S1 Supplementary Meterial

S1.1 Planetary Waves

The azonal component describes quasi-stationary planetary waves. The calculation depends on the level of height. At the equivalent barotropic level, azonal components of horizontal velocity are computed employing the definition of the stream function

$$\langle u_{EBL}^{*}(z)\rangle = -\nabla_{\phi}\langle\psi_{EBL}^{*}\rangle \tag{S1}$$

$$\langle v_{EBL}^{*}(z)\rangle = \nabla_{\lambda}\langle\psi_{EBL}^{*}\rangle \tag{S2 }$$

whereby the azonal component are computed assuming isothermal expansion of air parcels in planetary waves

$$\langle p_{z}^{*}\rangle = \langle p_{EBL}^{*}\rangle \exp[(z - z_{EBL})/H_0] + \frac{p_{*}g}{\Gamma R}\exp[-z/H_0]\left\{\ln\left[\frac{T(z)}{T(z_{EBL})}\right] - \ln\left[\overline{\frac{T(z)}{T(z_{EBL})}}\right]\right\} \tag{S3 }$$

and

$$\langle p_{EBL}^{*}\rangle = \rho\left\{\overline{\langle u_{EBL}\rangle}\right\}\nabla_{\phi}\langle\psi_{EBL}^{*}\rangle + 2\left(\frac{\overline{\langle u_{EBL}\rangle}}{a\cos(\phi)} + \Omega\right)\sin(\phi)\langle\psi_{EBL}^{*}\rangle \tag{S4 }$$

S1.1.1 Planetary Waves – orographically stream function:

For the waves excited by the orography, the stream function is calculated by

$$\beta\nabla_{\lambda}\langle\psi_{or,0,EBL}^{*}\rangle = -\frac{f}{H_0}\langle w_{or}\rangle + \frac{f^2}{g}\frac{\partial\langle u'v'\rangle^{*}}{\partial z} \tag{S5 }$$

where f is the Coriolis parameter and $\beta = \nabla_{\phi}f$ and

$$w_{or} = \langle u\rangle\nabla_{\phi}h_{or} + \langle v\rangle\nabla_{\lambda}h_{or} + a_{std}(\langle u\rangle^2 + \langle v\rangle^2 + \langle u'^2\rangle + \langle v'^2\rangle)^{1/2}h_{std}. \tag{S6}$$

The variable h_{or} describes the grid cell averaged orography height h_{std} the subgrid scale standard deviation of the height of mountains, and $a_{std}$is an additional tuning parameter.

The azonal component describes quasi-stationary planetary waves and is subdivided into a geostrophic and ageostrophic term:

$$u^* = u^*_{geos} + u^*_{ageos}$$

$$v^* = v^*_{geos} + v^*_{ageos}$$

S1.2 Derivation of the zonal mean meridional wind velocity

The zonal mean meridional wind velocity $\overline{\langle v(z,\phi)\rangle}$ which also accounts for convective contribution is calculated by

$$\overline{\langle v(z,\phi)\rangle} = \frac{d1 * (-2\tan(\phi)\left(\overline{\langle u^*v^*\rangle} + \overline{\langle u'v'\rangle}\right)) + d2 * (\frac{\partial}{\partial\phi}\left(\overline{\langle u^*v^*\rangle} + \overline{\langle u'v'\rangle}\right)) + d3 * ((-\frac{dK_z}{z} + \frac{K_z}{H_0})\frac{\partial\overline{\langle u\rangle}}{\partial z}a) + d4 * (A)}{n1 * (\tan(\phi)\overline{\langle u\rangle}) + n2 * \left(-\frac{\partial\overline{\langle u\rangle}}{\partial\phi}\right) + n3 * (2\Omega a\sin(\phi))} \tag{S7}$$

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With $K_z = 0.005\,z$ and In (4)

$$A = \left(\frac{P_{conv}\,L}{(\Gamma_a - \Gamma)} - \frac{1}{H_0}\right)\langle u_{s\_profile}\rangle$$

Whereby Γ is the lapse rate in the troposphere calculated by using the formula from Petoukhov (Petoukhov et al., 2000) , P_{conv} by the cloud module implemented by Eliseev at al. (Eliseev, n.d.) and

$$\langle u_{s\_profile}\rangle = \begin{cases} 2, & |\phi| > 40 \\ -2\ \cos\left(\phi\frac{\pi}{40°}\right), & \text{otherwise} \end{cases}$$

The additional calculating of $\langle u_{s\_profile}\rangle$ instead of using the calculated surface zonal velocity is done to avoid instabilities.

