Reply to Maarten Krol:

This is a well-prepared manuscript with a good focus. It is ideally suited for GMD, since it describes the development of an off-line model suitable for inversions of GHG emissions. The focus is on errors due to the time resolution of the meteorological driver data, and on the general validation of the adjoint code. The validation sites are well chosen.

We are grateful for you time to review our paper and for giving us fruitful comments and suggestions. Our replies to the comments are described below with line numbers/pages of the supplementary manuscript. The modifications we made are colored in red in the supplementary manuscript.

There is only one major comment that I would like to make. The numerical errors due to the use of a flux-limiter and due to low temporal resolution of the driver meteo are typically in the order of ~1 ppm for CO2 (and mostly smaller). It is difficult to place these numbers in a proper context. I advise the authors to provide this context by (i) report typical RMSD differences between different models, e.g. the TRANSCOM ensemble (ii) provide a typical "error" in the NICAM-TM that can be obtained by running in a different temporal or spatial resolution, or by other means (e.g. sampling error)

We agree in that the numerical errors are quite small in the most cases and it was difficult to place those in a proper context. According to your comment, we have added some discussions. For the suggestion (i), a typical error in NICAM-TM, we have performed an additional on-line simulation in which different wind data (JRA-55) are used for the nudging and calculated the RMSDs between the two on-line models (JCDAS versus JRA-55). Furthermore, we have added a table (Table 3) to show temporal correlation coefficients of synoptic variations between the models and the observation [Page 9, line 25-28]. Comparing with the model error (JCDAS versus JRA-55), this also shows implications for the magnitudes of the data thinning errors (corresponding descriptions are from Page 9, line 32 to Page 10, line 4). For the suggestion (ii), typical difference between different models, we have

cited Patra et al. (2008), which shows a correlation coefficient (not RMSD) range among the TransCom models [Page 10, line 4-6].

Accordingly, we have changed the word "underestimation" in several places [Page 9, line 2-3; Page 10, line 12-13; Page 13, line 13]. Furthermore, we have added further descriptions about the A6V6C6 performance at Karasevoe as below:

"*This lower $CO_2$ of A6V6C6 does not necessarily cancel out the positive deviations of the on-line model from the observation (Fig. 4b) and hence is not closer to the observation than the on-line model. In fact, the correlation coefficient of the synoptic variation reduces from the on-line model to A6V6C6 (from 0.610 to 0.579), whose magnitude is relatively large compared to the other changes (Table 3).*" [Page 10, line 14-17]

We agree in that a 20-year-long assimilation is very difficult. Here, we wanted to mention the need for an analysis on interannual variations (IAVs), which does not necessarily require one 20-year-long assimilation window. Consecutive several-months-long assimilation windows could be one way to estimate GHG IAV fluxes, but it still requires a number of forward/adjoint simulations. To clarify this, we have modified the text as follows:

"*Moreover, a global inversion calculation of an atmospheric greenhouse gas requires a long time analysis (~20 years; e.g. Chevallier et al., 2010) to figure out*

*interannual variations of surface fluxes, resulting in at least hundreds of years of model simulations in total.*" [Page 2, line 13-15]

Page 3, line 6: "The discrete adjoint is linear but reduces the accuracy of the numerical scheme, while the continuous adjoint is non-linear but maintains the numerical accuracy." The fact that numerical wiggles as "fixed" does not necessarily mean that the numerical accuracy is higher, because it implies also numerical diffusion. See also page 6, lines 14-15.

We agree in that a flux limiter suppresses numerical wiggles and does not improve the model accuracy in general. Accordingly, we have modified the text as follows:

"*the continuous adjoint is non-linear but maintains the numerical accuracy*"

=>

"*… maintains the monotonicity*" [Page 3, line 8-9]

and

"*the discrete adjoint produces negative (or oscillatory) sensitivities and reduces the model accuracy to some extent.*"

=>

"*… (or oscillatory) sensitivities.*" [Page 6, line 22-23]

Nevertheless, we think the flux limiter we use could improve the model accuracy, as shown in Miura (2007) (we have added a description in Page 6, line 15-16). In fact, we obtained larger correlation coefficients against the observations when the flux limiter was used, though they are minute changes (corresponding descriptions are added in Page 10, line 21-23).

Page 7, line 18: 7 minutes: please specify which configuration (frequency of meteo input, I assume A6V6C6)

We used A3V1C3 for this calculation. However, the computational time does not depend on the temporal resolution of the input meteorological data.

Descriptions are added in:

[Page 7, line 31 – Page 8, line 2]

[Page 13, line 7]

Please also note the supplement to this comment: http://www.geosci-model-dev-discuss.net/gmd-2016-231/gmd-2016-231-RC1- supplement.pdf

We very appreciate your many suggestions. We have modified the text according to all the suggestions (colored in red in the supplementary manuscript).

