dual operation”, Added for clearer statement

57. P7L19. “that of the elegant clustering approach (also has kd-tree fast location embedded)”, Added statement for clarification

58. P8L18. “For effectiveness”, Modified from “As for effectiveness”

59. P8L19. “set the transformation scale to”, Modified from “set the transformation scale at”

60. P8L19. “there are approximately”, Modified from “there are about approximately”

61. P8L21. “boundary points”, Modified from “boundary”

62. P8L23. “from which”, Modified from “by which”

63. P9L3. “which linearly interpolated the high-resolution surface”, Modified from “which linearly interpolated the high-resolution surface literally.”

64. P9L6. “will result in”, Modified from “warrants”

65. P9L14. “the”, Added

66. P9L14. “dataset”, Added

67. P9L18. “labelled”, Modified from “labelled as”

68. P9L18. “so”, Added

69. P9L20. “between”, Added

70. P9L20. “southeastern”, Modified from “south eastern”

71. P9L22. “northwestern”, Modified from “north eastern”
72. P9L23. “due to”, Modified from “for”
73. P9L25. “variations”, Modified from “variation”
74. P9L27. “of”, Added
75. P10L2. “As previously mentioned”, Modified from “As aforementioned”
76. P10L2. “and”, Modified from “with”
77. P10L2. “have been”, Added
78. P10L8. “according to”, Modified from “by”
79. P10L9. “highest”, Modified from “largest”
80. P10L9. “modified”, Modified from “modified by”
81. P10L10. “repeats”, Modified from “loops”
82. P10L21. “Error estimates for”, Modified from “Error estimates of”
83. P10L26. “the”, Added
84. P10L27. “obtained”, Modified from “from”
85. P10L27. “In all cases”, Modified from “While all”
86. P10L30. “from the aspect of terrain structure retention.”, Modified from “from terrain structure retention aspects”
87. P10L30. “resulting”, Modified from “result”
88. P10L31. “datasets”, Modified from “datesets”
89. P10L31. “performed well based on visual examination”, Modified from “performed well from visual examination”
90. P11L8. “detail losses by”, Modified from “detail loss from”
91. P11L10. “and”, Added
92. P11L11. “For”, Modified from “Upon”
93. P11L13. “overlaid”, Modified from “overlapped”

94. P11L14. “cCVT-generalised surface accurately conform with”, Modified from “cCVT-generalised surface are more accurately conform with”

95. P11L17. “near”, Modified from “to”

96. P11L20. “Topography transformation of DEM surfaces has been a deeply studied topic in geoscience, simplification techniques and generalisation principles are widely realised and adopted. Extracting terrain feature points and using these points to construct a generalised surface has proven to be one valuable approach;”
Correspond to Change #25

97. P11L24. “Take the mountain equation in Figure 1 as an example”, Modified from “Taking the mountain equation in Figure 1 for example”

98. P11L25. “the”, Added

99. P11L25. “Assume that the”, Modified from “Presume”

100. P11L27. “This observation implies that if global surface interpolation precision is of more importantly demanded, a robust approach that has overall considerations on surface approximation and terrain feature retention should be adopted”
Correspond to Change #26

101. P12L1. “However, the impact of the insertion of a new feature point on the inserted feature points is not considered due to computational burden. That is, modifications are only taken place on the triangle where the point with largest vertical error locates on. As a result, feature points may cluster around reliefs with sharp variations”
Correspond to Change #27

102. P12L10. “cCVT starts by constructing a terrain-adaptive variable-resolution grid. The cCVT iteration uses a robust mean curvature as density function which is based on the curvature’s capability to characterise shapes and conduct shape evolution. Under the curvature guidance, the two-step optimisation (c.f. Algorithm1 in Section 2.2) loops both to spatial equalisation and frequency equalisation. The process of spatial equalising of feature points has been seldom considered by classical approaches, which may explain why cCVT generally prevails over HFPRs (c.f. Table 1). Notably, the triangles from the spatial equalisation exhibited a maximum aspect ratio that was less than 5.0, which implies that the constructed terrain grid satisfied the numerical stability requirement from classical finite element or finite volume computations.”
Correspond to Change #28

103. P12L17. “On the other hand, CVT is an approach within variational framework. The result of the iteration depends on the boundary conditions and initial conditions. Hence, this article employed a feature point scheme (with additional input points considered) as a relatively stationary initial condition to maintain algorithm stability. The requirement of embedding feature points of interest, along with consideration of avoiding the problematic k-means like clustering, prompted us to develop a non-clustering approach with an exact energy referring method. Experiments on ten million DEM points demonstrated that the exact clipping approach performed comparably to the elegant clustering approach”.
Correspond to Change #29

104. P12L24. "In this article, a high-fidelity multiresolution DEM model was proposed. The variable-resolution with high-fidelity was achieved by the developed curvature-based CVT. cCVT optimisation increases the precision of surface approximation compared to existing heuristic DEM generalisations, while the equalisation of feature points from the spatial domain guarantees a high-quality grid".

Correspond to Change #30

105. P12L28. "Multiresolution models are essential tools to incorporate more scales, while a high-fidelity generalised DEM model can be used to construct a concrete topographic layer from which fine endogenous or exogenous processes can be assessed under proper scale conditions. Evaluation of the cCVT multiresolution DEM model on Earth and environmental systems in wide-ranged domains and scales is a topic for future studies. Furthermore, considering the tyranny of global modelling of the Earth systems (Ringler et al., 2011), this may imply a consideration of curvature of the Earth itself into the cCVT model".

Correspond to Change #31

106. P13L2. "We implement the cCVT and the comparing classical k-means clustering CVT algorithm using c++ on the open-source Visualisation Toolkit (The VTK, <https://www.vtk.org>). The approximated dual operation through edge bisectioning is based on the implementation of Valette et al. (2008). If anyone is interested in the technical details of the implementation, please contact the corresponding author for the source codes, demo datasets and necessary guide for compilation".

Response to Special Comment about Code Availability

107. P18, Caption for Figure 1. "depiction", Modified from "depicting"

108. P19, Caption for Figure 3. "The triangles incident towards the first vertex", Modified from "The incident triangles toward"

109. P20, Caption for Figure 7. "and the projection on oriPd of the newly computed site", Modified from "the newly computed site"

110. P22, Caption for Figure 8. "Comparison of converged results", Modified from "converged results comparison"

111. P26, Caption for Figure 14, "close-up view", Modified from "close view"

112. P27, Caption for Figure 15, "was", Modified from "were"

A high-fidelity multiresolution DEM model for Earth systems

Duan Xinqiao¹, Li Lin^{1,2,3}, Zhu Haihong^{1,3}, Ying Shen^{1,3}

¹Geographical Information Science Faculty, SRES School, Wuhan University, Wuhan, 430079, China
²Geospatial information science Collaborative Innovation Centre of Wuhan University, Wuhan, 430079, China
³The Key Laboratory for Geographical Information System, Ministry of Education, Wuhan, 430079, China

Correspondence to: Li Lin (lilin@whu.edu.cn)

Abstract. The impact of topography on Earth systems variability is well recognised. As numerical simulations evolved to incorporate broader scales and finer processes, accurately assimilating or transforming the topography to produce more exact land-atmosphere-ocean interactions, has proven to be quite challenging. Numerical schemes of Earth systems often use empirical parameterisation at sub-grid scale with downscaling to express topographic endogenous processes, or rely on insecure point interpolation to induce topographic forcing, which creates bias and input uncertainties. DEM (digital elevation model) generalisation provides more sophisticated systematic topographic transformation, but existing methods are often difficult to be incorporated because of unwarranted grid quality. Meanwhile, approaches over discrete sets often employ heuristic approximating which are generally not best performed. Based on DEM generalisation, this article proposes a high-fidelity multiresolution DEM model with guaranteed grid quality for Earth systems. The generalised DEM surface is initially approximated as a triangulated irregular network (TIN) via selected feature points and possible input features. The TIN surface is then optimised through an energy-minimised centroidal Voronoi tessellation (CVT). By devising a robust discrete curvature as density function and exact geometry clipping as energy reference, the developed curvature CVT (cCVT) converges, the generalised surface evolves to a further approximation to the original DEM surface, and the points with the dual triangles become spatially equalised with the curvature distribution, exhibiting a quasi-uniform high quality and an adaptive variable-resolution. The cCVT model was then evaluated on real LiDAR-derived DEM datasets and compared to the classical heuristic model. The experimental results show that the cCVT multiresolution model outperforms classical heuristic DEM generalisations in terms of both surface approximation precision and surface morphology retention.