For the derivation we start with the differential equation of the zonal wind component

$$\frac{du}{dt} = \frac{\tan\phi}{a} uv + fv - \frac{1}{\rho}\Delta_\lambda p + F_u \tag{S9 }$$

Whereby a is the Earth radius, f is the Coriolis factor and F_u is the frictional force in u-direction. Multiplying the equation with ρ and using that $\rho\frac{du}{dt} = \frac{d(\rho u)}{dt} - u\frac{d\rho}{dt}, \frac{d(\rho u)}{dt} = \frac{\partial(\rho u)}{\partial t} + \boldsymbol{V}\cdot\boldsymbol{\Delta}\,(\rho u)$ and $\boldsymbol{V}\ \cdot\boldsymbol{\Delta}\,(\rho u) = \Delta\left(\rho u\boldsymbol{V}\right) - (\rho u)\boldsymbol{\Delta}\cdot\boldsymbol{V}$, we get

$$\frac{\partial(\rho u)}{\partial t} + \Delta\,(\rho u\boldsymbol{V}) - u\left(\frac{d\rho}{dt} + (\rho u)\boldsymbol{\Delta}\cdot\boldsymbol{V}\right) = \frac{\tan\phi}{a}\rho uv + f\rho v - \Delta_\lambda p + \rho F_u$$

With the continuity equation and using spherical coordinates, the equation simplifies to

$$\begin{aligned}\frac{\partial(\rho u)}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial(\rho u^2)}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial(\rho\cos\phi\, uv)}{\partial\phi} + \frac{\partial(\rho wu)}{\partial z} \\ = \frac{\tan\phi}{a}\rho uv + f\rho v - \frac{1}{a\cos\phi}\frac{\partial p}{\partial\lambda} + \rho F_u\end{aligned} \tag{S10 }$$

We calculate the zonal average $(\overline{\dots})$, take into account that $\frac{\overline{\partial x}}{\partial\lambda} = 0$ and assume a vertical dependence of the density $\left(\rho = \rho_0\,(z)\right)$:

$$\overline{\frac{\partial(\rho_0 u)}{\partial t}} + \frac{1}{a}\overline{\frac{\partial(\rho_0 uv)}{\partial\phi}} + \overline{\frac{\partial(\rho_0 wu)}{\partial z}} = 2\frac{\tan\phi}{a}\rho\overline{uv} + f\rho\bar{v} + \rho_0\bar{F_u}$$

We split the wind variables into an synoptic scale waves , planetary waves and zonal mean wind ($u = \bar{u} + u^* + u'$) . Under the assumption that $\bar{u}$ and v^* are independent, the result of the zonal mean over the azonal component is zero:

$$\begin{aligned}\overline{uv} &= \overline{\bar{u}\bar{v} + \bar{u}v^* + \bar{u}v' + u^*\bar{v} + u^*v^* + u^*v' + u'\bar{v} + u'v^* + u'v'} \\ &= \overline{\bar{u}\bar{v} + \bar{u}v' + u^*v^* + u'\bar{v} + u'v'}\end{aligned}$$

We average eq. (10) over time and phase speed ($\langle\dots\rangle$). By assuming independency of the variables, we can simplify the terms $\langle\bar{u}v'\rangle = \langle u'\bar{v}\rangle = 0$. In addition, it is $\overline{\bar{u}\bar{v}} = \bar{u}\bar{v}$ due to quasi stationarity of both terms. It is also $\langle\frac{dx}{dt}\rangle = 0$

and $\langle\bar{u}\bar{v}\rangle = \langle\bar{u}\rangle\langle\bar{v}\rangle$ since the oscillations of $\bar{u}$ and $\bar{v}$ are very small and independent of each other. By using the continuity equation $\frac{\rho_0}{a}\frac{\partial\langle\bar{v}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\rho_0\bar{v} + \frac{\partial(\rho_0\langle\bar{w}\rangle)}{\partial z} = 0$, we can simplify eq. (S10) to

$$\frac{1}{a}\rho_0\langle\bar{v}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial\phi} + \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \langle\overline{v'u'}\rangle\right)}{\partial\phi} + \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} + \frac{\partial\left(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle\right)}{\partial z} = \frac{\tan\phi}{a}\rho_0\langle\bar{u}\rangle\langle\bar{v}\rangle + 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) + f\rho\bar{v} + \rho_0\bar{F_u} \tag{S11}$$

