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Interactive comment on "Locally-orthogonal unstructured grid-generation for general circulation modelling on the sphere*" *by* Darren Engwirda

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Dear reviewer,

Thank you for your comments and recommendations regarding the draft manuscript. I propose to incorporate many of your suggested changes 'as-is' in the revised submission. In some cases, as detailed below, I do not fully agree with the comments made, and have attempted to reply in detail to some of the points raised. In all cases I found the suggestions helpful, and look forward to amending the revised manuscript in response to your review.

Overall, I agree with the suggestion that the revised paper should better articulate the

impact of the techniques described in the present work, and better differentiate them from existing approaches. I include a detailed set of proposed changes below.

Reviewer comments are presented in italics, my responses are included in plain-text.

The current paper deals with high quality surface triangulation applied to general circulation modelling. The paper bypasses a parametric representation of an arbitrary surface by limiting the surface definition to an ellipsoid representing the earth. The main algorithm relies on a coupled Frontal-Delaunay approach. Various examples are provided to illustrate the method.

Overall, the paper is clear and there is obviously a lot of work in it. However, my main critic is that there is not much new brought by the paper, as opposed to what is claimed in the conclusion, except for the fact of applying it to a general circulation modelling. The curvature of the Earth is almost constant so technically it is not difficult to surface mesh it. The paper introduces a lot of concepts such as restricted Delaunay, Frontal Delaunay, Hill-climbing, Gradient based smoothing, etc, which are very classic and well established unstructured mesh techniques. At least, it should be clearly stated what is new.

1. The claim that Voronoi edges are always perpendicular to mesh edges is wrong. It is only valid for an acute triangulation, which is not true in general, particularly because of the boundaries. This is a property that has been pursued by the electromagnetic solvers for a long time for the same reason but only partially reached. This is only briefly mentioned in Section 4.

With respect, I don't believe the reviewer to be correct here. Voronoi edges are indeed (always) perpendicular to their associated Delaunay edges, even when the triangulation is non-acute. This local orthogonality is a fundamental aspect of the Voronoi-Delaunay geometric duality, and lies at the heart of many unstructured numerical formulations that are based on a Voronoi-Delaunay grid staggering, including the discretisation scheme employed in the MPAS framework referenced in the current paper.

What the reviewer may be referring to is the notion of ‘well-centred’ Voronoi-Delaunay grid staggering, which is a property only realised for ‘acute’ triangulations (all angles less than 90 degrees). The properties and benefits of ‘well-centred’ Voronoi-Delaunay staggered grids are discussed in the present paper in Section 4.

Voronoi vertices (the circumcentres of Delaunay triangles) only lie ‘within’ their associated Delaunay triangles when the triangulation is acute. As a result, Delaunay triangles containing obtuse angles induce an undesirable grid-staggering, with the Voronoi edges associated with these large angles failing to intersect their paired Delaunay edges.

In the context of unstructured general circulation models (i.e. the MPAS model) building such ‘well-centred’ Voronoi-Delaunay grids is (a) important (the MPAS framework, for instance, requires such constraints to be satisfied to ensure a correct evaluation of the vertex-centred vorticity distribution), and (b) non-trivial (a majority of conventional unstructured meshing algorithms do not produce such grids).

As such, the development of algorithms designed to produce high-quality well-centred grids is, in my view, a new and useful result. I am not aware of other publicly available software for the oceanic/atmospheric modelling communities that offers similar functionality.

In the current work, it’s shown that a combination of Frontal-Delaunay refinement and hill-climbing optimisation is an effective strategy — able to produce very high-quality well-centred Voronoi-Delaunay grids even when complex, highly non-uniform grid sizing constraints are imposed. I believe this to be a new result of benefit to the unstructured oceanic/atmospheric modelling communities. Public availability of the associated JIGSAW-GEO grid-generator is also thought to be a further benefit to the community.

Noting the reviewers concerns, and subsequent comments below, I propose to re-work the paper to make the arguments above more clearly, and earlier in the manuscript. I propose explicitly defining the notion of ‘well-centred’ grids in Section 2, and to amend

the abstract/introductory sections accordingly. I propose to include a new figure illustrating the distinction between ‘well-centred’ and ‘non-well-centred’ Voronoi-Delaunay grids.

2. Line 28 (page 4). It is not clear at all in general that maximisation of the minimum angle is beneficial. Add references.

Agreed. I will include additional references/explanation in the revised manuscript.

3. The pictures do not clearly show the mesh transitions for size variations in details.

Agreed. I will include better images of transitional mesh regions.

4. The abstract mentions a-priori guaranteed quality bounds while nothing is proved. Empirical studies show good results but no bounds are provided.

Such bounds are rigorously established in the companion paper (Engwirda and Ivers, 2016) that describes the Frontal-Delaunay refinement algorithm in full, with this point mentioned in Section 2.5 (pages 7-8) where references to the companion paper are given and results of the proofs summarised.

The present paper aims to address the question of grid-generation for oceanic/atmospheric modelling from a pragmatic, rather than theory-laden perspective. As such, it was not felt that a reproduction of existing proofs would necessarily aid the reader in this regard.

I propose to amend Section 2.5 (around line 3, page 8) to more explicitly refer interested readers to the companion paper (Engwirda and Ivers, 2016), and offer a more formal summary of theoretical results/proofs.

5. The abstract is misleading. The code may be recently developped but the techniques used in it are not recent.

I do not believe that the methods presented in the present work are preexisting.

The hybrid Frontal-Delaunay surface meshing techniques described here, able to guarantee worst-case bounds on element quality and sizing conformance are, in my view, new. I am not aware of another algorithm with the same properties — able to produce smoothly varying Voronoi-Delaunay grids with very high mean element quality (similar to advancing front type schemes), while also guaranteeing worst-case bounds on element angles and conformance (a'la standard Delaunay-refinement techniques).

Existing methods for unstructured oceanic/atmospheric modelling appear to either lack provable worst-case bounds [Jacobsen et al., 2013], or generally produce grids with somewhat lower overall quality [Lambrechts et al., 2008].

The combination of the Frontal-Delaunay scheme with a coupled hill-climbing optimisation strategy to generate 'well-centred' grids is also, in my view, new. A number of additional remarks regarding the generation and benefits associated with 'well-centred' grids are already included in response to the reviewers first comment above. As per my response to this earlier comment, I believe that by making these arguments more clearly and earlier in the revised submission, the impact and novelty of the approaches pursued in the present work will be more strongly articulated.

6. There is not detail about the initialization on the sphere of the algorithm. You mention that the algorithm scans the triangles that do not verify given criteria, but how are these initial triangles created?

Agreed. I propose amending Section 2.5 in the revised manuscript to contain the following information:

12 points describing a coarse regular icosahedron are initially projected onto the spheroidal surface at the beginning of the refinement process. Refinement then proceeds according to the Frontal-Delaunay scheme outlined in Section 2.5 (i.e. until all constraints — element shape, size — are satisfied).

Please let me know if you have further suggestions or comments regarding the sub-

mission or my responses to your review.

Kind regards,

Darren Engwirda

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