1 Introduction

1.1 Topography in Earth systems

Topography is one of the main factors controlling processes operating at or near the surface layer of the planet (Florinsky and Pankratov, 2015; Wilson and Gallant, 2000). With the success of Earth and environment systems with these scale diversified processes, there exist persistent demands for extending their utility to new and expanding scopes (Ringler et al., 2008; Tarolli, 2014; Wilson, 2012), as exemplified by lapse-rate controlled functional plant distributions (Ke et al., 2012), orographic forcing imposed on oceanic and atmospheric dynamics (Nunalee et al., 2015; Brioude et al., 2012; Hughes et al., 2015), topographic dominated flood inundations (Bilskie et al., 2015; Hunter et al., 2007), and many other geomorphological (Wilson, 2012), soil (Florinsky and Pankratov, 2015), and ecological (Leempoel et al., 2015) examples from Earth systems. However, as numerical simulation systems evolved to incorporate broader scales

批注 [o1]: Response to Comment #1

批注 [o2]: Correspond to Change #20

批注 [o3]: Response to Comment #2

批注 [o4]: Response to Comment #3

批注 [o5]: Response to Comment #3

批注 [o6]: Correspond to Change #21

批注 [o7]: Correspond to Change #22

批注 [o8]: Response to Comment #4

批注 [s9]: Modified from “exists”

批注 [s10]: Modified from “into”

and finer processes to produce more exact predictions (Ringler et al., 2011; Weller et al., 2016; Wilson, 2012; Zarzycki et al., 2014), how to accurately assimilate or transform the fine resolution topography has proven to be a quite difficult task (Bilskie et al., 2015; Chen et al., 2015; Tarolli, 2014).

Earth and environmental simulations usually adopt sub-grid schemes to express topography heterogeneous processes (Fiddes and Gruber, 2014; Kumar et al., 2012; Wilby and Wigley, 1997). The sub-grid schemes are designed for the empirical parameterisation rather than accurate topography representation, which often leads to mixed-up uncertainties and bias of endogenous variability (Jiménez and Dudhia, 2013; Nunalee et al., 2015). However, under-resolved representation could be improved by variable-resolution enhancement, and bias of simulations can be justified by more fidelity topography transformation (Nunalee et al., 2015; Ringler et al., 2011). Topography is also commonly treated as a static boundary layer in dynamics simulations, where different interpolation strategies and mesh refinement techniques are used to convey terrain variation (Guba et al., 2014; Kesserwani and Liang, 2012; Nikolos and Delis, 2009; Weller et al., 2016). But a mesh constructed from interpolated vertices does not necessarily comply with the terrain relief, and elevation errors are frequently reported as an input uncertainty (Bilskie and Hagen, 2013; Hunter et al., 2007; Nunalee et al., 2015; Wilson and Gallant, 2000). Although there are many situations where dynamic conditions are stressed for stronger impacts on modifying predictions (Cea and Bladé; Budd et al., 2015), the underlying topography is still very important due to its increasingly improved fidelity to the Earth's surface (Bates, 2012; Tarolli, 2014), and a sophisticated topography transformation would be beneficial to reduce discrepancies arisen from physical inconsistencies (Chen et al., 2015; Glover, 1999; Ringler et al., 2011).

批注 [o11]: Response to Comment #5

批注 [o12]: Correspond to Change #23

批注 [o13]: Response to Comment #6

批注 [s14]: Response to Comment #7

批注 [s15]: Response to Comment #8

批注 [s16]: Response to Comment 9#

批注 [s17]: Modified from “for”

1.2 Multiresolution DEM model

Systematic scale transformation of topographic data has long been studied under terrain generalisation, where precise surface approximation and terrain structural feature retention have both been pursued (Ai and Li, 2010; Chen et al., 2015; Guilbert et al., 2014; Jenny et al., 2011; Weibel, 1992; Zhou and Chen, 2011). Triangulated irregular networks (TIN) are generally chosen as a substitute for the regularly spaced grids (RSG), and terrain feature points (critical points or salient points from some significance metric) are selected for constructing the network. Triangular networks are used for their adaptiveness to locally enhanced multiresolution schemes. Critical points or salient points are selected because they can effectively improve the approximation precision (Heckbert and Garland, 1997; Zakšek and Podobnikar, 2005; Zhou and Chen, 2011).

As surface approximation precision and terrain feature retention are competitive for the redistribution of feature points, DEM (digital elevation model) generalisation is differentiated from terrain generalisation by its emphasis on surface approximation as a whole, with the aim of providing precise surface interpolation (Guilbert et al., 2014). Terrain generalisation emphasises geomorphology or landform depiction, where map generalisation measures (such as abstracting, smoothing) are drawn to produce progressive data reduction, with the effect that the main relief features are strongly stressed while non-structural details are massively suppressed (Ai and Li, 2010; Guilbert et al., 2014; Jenny et al., 2011). Since the static topographic layers are commonly composed directly from DEM datasets for diverse simulation interests, maintaining precise surface approximation for rigorous boundary conditions is often more important than ‘sparse’ geomorphology representation. While DEM datasets are usually used interchangeably with topography or terrain in Earth systems, we will use DEMs and topography indiscriminately hereafter.

Existing DEM generalisations can be catalogued into two broad classes, namely, heuristic refinements and smooth-fittings, according to differences in surface approximation strategy. The first class of approaches is due to the computational feasibility consideration, for selecting a TIN surface that best approximates the original DEM surface from exhaustively enumerating (of triangular combinations) requires exponential time (Chen and Li, 2012; Heckbert

批注 [o18]: Modified for clear statement from “and critical points or salient points with terrain significance are selected for the network feature points”

批注 [s19]: Modified from “shceme”

批注 [o20]: Response to Comment #11

批注 [s21]: Response to Comment #10

批注 [s22]: Response to Comment #12

批注 [o23]: Modified from “two broad groups”

批注 [o24]: Modified from “heuristic refinements and smooth fitting”

and Garland, 1997). It thus forces existing researches to employ some heuristic strategy, in which insertion (or deletion) refinements on feature points are adopted, to find a sub-optimal approximation that is computationally practical. In each insertion (or deletion), rearranging the entire existing grid to obtain a better approximation is also computationally prohibitive and thus not adopted (Chen et al., 2015; Heckbert and Garland, 1997; Lee, 1991); this may result in the clustering of feature points (Chen and Li, 2012). Among those existing heuristic approaches, trenching the pre-extracted terrain features (drainage streamlines, for example) into the TIN surface seems quite appealing (compound method) (Chen and Zhou, 2012; Zhou and Chen, 2011), but the quality of the generalised TIN surface cannot be guaranteed, and the existence of sliver triangles makes it difficult to be incorporated with simulation numerical stability (Kim et al., 2014; Weller et al., 2009). The second class of approaches recognised the TIN surface constructed from locally computed feature points as not-well approximation to the original DEM surface. Much research thus considered global approximation instead of relying on an elaborate feature point selection scheme, such as bi-linear, bi-quadric, multi-quadric, Kriging, or general radial base function-based fitting (Aguilar et al., 2005; Chen et al., 2015; Schneider, 2005; Shi et al., 2005). The proposed multi-quadric method (Chen et al., 2012; Chen et al., 2015), for example, well approximates the original DEM surface with a high-order smooth surface, and the smooth surface provides a kind of rejection mechanism to cure the feature point clustering problem. However, the high-order radial base function is computationally expensive when a broad scenario is involved (Chen et al., 2015; Mitášová and Hofierka, 1993). In brief, existing DEM transformations are neither well performing with respect to loyalty to the original terrain surface nor easily incorporated by the numerical schemes.

- 批注 [o25]: Response to Comment #13
- 批注 [s26]: Modified from “which insertion (or deletion) refinements on feature points”
- 批注 [s27]: Modified from “inhibitive”
- 批注 [s28]: Modified from “and this”
- 批注 [o29]: Response to Comment #15
- 批注 [s30]: Response to Comment #14
- 批注 [o31]: Modified from “best performed with loyalty to the original terrain surface, nor easily incorporated by the numerical schemes”

1.3 Aims and contributions

The purpose of this article is to devise a multiresolution DEM model that optimises surface approximation and guarantees grid quality that can be easily incorporated into the simulation systems. Multiresolution is an effective paradigm to model scale diversity (Du et al., 2010; Guba et al., 2014; Ringler et al., 2011; Weller et al., 2016). Amongst a number of promising approaches, we are especially fascinated by the centroidal Voronoi tessellations (CVTs) method as an intuitive way to redistribute samples with a designated function (Du et al., 1999; Du et al., 2010; Ringler et al., 2011) to develop an optimised surface transformation method to realise multiresolution terrain models. CVT is essentially a two-step optimisation loop, i.e., spatial domain equalisation from Voronoi tessellation and property domain equalisation from barycentre computation (Du et al., 1999). To make this general optimisation method work for DEM transformation, we made the following contributions:

(1) The generalised DEM surface is initially approximated by a triangular grid constructed from selected feature points. The selection of feature points have important morphological structures embedded (in the form of serialised points sequences), for computed (such as D8 flow algorithm) or auxiliary input morphological lines has been proved to have significant influence on the quality of the transformed DEM surface (Zhou and Chen, 2011). The proposed method keeps the structural lines in the optimisation loop and makes it different to existing CVT implementations where stationary points are commonly not considered.

(2) For the discrete TIN surface, we compute robust mean curvature on each facet. The attached curvature acts as a frequency distribution. In this discrete spatial domain and frequency domain, the CVT loops and makes sample facets equalised from both domains. Spatial equalisation warrants a quasi-uniform grid quality, while the curvature

- 批注 [o32]: Response to Comment #16
- 批注 [o33]: Modified from “this general purposed optimization method”
- 批注 [o34]: Modified from “we made a few contributions as follow”
- 批注 [o35]: Response to Comment #17

domain equalisation warrants adaptive distribution conforming to the terrain relief. It is thus a totally different DEM generalisation approach, and we called it curvature CVT (cCVT).

批注 [o36]: Response to Comment #18

(3) Existing CVT implementations often undertake a clustering approach. However, clustering over discrete sets suffers from numerical issues such as zigzag boundaries, invalid cluster cells (Valette et al., 2008) and limited grid quality.