**Reference:**
Miura, H.: An Upwind-Biased Conservative Advection Scheme for Spherical Hexagonal–Pentagonal Grids, Mon Weather Rev, 135, 4038–4044, doi:10.1175/2007MWR2101.1, 2007.

Reply to Arne Babenhauserheide

The manuscript describes the implementation of the offline forward and adjoint transport for the NICAM-TM 4D-Var system with two different approaches: A linear forward mode with an exact adjoint and a non-linear forward mode with flux limiter using an approximate adjoint.

The manuscript is well writen, relevant and well-suited for GMD. The validation methods are well-chosen and the tradeoffs in both modes are explained.

We are grateful for you time to review our paper and for giving us fruitful comments and suggestions. Our replies to the comments are described below with line numbers/pages of the supplementary manuscript. The modifications we made are colored in red in the supplementary manuscript.

As a minor revision I would suggest adding a comparison between the magnitude of the error introduced by data-thinning and the error introduced by using the approximated adjoint (with the non-linear forward equations) on page 10, subsection 3.4 (validation of the adjoint model) to complement the estimate in the companion paper (Niwa et al., 2016). By eye Figure 6 seems to show deviations with a magnitudes up to 10%.

Thank you for your suggestion. We contemplated whether to add the comparison between the data-thinning error and the non-linear effects as suggested. However, we have decided not to add that comparison, for simplicity. In fact, the non-linear effect can be translated to that of the flux limiter, which is already compared with the data-thinning errors in the previous sections (3.2 and 3.3). Furthermore, in the inversion experiment of the accompanying paper, we used the same temporal resolution of the input meteorological data for the forward and adjoint simulations. Therefore, the data-thinning error does not affect the forward-adjoint relationship.

Improving the reproducibility would require providing the source code of the described model versions (offline and online) and the required data which is not yet

common practice for weather models and would likely face institutional hurdles.

At this stage, the source code of NICAM(-TM) cannot be available through a web-page, but we can provide it upon request as stated in Code availability. We are happy to share our model code and the dataset with researchers who are interested.

Line-notes:

• page 2 line 13: Chevallier et al. (2010) used 21 years of data but note that at this scale "modeling and representativeness errors exceed the measurement errors by an order of magnitude", therefore the reference does not support the argument. Babenhauserheide et al. (2015) show that a 10 to 15 weeks assimilation window suffices to represent remote fluxes, i.e. in the antarctic (sorry for the self-reference here).

We agree in that a 20-year-long assimilation is very difficult. Here, we wanted to mention the need for an analysis on interannual variations (IAVs), which does not necessarily require one 20-year-long assimilation window. Consecutive several-months-long assimilation windows could be one way to estimate GHG IAV fluxes, but it still requires a number of forward/adjoint simulations. To clarify this, we have modified the text as

"*Moreover, a global inversion calculation of an atmospheric greenhouse gas requires a long time analysis (~20 years; e.g. Chevallier et al., 2010) to figure out interannual variations of surface fluxes, resulting in at least hundreds of years of model simulations in total.*" [Page 2, line 13-15]

- page 2 line 14: resulting **in** at least . . .

- page 2 line 15: of making **the** computation . . .

- page 2 line 28: loses → **losses**

- page 3 line 34: This avoids the pole problem **inherent in latitude-longitude**

> **grids** and . . .

We have modified the text according to the above comments.

[Page 2, line 15], [Page 2, line 16], [Page 2, line 29], [Page 4, line 5]

- page 3 line 34: simulations. **Therefore** ← seems not to follow from the previous sentence. Maybe **The** instead?

We have modified the text as "*Owing to the feasibility of high-resolution simulations, the dynamical core...*", because the high-resolution and the non-hydrostatic are linked. [Page 4, line 6]

- page 4 line11:"240km...comparable or finer than previous inversion studies" CarbonTracker CTE2016-FT used in the Global Carbon Budget 2016 (Le Quéré et al., 2016) uses 1x1 degree resolution over Europe and North America1, which is approximately 100x100km resolution.

Thank you for your information. We have modified the text and added Le Quéré et al. (2016) in the reference. [Page 4, line 17-18]

- page 5 line 20: can be easily shown → can **easily be shown**

- page 6 line 16: second approach **a** continuous

- page 6 line 19: is no longer inexact → **exact**

- page 6 line 23: detail derivation → **a detailed** derivation . . .

- page 6 line 30: to readily make **create** the adjoint model, but it sometimes makes **this carries the risk of making** the model . . .

- page 12 line 13: coefficient, while → coefficient **and**

Thank you for the above corrections. Accordingly, we have modified the text.

