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Interactive comment on "Total energy and potential enstrophy conserving schemes for the shallow water equations using Hamiltonian methods: Derivation and Properties (Part 1)" *by* Christopher Eldred and David Randall

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In this paper, the authors introduce some new aspects of C-grid and Z-grid schemes for the rotating shallow water equations on polygonal grids. The paper starts each section by collecting together quite a few bits of mathematical structure and previous results about C-grid and Z-grid schemes. These can be found elsewhere but it is nice to see them collected together in this context.

For C-grid schemes, they introduce a new methodology for obtaining schemes that simultaneously conserve energy and enstrophy on arbitrary orthogonal grids, a previ-

ously unsolved problem. The authors take the approach of writing out the (possibly over- or under-determined) system of equations that form the constraints, applying the Thuburn et al (2009) decoupling formula and solving the resulting system numerically. There is no proof of solvability for this system but the authors report that a unique solution is obtained in their numerical tests. For Z-grid schemes, the authors extend the Nambu bracket approach to arbitrary orthogonal grids, obtaining energy-enstrophy conserving schemes by construction.

As described in the introduction, this paper is Part I of a series of 3 papers with Part II containing numerical tests and Part III containing linear dispersion analysis. Part I concentrates on exposition of the methods and discussion of their properties. Given the focus of GMD on documenting and describing models and model software, I think that Part I requires:

(a) some evidence that the schemes are practically useful, i.e. that they do not do anything obviously weird. Having conservation properties is a good sign, but I have built plenty of schemes before that have good conservation properties but still lead to terrible numerics. If Part II promises to be a detailed comparison and analysis of results then Part I should at least have some provisional examples that show that things are working as expected.

(b) some evidence that the code provided is a correct implementation of the algorithms described.

(c) for GMD, I would expect some discussion as to how the numerical methods were implemented and expressed as code. This is not the same as code documentation, but should describe the main data structures used and how they form an efficient implementation.

It may actually be that a merge of Parts I and II makes sense, or some partial repetition between Parts I and II to achieve (a) and (b). Perhaps the solid rotation test plus something else where we can easily check that things are working, like the mountain after the usual 12 days (or however long it is).

A few other remarks:

(1) It’s bad form for a referee to ask for a reference to their own paper so feel free to ignore, but you might like to mention that the problem of simultaneously conserving enstrophy and energy on arbitrary grids was solved in the context of compatible finite element methods in McRae, Andrew TT, and Colin J. Cotter. "EnergyâĂŘand enstrophyâĂŘconserving schemes for the shallowâĂŘwater equations, based on mimetic finite elements." Quarterly Journal of the Royal Meteorological Society 140.684 (2014): 2223-2234.

(2) The paper suddenly jumps in with 1-forms, 2-forms, Hodge stars etc without any warning to the reader! You should at least provide some references and a bit of a guide to what is going on, and maybe consider whether the language of differential forms is really necessary for this paper in terms of accessibility to a more general numerics audience.

(3) I would like to see some more description of how Equation (61) decouples the problem and how big the resulting uncoupled systems are. Why does it take so long to solve these systems? Why can’t they be analysed to check if there is a unique solution?

(4) Please can you do a consistency test e.g. on the sphere for the Q operator? That is, take an analytic formula for u,h, interpolate to the grid and apply the Q operator, then analytically compute Q and interpolate to the grid, and compare errors in the L2 norm. I’d be especially interested in the cubed sphere case, where we observed lack of consistency for the Coriolis operator in our non-orthogonal scheme.

(5) If I’m thinking about this correctly, then the Q operator should imply a Coriolis reconstruction operator for the linear equations. Is this operator consistent in the limit as dx->0 on e.g. a cubed sphere mesh?

(6) There is no mention of timestepping anywhere. What do you do about timestepping in the code? How do time series of energy and enstrophy look?

(7) What is the relationship of the Z-grid scheme to Heikes et al (2013)? Is it a straightforward extension of the same formulae to arbitrary grids or is another idea needed?

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