批注 [o37]: Response to Comment #19

5 By devising an exact geometry clipping technique, this article develops a dedicated CVT algorithm for DEM transformation that helps to improve or avoid the numeric problems listed above.

The cCVT works on discrete sets but has a global optimisation mechanism. It promises an optimised surface approximation and quality grid, which can be used to build a high-fidelity multiresolution terrain model. From this terrain model, reliable surface variables can be estimated under a coupled system, or improved computational mesh can be constructed and refined to possible dynamic conditions.

批注 [o38]: Modified from “but has global optimization mechanism”

批注 [o39]: Added “can be used to”

1.4 Organisation of the article

The rest of the article is organised as follows. In Section 2, the theory behind CVT for optimised DEM surfaces is introduced, techniques for incorporating DEM generalisation principles and fast convergence are presented, and the differences between the cCVT implementation and classical clustering approach are discussed. In Section 3, the cCVT model is tested with real LiDAR-derived terrain datasets. Section 4 discusses some considerations, comparable results, possible causes, and interpretations of the cCVT model. Finally, Section 5 briefly presents a short conclusion and outlook.

批注 [o40]: Edited from “Section 5 presents a short conclusion and outlooks briefly.”

2. Curvature centroidal Voronoi tessellation on DEM surface

2.1 Definition

Centroidal Voronoi tessellation is a space tessellation for each Voronoi cell’s geometrical centre (in the spatial domain) that coincides with its barycentre from the abstract property domain (Du et al., 1999). Here, the property domain is analogous to the frequency domain. For the vertex set $\{v_i\}_k^1$ in $\Omega \subset R^3$, the Voronoi tessellation graph is defined as

$$V_i = \{p \in \Omega: |p - z_i| < |p - v_j|, j = 1..k, j \neq i\}, i = 1..k. \quad (1)$$

That is, a Voronoi cell V_i is the set of points whose distance to v_i is less than that to any other vertices. $|\cdot|$ is the Euclidean norm. Every vertex and its corresponding dual cell commonly have some intensity scalar ρ attached from some abstract property domain, which is called a density function. The total potential energy of the Voronoi graph V of a terrain surface can be computed by summing up the potential energies of all cells V_i :

$$E = \sum_{i=1}^k \iint \rho \cdot |p - v_i|^2 \cdot d\sigma. \quad (2)$$

批注 [o41]: Modified from “by summing up every cell V_i ’s potential energy”

The energy minimiser is

批注 [o42]: Added “The”

$$\bar{x} = \frac{\iint x \cdot \rho \cdot d\sigma}{\iint \rho \cdot d\sigma}; \bar{y} = \frac{\iint y \cdot \rho \cdot d\sigma}{\iint \rho \cdot d\sigma}; \bar{z} = \frac{\iint z \cdot \rho \cdot d\sigma}{\iint \rho \cdot d\sigma}, \quad (3)$$

which minimises the surface’s total potential, since $\bar{\mathbf{v}}_i = (\bar{x}, \bar{y}, \bar{z})$ satisfies

$$\frac{\partial E}{\partial p} = 2(p - v_i) \cdot \iint \rho \cdot d\sigma = 0. \tag{4}$$

In other words, when \mathbf{v}_i coincides with the barycentre $\bar{\mathbf{v}}_i$, each cell’s potential effect on the property domain (gravity) becomes equalised to a stable energy state.

批注 [o43]: Modified from “for”

5

2.2 Lloyd Relaxation

The most classical energy minimisation process of centroidal Voronoi tessellation is expressed by *Lloyd’s Relaxation* (Lloyd, 1982). The main idea of this algorithm is to first tessellate the surface, and then perform density integration over the area to find a ‘gravity’ barycentre for each tessellated cell, which is used as the new site for the iteration. The pseudo code of this procedure is shown below.

批注 [o44]: Correspond to Change #24

```
10 Algorithm1 Lloyd_relaxation
   Inputs: vertex set  $\mathbf{N} = \{\mathbf{v}_i\}_k^1$ 

   while ( $\Delta E > \mathbf{Threshold}$ )
   {
15     use the  $k$  vertices to tessellate the surface, obtain Voronoi cells  $\{\mathbf{V}_i\}$ ;
       clear  $\mathbf{N}$ ;

       for each  $\mathbf{V}_i$  in  $\{\mathbf{V}_i\}$ 
       {
20         Compute barycentre of  $\mathbf{V}_i$ :  $\bar{x} = \frac{\iint x \rho d\sigma}{\iint \rho d\sigma}$ ;  $\bar{y} = \frac{\iint y \rho d\sigma}{\iint \rho d\sigma}$ ;  $\bar{z} = \frac{\iint z \rho d\sigma}{\iint \rho d\sigma}$ ;
           push  $(\bar{x}, \bar{y})$  to  $\mathbf{N}$ ;
       }
       compute  $\mathbf{E}$ ;
   }
25 }
```

We follow Lloyd’s elegant idea. The barycentre of a 2-dimensional Voronoi cell may fall outside this surface patch, so an additional calculation may be needed. Du et al. suggested projecting the barycentre onto a nearest facet and using instead the constrained projection point for the new site (Du et al., 2003). Others suggest quadric interpolations over all the facets of the cluster for more accurate site calculations (Valette et al., 2008).

批注 [o45]: Modified from “project”

批注 [o46]: Modified from “the constrained projection point is used instead”

批注 [o47]: Modified from “for further accurate site calculation”

30

2.3 Fast convergence to DEM equilibria

批注 [s48]: Modified from “Fast converge to DEM equilibrium”

2.3.1 Clustering CVTs

Lloyd’s Relaxation requires Voronoi tessellation on a discrete 2-dimensional surface, but direct Voronoi tessellation on a piecewise smooth surface requires costly geodesic computation and may be challenged by complicated numerical issues (Cabello et al., 2009; Kimmel and Sethian, 1998). Du et al. suggested that CVT could be realised through some

35 clustering approaches (Cohen-Steiner et al., 2004; Du et al., 1999; Du et al., 2003), that is, using the associated property

as density function to cluster facets and then find the clustered cells' barycentres to create new clustering sites. Through this heuristic iteration, the new sites along with the new tessellations compose better and better approximation to the original surface, with their spatial distribution conforming to the pre-defined density function.

The clustering approach avoids geodesic tessellation by direct facet combination, which is computationally light. The greatest expenditure then comes from global distance computation for identifying every cell to its cluster centre. However, this *k-means* like clustering over discrete facets suffers from some numeric issues, such as zigzag cluster cell boundaries - since no geodesic Voronoi tessellation was used, and invalid clusters due to disconnected set of facets (Valette and Chassery, 2004; Valette et al., 2008). Furthermore, the key to the quick clustering algorithm is that it avoids generating new sites (to avoid surface reconstruction) and relies on existing sites (or facets). Thus, the generated grid cells may not be well qualified.

2.3.2 Curvature as density function

Terrain surface critical points such as peaks, pits, and saddles are treated as gravity equilibria and key elements depicting the surface geometry in the large (Banchoff, 1967; Milnor, 1963); a further extension of the critical points on a second-order surface derivatives will describe a more fundamental terrain geometry shape (Jenny et al., 2011; Kennelly, 2008).

When constructing a generalised DEM surface, these feature points are commonly used as a base set, and additional input points, or pre-extracted terrain structures are embedded for further approximation (Guilbert et al., 2014; Zakšek and Podobnikar, 2005; Zhou and Chen, 2011). The additional input points or pre-extracted terrain structures of interest are also commonly required in numerical simulation setups for cross-checking or validation purposes.

Based on these observations, and considering requirements of the CVT variational framework, this article proposes a feature point based scheme (including boundary points, feature points, and pre-extracted structural points of interest) as initial Voronoi sites. For optimised spatial distribution of these sample points, we calculate a robust discrete mean curvature as density function, which is based on the recognition of curvature's flexibility in capturing shape characteristics and capability in conducting shape evolution (Banchoff, 1967; Kennelly, 2008; Pan et al., 2012). Curvature's ability to flexibly describe terrain morphology has been appreciated by many researchers. For example, P. J. Kennelly noted that, compared to the results of the flow accumulation model, curvature based delineation of drainage networks is not limited to one pixel thickness and requires no depression filling (Kennelly, 2008). The robust discrete curvature calculation is referred to Meyer et al. (2003).

2.3.3 Improved CVT implementation from approximation

The *Lloyd Relaxation* demonstrates an effective way for heuristic approximation. To follow this elegant approximation, an edge bi-sectioning based dual operation (Du et al., 2010) approach is utilised. That is: from the sample points, an initial TIN surface is constructed. We compute its dual mesh and take this space tessellation as approximated Voronoi cells. The approximated Voronoi tessellation is then optimised within the cCVT iteration. But different to clustering approach, we use each approximated Voronoi cell to (vertically) clip the original dense DEM surface, called referring patch. From this exactly clipped referring patch, we compute accurate energy estimations for the new sites. The global clipping computation is localised using a *kd-tree* structure.