[Page 5, line 27], [Page 6, line 24], [Page 6, line 27], [Page 6, line 31], [Page 7, line 7], [Page 13, line 15]

# A 4D-Var inversion system based on the icosahedral grid model (NICAM-TM 4D-Var v1.0): 1. Off-line forward and adjoint transport models

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**Abstract.** A 4-dimensional variational (4D-Var) method is a popular algorithm for inverting atmospheric greenhouse gas (GHG) measurements. In order to meet the computationally intense 4D-Var iterative calculation, off-line forward and adjoint transport models are developed based on the Nonhydorstatic ICosahedral Atmospheric Model (NICAM). By introducing flexibility into the temporal resolution of the input meteorological data, the forward model developed in this study is not only computationally efficient but is also found to nearly match the transport performance of the on-line model. In a transport simulation of atmospheric carbon dioxide ($CO_2$), the data-thinning error (error resulting from reduction in the time resolution of the meteorological data used to drive the off-line transport model) is minimized by employing high temporal resolution data of the vertical diffusion coefficient; with a ~~lower~~ low 6-hourly temporal resolution, significant concentration biases near the surface are introduced. The new adjoint model can be run in discrete or continuous adjoint mode for the advection process. The discrete adjoint is characterized by perfect adjoint relationship with the forward model that switches off the flux limiter, while the continuous adjoint is characterized by an imperfect but reasonable adjoint relationship with its corresponding forward model. In the latter case, both the forward and adjoint models use the flux limiter to ensure the monotonicity of tracer concentrations and sensitivities. Trajectory analysis for high-$CO_2$ concentration events are performed to test adjoint sensitivities; we also demonstrate the potential usefulness of our adjoint model for diagnosing tracer transport. Both the off-line forward and adjoint models have computational efficiency about ten times ~~more than that of~~ higher than the on-line model. A description of our new 4D-Var system that includes an optimization method, along with its application in an atmospheric $CO_2$ inversion and the effects of using either the discrete or continuous adjoint method, is presented in an accompanying paper (Niwa et al., 2016).

# 1 Introduction

We have developed a new 4-dimensional variational (4D-Var) inversion system for estimating surface fluxes of greenhouse gases (GHGs: present primarily targets are carbon dioxide ($CO_2$) and methane ($CH_4$)). The new system is referred to as NICAM-TM 4D-Var. It consists mainly of forward and adjoint transport models and an optimization scheme. This paper presents derivation of the transport models and evaluate their performances. The accompanying paper (Niwa et al., 2016) describes the optimization scheme and demonstrates the application of the new system to an atmospheric $CO_2$ inversion problem.

The 4D-Var inversion method has evolved over the years to achieve higher spatiotemporal resolution in inverse calculations of various atmospheric trace gas measurements (Rödenbeck, 2005; Chevallier et al., 2005; Baker et al., 2006; Meirink et al., 2008; Basu et al., 2013; Wilson et al., 2014; Liu et al., 2014) that include continuous measurements at the surface, as well as aircraft (Machida et al., 2008; Sawa et al., 2015; Matsueda et al., 2015) and satellite observations (e.g. Yokota et al., 2009). The 4D-Var method is an iterative method requiring model simulations many times, not only forward but also backward using an adjoint model. Moreover, a global inversion calculation of an atmospheric greenhouse gas requires a long time analysis window ($\sim$20 years; e.g. Chevallier et al., 2010) owing to the gas' long atmospheric lifetime to figure out interannual variations of surface fluxes, resulting in at least hundreds of years of model simulations in total. This motivates us to develop ways of making the computations more efficient.

For GHG simulations, there are two types of atmospheric transport models: one is on-line (e.g. Patra et al., 2009) and the other is off-line (e.g. Kawa et al., 2004; Krol et al., 2005). On-line models include atmospheric general circulation models (AGCMs) which incorporate passive tracers of GHGs and simulate their movements. Off-line models are those that simulate transport of tracer gases using archived meteorological data (e.g. temperature, wind velocity, humidity). Therefore, an off-line model is computationally much more efficient than an on-line model and hence is favored for the 4D-Var calculation. However, archived meteorological data usually consist of reanalysis data with limited spatiotemporal resolution. Furthermore, temporal snapshots of reanalysis data are not physically consistent with each other (Stohl et al., 2004). Therefore, in off-line transport calculation, reanalysis wind data should be modified in advance to restore the dynamical consistency with pressure tendencies, otherwise the tracer mass cannot be conserved (Heimann and Keeling, 1989; Heimann, 1995; Bregman et al., 2003).

