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Geoscientific Model Development Discussions

Interactive comment on "Quantitative evaluation of numerical integration schemes for Lagrangian particle dispersion models" *by* H. Mohd. Ramli and J. G. Esler

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Thank you for the encouraging and positive review. A point-by-point response follows:

1. Calculation times for different schemes: We considered making further comment on calculation speeds in the original version, but were concerned about reproducibility due to different programming languages, machine architectures etc.. Encouraged by the referee, however, we have now added a table comparing computational clock times, measured relative to E-M, for all of the schemes considered. This is table 4 in the new version and it has the following description in the text.

'In the results below, in the interests of reproducibility, the error is presented as a function of the fixed time-step Δt for each scheme. However, the schemes have different computational costs per time-step, which will depend on both the method of implementation of each algorithm, and on the machine used for the simulations. To give a rough idea of representative computational costs, in Table 4 the relative cost, measured with reference to the E-M scheme is shown for our calculations. Following best practice in large operational calculations (see e.g. Stohl et al. 2005), the random numbers used to simulate the Wiener processes are pre-calculated so the costs of their generation are not included in the comparison.'

2. Variable versus fixed time steps: For a comparison between schemes it seemed to us that using a fixed time step was a sensible starting point. We agree with the referee, however, that it is interesting to compare the schemes with fixed and variable time-steps. We have therefore made some variable time-step calculations (with $\Delta t \propto \tau$) and have summarised our findings in the following paragraph added to the last paragraph in section 4.1
'Another possible computational saving comes from the use of variable time-steps. To test whether or not a significant computational saving is easily attainable, we have made some calculations in which $\Delta t \propto \tau$ (the local Lagrangian decorrelation time). For each scheme tested, the use of variable time-steps was found to lead to a computational saving of a factor of around two to three compared to fixed time-steps, with the schemes otherwise performing as detailed below. More details on variable time-stepping schemes will be given elsewhere.'

3. Gram-Charlier series of Type A: We have added a comment about this on pg. 5. 'In statistics this expansion is also known the Gram-Charlier series of Type A (see pg. 23 of Barndorff-Nielsen and Cox, 1989)'.

4. Negative probability in the FPE solution: We agree that in theory a truncated se-

ries can certainly lead to negative values in the approximation to $p(\omega, z, t)$. However, we do not use $p(\omega, z, t)$ explicitly anywhere, and the only practical problem in which the very small errors (order 10^{-16}) associated with our truncation might be important, is if one were interested in the probability of trajectories reaching a very high velocity (i.e. extreme value statistics).

5. Reference solution from a large SDE ensemble and small time-step: Yes, one could certainly construct the reference solution using the SDEs (in fact we have effectively demonstrated this in our paper), but for a given degree of accuracy this would be necessarily be much more computationally expensive compared to the FPE method we have chosen.

6. Derivation of the diffusion equation: Our derivation is to replace the $k = 1$ equation in our hierarchy (11) with the quasi-steady approximation in equation (14). This assumes that both $C_2 \approx 0$ and $\partial_t C_1 \approx 0$ and we do not justify it formally (it is just a truncation of the series). However, we have added a reference to the formal derivation of the diffusion equation from the LPDM (see reply 4 to reviewer 1).

7. Reference to equation (15)/(16): Fixed, thanks.

8. Pg. 11. Particle concentration versus concentration of tracer: After equation (6) we have added the comment in parenthesis '(In general, tracer concentrations and the marginal probability given in (6) can differ by a normalisation constant.)'

9. Pg. 18. typo. Fixed, thanks.

10. Pg. 19. l. 5-6 phrase. Fixed, thanks.

11. Repeated symbol 'h' for bandwidth: h is now changed to h_b for bandwidth, and h_* remains as optimal bandwidth.

12. Definition of c_{zz}: This is simply the second derivative of the particle concentration $c(z, t)$ with respect to z. This symbol has now been changed to $\partial_{zz} c(z, t)$ in the text.

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