The localisation and accurate referring energy computation makes the cCVT iteration converge fast. The efficiency of the cCVT approximation as a whole is comparable to that of the elegant clustering approach (also has *kd-tree* fast location embedded). We go no further with the complexity analysis, but provide an implementation of the classical

批注 [o49]: Edited from "zigzag cluster boundaries"

批注 [o50]: Added for smoother statement

批注 [o51]: Modified from "derivative"

批注 [o52]: Edited from "will be used to describe a more detailed set of terrain surface parameters"

批注 [s53]: Modified from "consider"

批注 [o54]: Edited from "Curvature's flexible ability on depicting terrain morphology"

批注 [s55]: Modified from "pointed out"

批注 [s56]: Modified from "compared to the result of"

批注 [s57]: Modified from "has not limit to"

批注 [o58]: Added for clearer statement

批注 [o59]: Added statement for clarification

clustering algorithm with the same settings as the cCVT method in the attachment. The pseudo-code of this improved cCVT iteration is described as follows.

Algorithm2 cCVT_iteration

```

5  Input: vertices  $N = \{v_i\}_K^1$ , scale-transformation Ratio.

1) Construct the original DEM surface oriPd from vertices N, compute density function  $\rho$  based on robust mean-curvature estimation;
2) Extract and mark boundary points B, mark stationary control points, check points as C, extract and mark the feature points F;
3) Perform constrained Delaunay triangulation on point set  $\{B, C, F\}$ , with boundary  $\{B\}$  and structural terrain features  $\{C\}$  as constraints;
10 obtain an initial approximated TIN surface;
4) While ( $\Delta E > \text{Threshold}$ )
{
    4.1) Compute TIN's dual TD;
    4.2) For the  $n$  vertices  $r_j$  in TD, extract its direct incident facets as  $FS = \{T_i\}_N^1$ ;
    4.3) For each  $T_j$  in FS
15 {
        4.3.1) Compute its minimal bounding box BBoxj, quickly compute its intersection with oriPd using a kd-tree, obtain a narrowed
        reference geometry narrPd;
        4.3.2) Compute exact intersection of  $T_j$  and narrPd, push the result into reference sets  $REF=\{ref_j\}$ ;
20 }
    4.4) For each  $ref_j$  in REF
    {
        4.4.1) Compute approximated Voronoi barycentre:  $\bar{x} = \frac{\sum \rho \cdot x \cdot \text{area}(ref_j)}{\sum \rho \cdot \text{area}(ref_j)}$ ;  $\bar{y} = \frac{\sum \rho \cdot y \cdot \text{area}(ref_j)}{\sum \rho \cdot \text{area}(ref_j)}$ ;  $\bar{z} = \frac{\sum \rho \cdot z \cdot \text{area}(ref_j)}{\sum \rho \cdot \text{area}(ref_j)}$ ;
        4.4.2) Use kt-tree for fast intersection computation of point  $(\bar{x}, \bar{y}, \bar{z})$  and oriPd, with the result used as the projected nearest
25 point; push it into the new candidate point set F;
    }
    4.5) Using  $\{B, C\}$  as constraints, Delaunay triangulate point set  $\{B, C, F\}$  and obtain reconstructed TIN';
    4.6) Compute E on TIN';
}

```

30 Here, we illustrate this algorithm using a numerical mountain model. The analytic equation is

$$z = (4x^2 + y^2) \cdot e^{-x^2 - y^2} . \quad (5)$$

It has two peaks, two saddles and a pit. We rasterise it with a 49×49=2401 regular grid (Figure 1, left). For effectiveness, we set the transformation scale to 0.1 (**Ratio** = 0.1), that is, there are approximately 240 points left. The sample set includes 56 boundary points, 5 critical points, and an additional 169 random points for visual saturation purposes (Figure 1, left; the red points are randomly generated points, the blue points are boundary points, and the green points are critical points). Relief feature points are always abundant in a real terrain dataset, so additional random points are rarely needed. A robust mean curvature estimation is computed on the original high-resolution surface **oriPd** (Figure 1, right), from which we can clearly distinguish critical points as peaks, saddles, and pits. The initially approximated TIN surface from the sample set is shown in Figure 2 (left), and its generated dual mesh is shown in Figure 2 (right), which corresponds to step 3 of Algorithm2. Figure 3 shows the dual cell of the sample points, which is the key idea of the cCVT approximation. Figure 4 and Figure 5 show the algorithm steps 4.3.1 and 4.3.2, respectively, where the exact clipping is completed on the original DEM surface. Figure 6 and Figure 7 show the final computation on the reference patch of the first sample point, which corresponds to the algorithm steps 4.4.1 and 4.4.2. Figure 8 exhibits the result of the first iteration, compared to that of the final iteration, with the initial sample points included (top). A comparison of the constructed approximate TIN grids of the initial state and final state is illustrated in the middle, while the curvature distribution that represents the terrain feature comparison is illustrated at the bottom.

The results show how the embedded stationary points (control points and boundary points), feature points, and

批注 [o60]: Modified from "As for effectiveness"

批注 [s61]: Modified from "set the transformation scale at"

批注 [s62]: Modified from "there are about approximately"

批注 [o63]: Modified from "boundary"

批注 [o64]: Modified from "by which"

random points are spatially equalised (Figure 8). Additionally, the cCVT generated a variable-resolution terrain grid (middle right); the convergent TIN grid exhibited nearly uniform high quality, and the convergence process generally resembled *Lloyd's Relaxation* (Figure 9).

5 Notably, the direct reference on the original DEM surface is realised by the exact geometry clipping, which linearly interpolated the high-resolution surface. This clipping technique has several important benefits: it guarantees accurate energy estimation, it avoids the generation of invalid clustering cells or zigzagging cells, and it promises exact site position calculation, which will result in improved grid quality.

批注 [s65]: Modified from “which linearly interpolated the high-resolution surface literally.”

批注 [s66]: Modified from “warrants”

3 Multiresolution DEM Experiments

3.1 Experimental datasets

10 Two sites with significant geomorphological characteristics were selected. Experimental site 1 is Mount St. Helens, located in Skamania County, W.A., USA. This mountain is an active volcano whose last eruption occurred in May 1980, and deep magma chambers have been observed recently (Hand, 2015). This site was selected for its typical mountain morphology along cone ridges and evident fluvial features downhill, where heavy pyroclastic materials and deposits are present. These two distinctively different terrain structures mingle together, posing challenges for DEM

15 generalisation. The St. Helens dataset was selected from the Puget Sound LiDAR dataset (http://wagda.lib.washington.edu/data/type/elevation/lidar/st_helens/). This LiDAR dataset was collected in late 2002. The selected dataset is a 2924×3894 regular grid with a 3 m cell size and covers an area of approximately 102 km². The elevation ranges from 855.32 to 2539.34 m. The image and hillshade views of these data are illustrated in Figure 10.

批注 [s67]: Added

批注 [s68]: Added

20 Experimental site 2 is the Columbia Plateau, USA. This area has been labelled UTM zone 11, so we hereafter call it UTM11 (<http://gis.ess.washington.edu/data/raster/tenmeter/>). This LiDAR dataset was collected in 2009. The selected site is located on the border between Columbia County and Walla Walla County, WA. The southeastern corner is located in the Wenaha-Tucannon Wilderness, Umatilla National Forest. This area contains rugged basaltic ridges with steep canyon slopes at high elevations (average of 1700 m). The northwestern area is located near Dayton City, which is a

批注 [s69]: Modified from “labelled as”

批注 [s70]: Added

批注 [s71]: Modified from “of”

批注 [s72]: Modified from “south eastern”

批注 [s73]: Modified from “north eastern”

批注 [s74]: Modified from “for”

批注 [s75]: Modified from “variation”

25 a vast agricultural and ranching area with relatively smoother morphology at low elevations (average of 500 m). This site is selected due to the coexistence of these two different prominent surface morphologies. If the generalisation scheme emphasises the high-elevation areas with sharp variations, the surface interpolation as a whole might be unbalanced, which may result in smoothing of the low-elevation areas.

批注 [s76]: Added

30 The selected UTM11 dataset is a 3875 × 3758 grid with a 10 m cell size and covers an area of over 1456 km². The elevation ranges from 3533 to 19340 cm. The image and hillshade views of these data are shown in Figure 11.

批注 [s77]: Added

3.2 Comparison method

As previously mentioned, DEM generalisation has long been studied in geoscience, and numerous methods have been proposed over time. One of the most classical approaches is the hierarchical insertion (or decimation) of feature points to construct a TIN grid under a destination scale. This type of heuristic feature point refinement (HFPR) performs very well in terms of surface approximation and terrain structure retention. For this reason, although HFPR methods generally cannot guarantee high-quality grids, these methods are suitable for comparison purposes.

批注 [s78]: Modified from “As aforementioned”

批注 [s79]: Modified from “with”

批注 [s80]: Added

5 A typical HFPR starts with four corner points from a dense DEM image and constructs a Delaunay triangular grid that contains two triangles. The rest of the points are weighted according to their distances from the triangular surface or other error criteria and then queued. The point with the highest priority in the queue is selected, and the grid is modified using incremental Delaunay triangulation. This process repeats until some error threshold is satisfied (Heckbert and Garland, 1997). Michael Garland provided a classical HFPR implementation (<http://mgarland.org/software.html>), and many other variants are available in GIS, meshing, and visualisation tool suites.