With the assumption that $\rho_0 = e^{-z/H_0}$ and $\rho_0\bar{F_u} = \frac{\partial\,\bar{\tau}}{\partial z} = \frac{\partial}{\partial z}\left(\kappa\rho_0\frac{\partial\langle\bar{u}\rangle}{\partial z}\right) = \kappa\frac{\partial\rho_0}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\kappa\frac{\partial^2\langle\bar{u}\rangle}{\partial z^2} = -\kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$, we obtain

$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \langle\overline{v'u'}\rangle\right)}{\partial\phi} - \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial\left(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle\right)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \tag{S12}$$

The contribution to the vertical exchange of the atmospheric momentum from stationary eddies described in our case by zonally averaged $\langle\overline{w^*u^*}\rangle$ is shown negligibly small (Hantel and Hacker, 1978). Also, the scale analysis attests that Also, the scale analysis attests that $\langle\bar{w}\rangle\frac{\partial\langle u\rangle}{\partial z}$ are small:

$$-\rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial\left(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle\right)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \approx -\frac{\partial\left(\rho_0\langle\overline{w'u'}\rangle\right)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$$

Hence the eq. (S12) can be rewritten into

$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\,\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \overline{\langle v'u'\rangle}\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \langle\overline{v'u'}\rangle\right)}{\partial\phi} - \rho_0\langle\bar{w}\rangle\frac{\partial\langle\bar{u}\rangle}{\partial z} - \frac{\partial\left(\rho_0\langle\overline{w^*u^*}\rangle + \langle\overline{w'u'}\rangle\right)}{\partial z} - \kappa\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial\kappa}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z} \tag{S13}$$

With $\langle\overline{u'w'}\rangle = -\kappa'\frac{\partial\langle\bar{u}\rangle}{\partial z}$, whereby κ' is the coefficient of large-scale turbulent exchange for the momentum due to transient synoptic eddies (Williams and Davies, 1965), we get

$$\rho_0\langle\bar{v}\rangle\left(\frac{1}{a}\frac{\partial\langle\bar{u}\rangle}{\partial\phi} - \frac{\tan\phi}{a}\langle\bar{u}\rangle - f\right) = 2\frac{\tan\phi}{a}\left(\overline{\langle v^*u^*\rangle} + \langle\overline{v'u'}\rangle\right) - \frac{\rho_0}{a}\frac{\partial\left(\overline{\langle v^*u^*\rangle} + \langle\overline{v'u'}\rangle\right)}{\partial\phi} - (\kappa+\kappa')\frac{\rho_0}{H_0}\frac{\partial\langle\bar{u}\rangle}{\partial z} + \rho_0\frac{\partial(\kappa+\kappa')}{\partial z}\frac{\partial\langle\bar{u}\rangle}{\partial z}$$

With $\tilde{\kappa} = \kappa + \kappa'$ we can simplify the equation to

$$\overline{\langle v(z,\phi)\rangle} = \frac{-2\tan(\phi)\left(\overline{\langle u^{*}v^{*}\rangle} + \overline{\langle u'v'\rangle}\right) + \frac{\partial}{\partial\phi}\left(\overline{\langle u^{*}v^{*}\rangle} + \overline{\langle u'v'\rangle}\right) + \left(-\frac{dK_z}{z} + \frac{K_z}{H_0}\right)\frac{\partial\overline{\langle u\rangle}}{\partial z}a}{\tan(\phi)\overline{\langle u\rangle} - \frac{\partial\overline{\langle u\rangle}}{\partial\phi} + 2\Omega a\sin(\phi)} \qquad (\text{S13})$$

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