- 批注 [s81]: Modified from “by”
- 批注 [s82]: Modified from “largest”
- 批注 [s83]: Modified from “modified by”
- 批注 [s84]: Modified from “loops”

3.3 Quantitative comparisons

10 We performed the processes from Algorithm2 for the two experimental datasets, including triangulation and curvature estimation, boundary point extraction and marking, feature point extraction based on curvature significance and marking, and optimisation loop through cCVT. For effectiveness, the transformation **Ratio** was set to range from 5% to 0.1% point density (comparable to the 3.1% to 0.6% setting in (Zhou and Chen, 2011)).

15 The accuracy of the surface approximation determines the final surface interpolation precision and is thus a basic quality comparison index. Here, we applied a statistical interpolation method to measure the surface approximation precision. From each triangle on the cCVT-generated quasi-uniform TIN grid, a random point is selected and a vertical line is introduced to intersect the original dense DEM surface and the HFPR generalised TIN at the same time. Error estimates for the surface approximation could be obtained from these intersection points. We computed the mean error, maximum error, and root mean squared error (RMSE) for this elevation interpolation (TIN interpolation); the results are listed in Table 1. Furthermore, we computed the aspect ratios of the triangles for both generalised TIN

20 surfaces to measure the grid quality, which are also listed in Table 1. RMSEs with various transformation **Ratios** are listed in independent rows and columns in Table 1.

From the results in Table 1, we can conclude that under the same resolution (point density), the transformed DEM surface obtained using the cCVT method is generally more precise than that obtained using the HFPR method. In all cases, surface approximation precision (compared to the original) decreased as the resolution coarsened.

- 批注 [s85]: Modified from “Error estimates of”
- 批注 [s86]: Added
- 批注 [s87]: Modified from “from”
- 批注 [s88]: Modified from “While all”

25 3.4 Qualitative comparison

A qualitative index is usually measured from the aspect of terrain structure retention. According to the resulting TINs from the two experimental datasets, both the cCVT and HFPR methods performed well based on visual examination. However, upon closer inspection, the surface generated by cCVT has a smoother transition effect than that generated by HFPR (**Figure 12**). HFPR accumulated more samples around sharp features (c.f. **Figure 13**), and its surface exhibited clearer impressions because flat details were smoothed out. From visual examination, it may be concluded that under the same transformation conditions, HFPRs may exert a stronger generalisation effect than cCVTs.

30 However, a stronger generalisation effect actually decreases the precision of the general approximation, which may result in structural distortion or misconfiguration. **Figure 14** illustrates a closer examination of St. Helens. Some structural details on the original surface were recovered by the cCVTs but not by the HFPRs. This terrain structure loss occurred on both smooth areas and steep areas, as illustrated in **Figure 14**. **Figure 15** illustrates similar structural detail losses by the HFPRs in the UTM11 dataset.

35 Terrain structural features could also be measured from DEM derivatives such as the slope, aspect, and hydrological structural lines. Here, we used contours to compare the generalisation accuracy using experimental site 2. For the same

- 批注 [s89]: Modified from “from terrain structure retention aspects”
- 批注 [s90]: Modified from “result”
- 批注 [s91]: Modified from “datesets”
- 批注 [s92]: Modified from “performed well from visual examination”
- 批注 [s93]: Modified from “detail loss from”
- 批注 [s94]: Added
- 批注 [s95]: Modified from “Upon”

configurations (80 m elevation increments), we generated contours for the original dense TIN (rendered in red), the cCVT-generated TIN (rendered in blue), and the HFPR-generated TIN (rendered in black), and overlaid the three sets of contours for comparison (Figure 16). The illustrations demonstrate that in most cases (Figure 16 b, c), the contours from the cCVT-generalised surface accurately conform with those from the original dense surface, while the contours from the HFPR-generalised surface generally did not, except for some cases in steeper areas with sharp curvature variations (Figure 16 d). This result can be explained by the HFPR's stronger accumulation of sample points near sharp features, which guaranteed an edging out, if we noticed that the inspection area d is much smaller than b or c.

- 批注 [s96]: Modified from “overlapped”
- 批注 [s97]: Modified from “cCVT-generalised surface are more accurately conform with”
- 批注 [s98]: Modified from “to”

4 Discussion

Topography transformation of DEM surfaces has been a deeply studied topic in geoscience, simplification techniques and generalisation principles are widely realised and adopted. Extracting terrain feature points and using these points to construct a generalised surface has proven to be one valuable approach; its success may be due to the feature points' strong capability to capture terrain structures. However, discrete surface as TIN grids that are constructed purely from feature points may not be best approximated to the original high-resolution surfaces. Take the mountain equation in Figure 1 as an example. It has at least two peaks, two saddles, and one pit close to the zero level. Assume that the scale transformation requires that only two critical points are left; selecting both peak points is more reasonable than selecting the pit point, even if the pit point has a stronger quantitative index (curvature in this case) than the peak points. This observation implies that if global surface interpolation precision is of more importantly demanded, a robust approach that has overall considerations on surface approximation and terrain feature retention should be adopted. Among those classical DEM generalisation approaches, heuristic feature point refinement is an outstanding example. As illustrated by Table 1, Figure 12, and Figure 16, HFPR methods perform excellently in terms of surface approximation and surface morphology retention. For the treatment of feature points, these methods use a heuristic strategy by incremental Delaunay triangulation, which considers the point with the largest error with respect to the constructed TIN. However, the impact of the insertion of a new feature point on the inserted feature points is not considered due to computational burden. That is, modifications are only taken place on the triangle where the point with largest vertical error locates on. As a result, feature points may cluster around reliefs with sharp variations, as shown in Figure 12. Too many feature points accumulating near sharp features means that relatively few feature points are present in flat areas, which will eventually lead to terrain structure misconfiguration, as shown in Figure 14, Figure 15, and Figure 16. Sometimes, this type of structural loss is unacceptable. For example, the terrain relief at high elevation under the studied scale (10 m cell spacing) in experimental site 2 exhibited a fiercer landform than at lower elevations. The accumulation of too many sample points in high-elevation areas may result in the distortion of the smooth anthropogenic terrain morphology in low-elevation areas. cCVT starts by constructing a terrain-adaptive variable-resolution grid. The cCVT iteration uses a robust mean curvature as density function which is based on the curvature's capability to characterise shapes and conduct shape evolution. Under the curvature guidance, the two-step optimisation (c.f. Algorithm1 in Section 2.2) loops both to spatial equalisation and frequency equalisation. The process of spatial equalising of feature points has been seldom considered by classical approaches, which may explain why cCVT generally prevails over HFPRs (c.f. Table 1). Notably, the triangles from the spatial equalisation exhibited a maximum aspect ratio that was less than 5.0, which implies that the constructed terrain grid satisfied the numerical stability requirement from classical finite element or finite volume computations. On the other hand, CVT is an approach within variational framework. The result of the iteration depends on the boundary conditions and initial conditions. Hence, this article employed a feature point scheme (with additional input points considered) as a relatively stationary initial condition to maintain algorithm stability. The requirement of

- 批注 [s99]: Correspond to Change #25
- 批注 [s100]: Modified from “Taking the mountain equation in Figure 1 for example”
- 批注 [s101]: Added
- 批注 [s102]: Modified from “Presume”
- 批注 [s103]: Correspond to Change #26
- 批注 [o104]: Correspond to Change #27
- 批注 [o105]: Correspond to Change #28

embedding feature points of interest, along with consideration of avoiding the problematic *k-means* like clustering, prompted us to develop a non-clustering approach with an exact energy referring method. Experiments on ten million DEM points demonstrated that the exact clipping approach performed comparably to the elegant clustering approach.

批注 [o106]: Correspond to Change #29

5 Conclusions

5 In this article, a high-fidelity multiresolution DEM model was proposed. The variable-resolution with high-fidelity was achieved by the developed curvature-based CVT. cCVT optimisation increases the precision of surface approximation compared to existing heuristic DEM generalisations, while the equalisation of feature points from the spatial domain guarantees a high-quality grid.

10 Multiresolution models are essential tools to incorporate more scales, while a high-fidelity generalised DEM model can be used to construct a concrete topographic layer from which fine endogenous or exogenous processes can be assessed under proper scale conditions. Evaluation of the cCVT multiresolution DEM model on Earth and environmental systems in wide-ranged domains and scales is a topic for future studies. Furthermore, considering the tyranny of global modelling of the Earth systems (Ringler et al., 2011), this may imply a consideration of curvature of the Earth itself into the cCVT model.

批注 [o107]: Correspond to Change #30

批注 [o108]: Correspond to Change #31

Code Availability

We implement the cCVT and the comparing classical *k-means* clustering CVT algorithm using c++ on the open-source Visualisation Toolkit (The VTK, <https://www.vtk.org>). The approximated dual operation through edge bi-sectioning is based on the implementation of Valette et al. (2008). If anyone is interested in the technical details of the implementation, please contact the corresponding author for the source codes, demo datasets and necessary guide for compilation.

批注 [o109]: Response to Special Comment about Code Availability

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Table 1 Quantitative comparison of the grid quality at scale transformation *Ratio* 1%

Dataset	Dense DEM Points	Random Interpolation Points	Approx. Method	Mean Error (m)	Max Error (m)	RMSE (m)	Max. Aspect Ratio
St. Helen 3 m	11,386,056	230,909	cCVT	0.0353	23.05	1.6145	3.23
			HFPR	0.1107	191.31	2.3714	9255
UTM11 10 m	14,562,250	301,255	cCVT	0.5773	37.70	3.77313	4.09
			HFPR	0.8765	487.81	6.71214	8426

Interpolated elevation RMSEs (m) at varied scale transformation *Ratios*

Dataset	Approx. Method	5% *	1%	0.5%	0.1%
St. Helens 3 m	cCVT	0.636	1.614	2.455	5.772
	HFPR	1.028	2.371	4.006	11.779
UTM11 10 m	cCVT	1.239	3.773	6.593	19.997
	HFPR	3.087	6.712	10.137	28.460

* The *Ratio* percent number means *n*% points left.

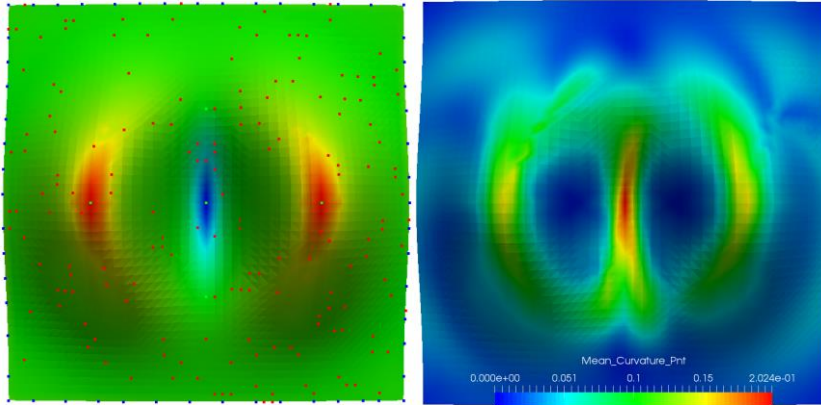


Figure 1 High-resolution grid of a mountain equation (5). Left: original grid *oriPd* in 49×49 resolution, rendered in mean curvature. The sample points were also rendered on *oirPd*; the blue points are boundary points, the green points are critical points, and the red points are random points. Right: robust mean curvature estimation. The saddle terrain features, peaks, and pits can be distinguished, compared to the depiction on the left.

批注 [s110]: Modified from “depicting”

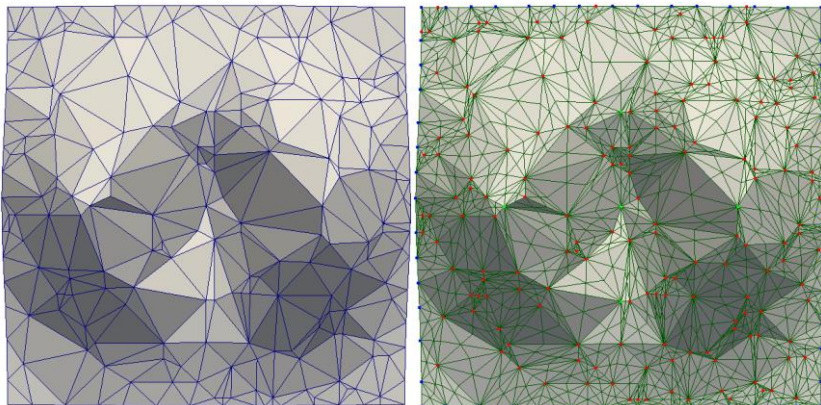


Figure 2 Initial *TIN* surface (left) and its dual grid *TD* (right). The initial sample points on the dual grid are also rendered.

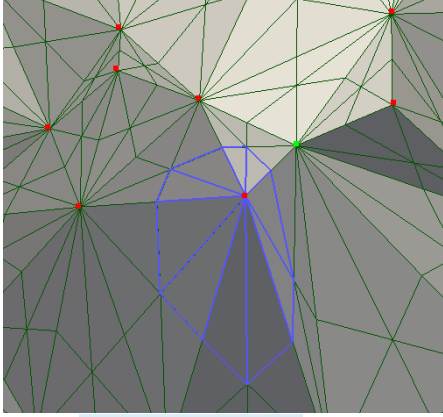


Figure 3 The triangles incident towards the first vertex r_i on the dual grid comprise an initial approximated Voronoi cell (rendered as a blue wireframe); the centre vertex is rendered in red.

批注 [s111]: Modified from "The incident triangles toward"

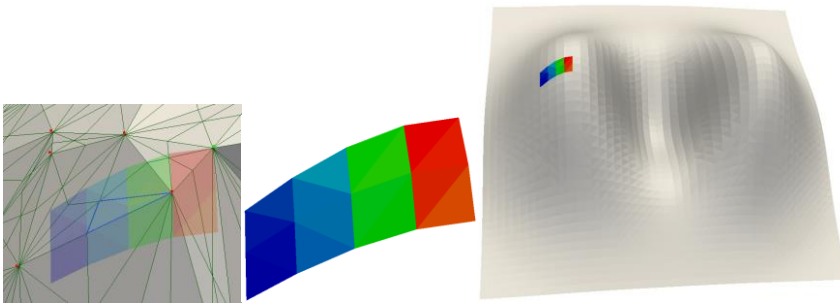


Figure 4 A triangle T_j (rendered in semi-transparent blue) with its minimal bounding box's intersection part $narrPd$ with the original grid on the approximate grid (left), the localised intersection part $narrPd$ (middle), and the intersection part on the original grid $oriPd$ (right).

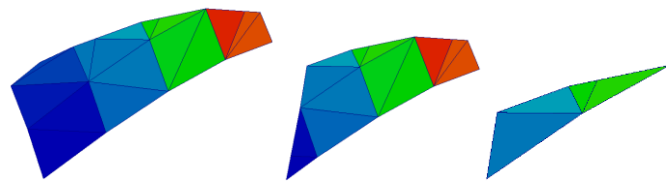


Figure 5 Exact clipping steps of $narrPd$ with T_i . The sequence from left to right illustrates the edge clipped results.

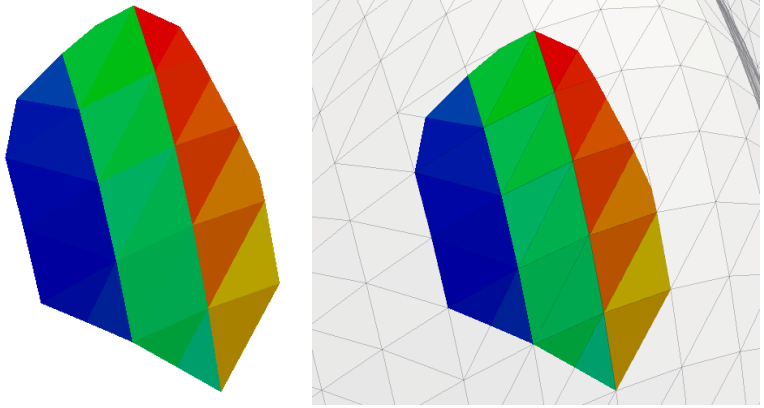


Figure 6 Reference patch ref_j on the original DEM surface of an initial Voronoi cell, with the centre at r_i (left). Right: ref_j on the original DEM surface $oriPd$.

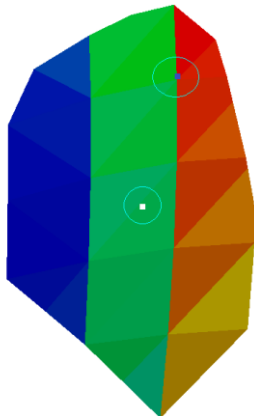


Figure 7 Barycentre computation based on the reference patch ref_j ; the original site is the white block in the circle, and the projection on $oriPd$ of the newly computed site is depicted as the blue block in the circle.

批注 [s112]: Modified from “the newly computed site”

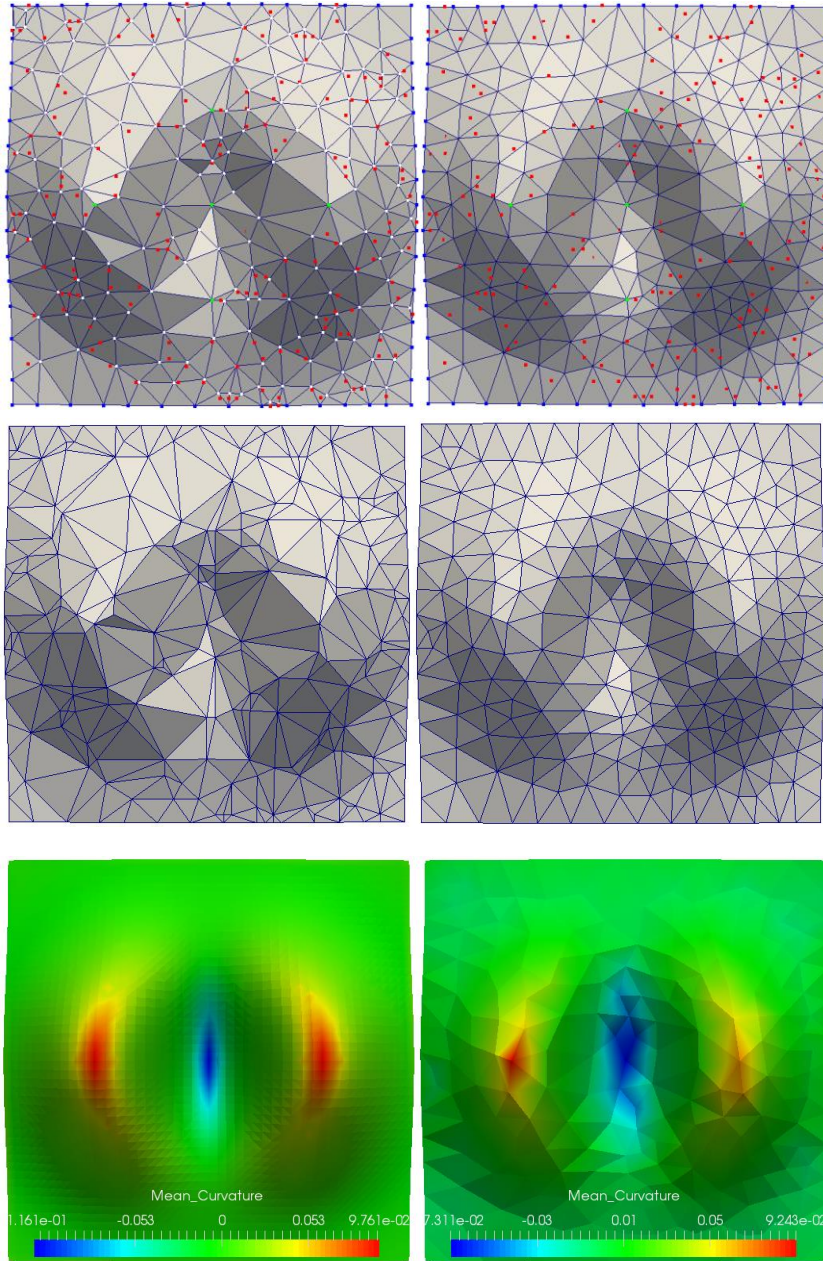


Figure 8 Comparison of converged results. Top left: reconstructed TIN surface from one iteration, with the initial points presented. Top right: the converged TIN surface with the initial sites, after approximately 140 loops. Middle: the initial approximated TIN surface (left) and the final TIN surface (right). Bottom: curvature distribution on the original surface (left) and the generalised grid (right).

批注 [s113]: Modified from "converged results comparison"

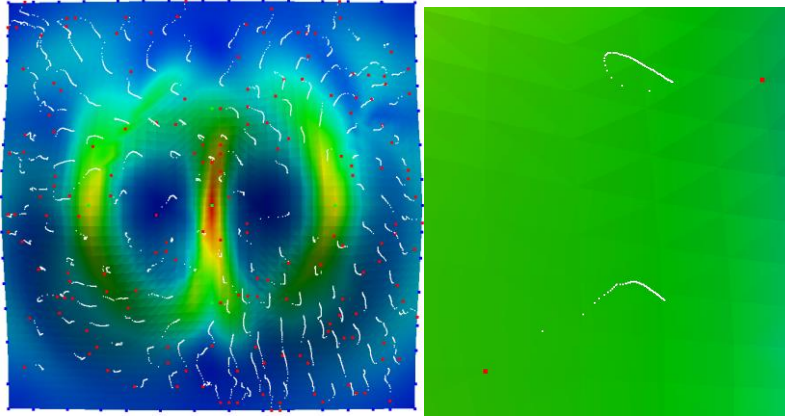


Figure 9 Trajectories of point convergences. The red points indicate the initial sample set, and the trajectories show the convergence trends, with closer gaps between candidate points. The right side shows a close view of the convergence of two points. These trends imply that the cCVT's convergence complies with *Lloyd Relaxation* linear convergence.

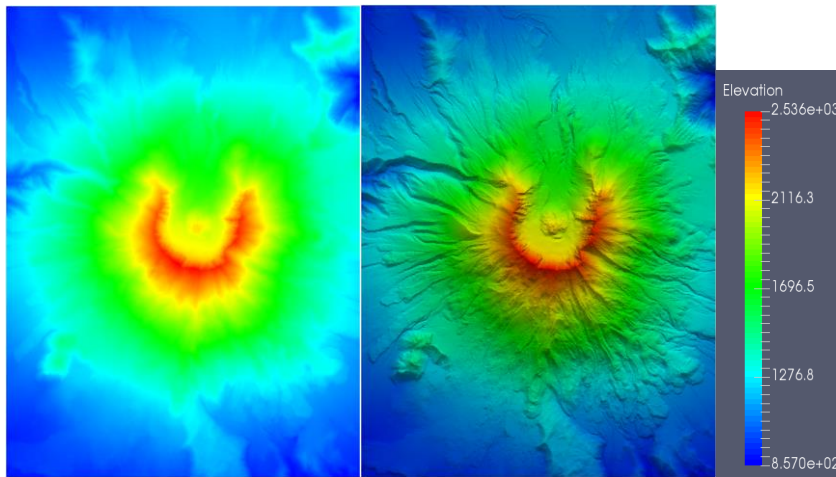


Figure 10 Experimental Site 1: Mount St. Helens. Left: image view. Middle: hillshade view. It is a 2924x3894 grid with a 3 m cell size.

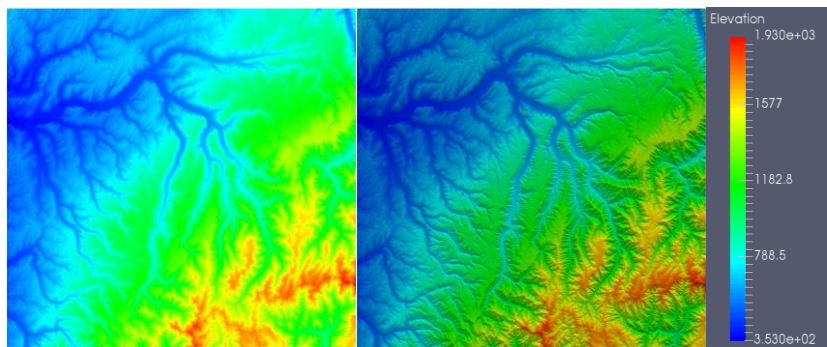
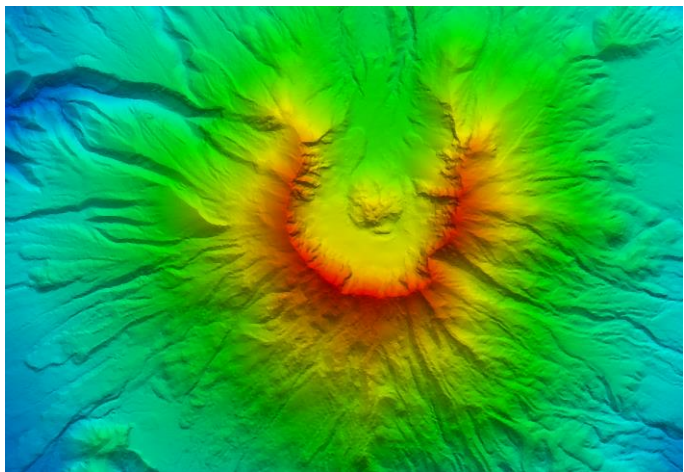


Figure 11 Experimental Site 2: UTM11 Zone. Left: image view. Middle: hillshade view. It is a 3875 x 3758 grid with a 10 m cell size.



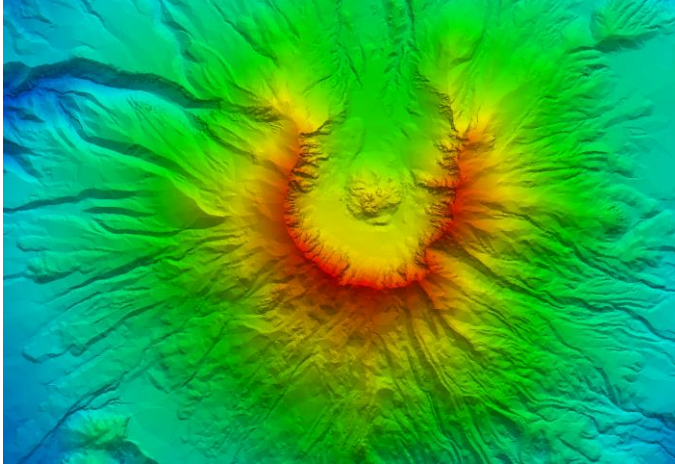
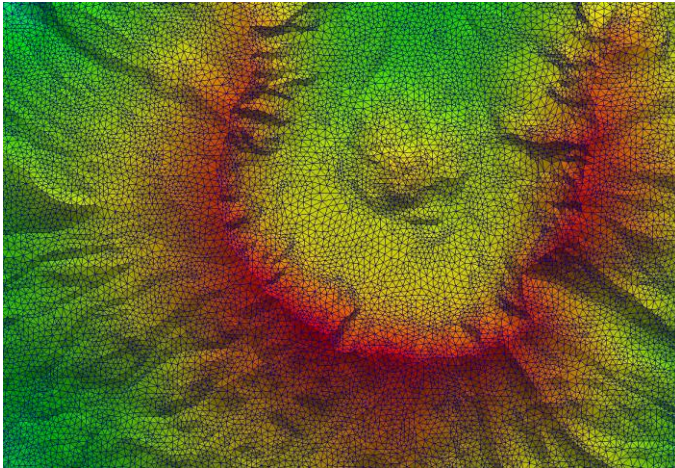


Figure 12 Visual examination of St. Helens. Top: cCVT grid. Bottom: HFPR grid. The latter grid appears more rigid than the former, which implies a stronger generalisation effect.



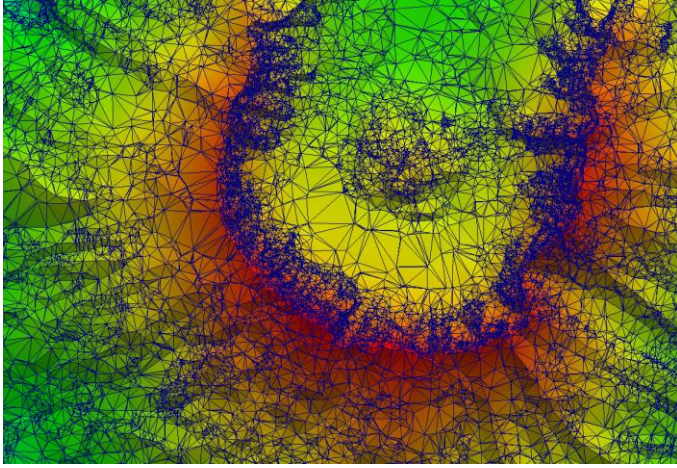


Figure 13 Grid quality from an intuitive comparison. Top: cCVT-generalised grid with nearly uniform triangles. Bottom: HFPR generalised grid with irregular triangles.

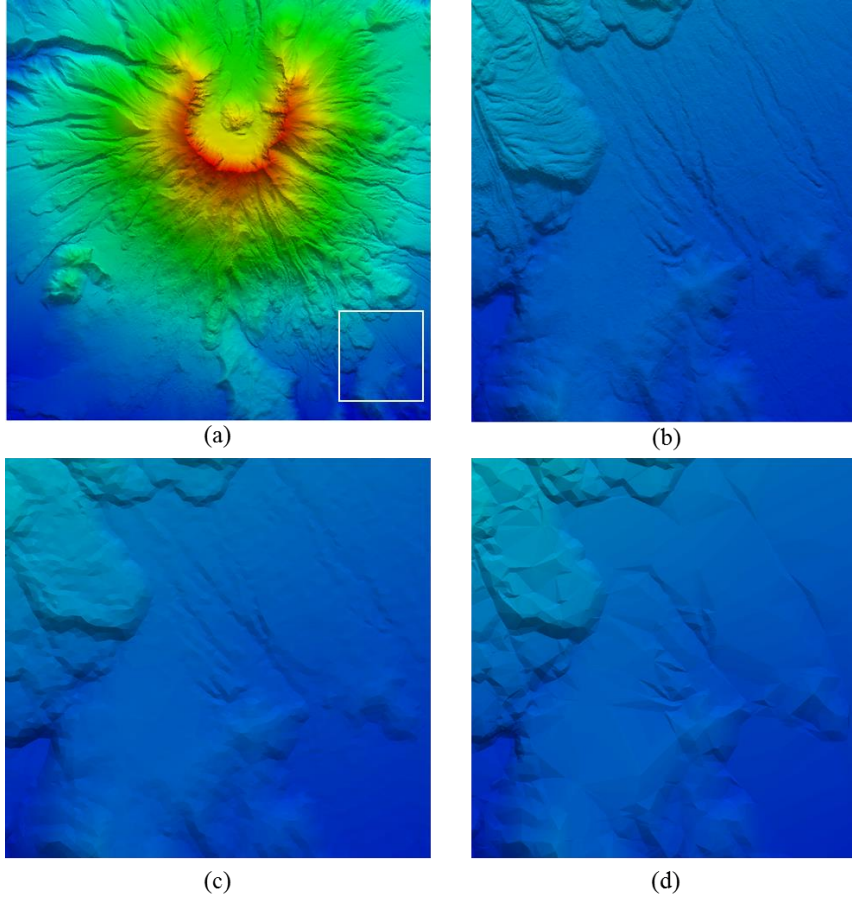


Figure 14 Detail loss of the HFPR generalisation grid. The inspected area in the experimental site is bounded by the white rectangle (a); the magnified inspection area on the original dense TIN (b); the area on the cCVT-generalised TIN (c); and the area on the HFPR-generalised TIN (d). From the close-up view, it can be seen that the HFPR method generates a rougher grid than the CVTs. Thus, structural distortion or misconfiguration might be introduced.

批注 [s114]: Modified from “close view”

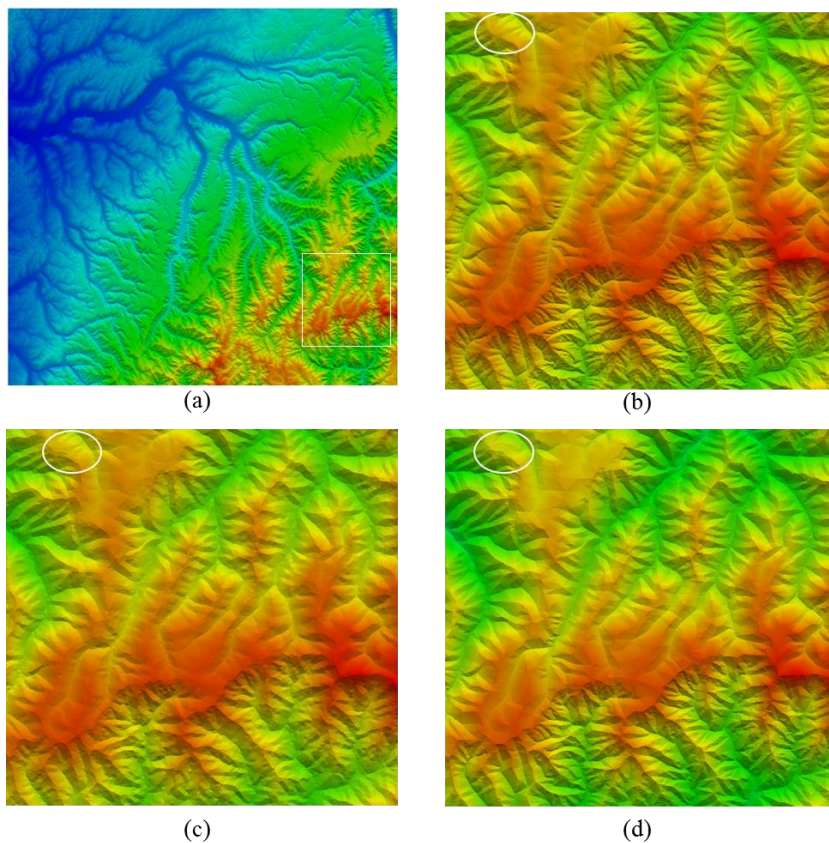


Figure 15 Detail loss from the HFPR method in the UTM11 dataset. (a) The inspection area in the entire experimental site. The magnified view of the inspection area is shown on the original dense DEM (b), on the cCVT-generalised TIN (c), and on the HFPR-generated TIN surface (d). The fold morphology in the white ellipse was recovered by the cCVT method but not by the HFPR method.

批注 [s115]: Modified from “were”

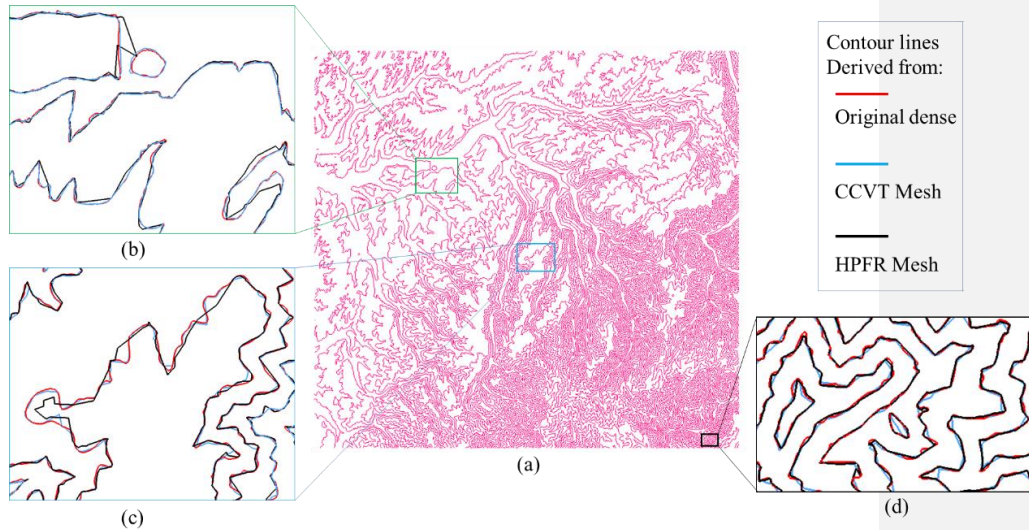


Figure 16 Comparison of the derived contour lines. The contours from the dense UTM11 dataset are shown in (a) and rendered in red. Three areas in the boxes were magnified to show the differences in the contour configurations. The blue contour lines are from the cCVT-generalised TIN surface, and the black lines are from the HFPR-generated TIN surface. According to the illustrations in (a) and (b), the contours that were derived from the cCVT grid (rendered in blue) are more in line with those from the original dense surface (rendered in red). The contours from the HFPR grid (rendered in black) may sometimes edge out on areas with steep slopes, as shown in (d), because the HFPR method accumulated a relatively abundant of sample points around these areas.