An adjoint model integrates variables backward in time to calculate sensitivities of a certain scalar variable against model parameters (Errico, 1997), with applications to data assimilation and inversion analyses. Furthermore, the adjoint sensitivity could be is a powerful tool to diagnose tracer transport mechanisms (e.g. Vukićević and Hess, 2000). For GHG inverse analyses, the atmospheric processes are considered to be all linear, with $CO_2$ and $CH_4$ transported as passive tracers and $CH_4$ losses calculated by simple linear equations with prescribed hydroxyl (OH) and chlorine (Cl) radicals and $O(^1D)$ (Patra et al., 2011). But in practice, non-linearity is introduced into the discretized model, hampering the development of an adjoint model which complicates adjoint model formulation. One prominent example is the discretization of an advection scheme. An advection scheme with higher than first order accuracy must employ a non-linear algorithm to preserve tracer monotonicity (Godunov, 1959). Therefore, an advection scheme often uses a non-linear flux limiter or fixer that depends on tracer quantities, introducing

non-linearity and discontinuity. However, the direct adjoint of such a non-linear code is computationally inefficient for a long simulation, because it requires several checkpoints from which time forward simulations are restarted to restore tracer quantities at every model time step. Furthermore, it has been found that such an adjoint model is ill-behaved due to ~~the discontinuity~~ discontinuities (Thuburn and Haine, 2001; Vukićević et al., 2001). Therefore, alternative approaches have been proposed at
5   the expense of linearity or the accuracy of numerical scheme (Vukićević and Hess, 2000; Vukićević et al., 2001; Sandu et al., 2005; Hourdin and Talagrand, 2006; Hakami et al., 2007; Gou and Sandu, 2011; Haines et al., 2014). Most studies use either the "discrete adjoint" or "continuous adjoint". However, which approach performs better is still controversial. The discrete adjoint is linear but reduces the accuracy of the numerical scheme, while the continuous adjoint is non-linear but maintains the ~~numerical accuracy~~ monotonicity.

10     In this study, we have achieved a level of computational efficiency to conduct a 4D-Var inversion of atmospheric GHGs using off-line forward and adjoint models. The off-line model is closely linked to the AGCM of Nonhydrostatic ICosahedral Atmospheric Model (NICAM: Tomita and Satoh, 2004; Satoh et al., 2008, 2014). In fact, the model can be considered as an off-line version of the on-line transport model of NICAM-based Transport Model (NICAM-TM: Niwa et al., 2011a, b). In the off-line model, tracer transport is calculated in the same way as in the on-line model, but driven by meteorological data
15   provided from the AGCM run of NICAM in which winds fields are nudged toward reanalysis data. Compared to the reanalysis data, the physical and dynamical consistency in the nudged AGCM data is maintained. Furthermore, the use of the AGCM enables us to change the spatiotemporal resolution of the meteorological input data. Similar AGCM-based off-line models have been developed by previous studies (Hourdin et al., 2006; Yumimoto and Takemura, 2013). In fact, the off-line NICAM-TM has already been used in a $CO_2$ inversion studies using the conventional matrix calculation method (Niwa et al., 2012). In
20   this study, we examine the relative impact of the meteorological driver data with different temporal resolutions in each of the transport processes (advection, vertical diffusion and cumulus convection) on model accuracies. Maintaining the same degree of flexibility in the time resolution of the off-line forward model, we develop a new adjoint model. The new adjoint model can be run in the discrete or continuous mode. In order to achieve the exact adjoint relationship with its corresponding forward model, the discrete adjoint method switches off the non-linear flux limiter in the advection scheme, while the continuous adjoint
25   utilizes the flux limiter to give preference to monotonicity over the adjoint exactness.

    Because thinning (i.e., reducing time resolution, resulting in a decreased number of data points) of the meteorological data might introduce some additional model errors in the off-line calculation, we evaluate those errors by comparing $CO_2$ concentrations simulated by the off-line model with those by the one-line model. In that evaluation, we test various temporal resolutions of the meteorological data, which are separately determined for each transport process. Also, we validate ~~the~~
30   ~~fundamental property~~ fundamental properties of the adjoint model and demonstrate the utility of the adjoint sensitivity in a back-trajectory analysis.

## 2 Methods

### 2.1 NICAM

The horizontal grid of NICAM has a distinctive structure. Different from the conventional latitude-longitude grid models, it has a quasi-homogenous grid distribution produced from an icosahedron obtained by a recursive division method (Stuhne and Peltier, 1996). This avoids the pole problem inherent in latitude-longitude grids and facilitates global high-resolution simulations. ~~Therefore~~ Owing to the feasibility of high-resolution simulations, the dynamical core of NICAM is constructed with nonhydrostatic equations (Tomita and Satoh, 2004). Furthermore, the model program is designed for an efficient parallel computation with Message Passing Interface (MPI) libraries (Tomita et al., 2008; Kodama et al., 2014). In fact, NICAM has been used for global nonhydrostatic high-resolution simulations with 14 km to 850 m grid resolutions (Miura et al., 2007; Miyamoto et al., 2013; Miyakawa et al., 2014). Nonetheless, in this study, we use a moderate resolution to reduce the high computational cost associated with the GHG inversion that requires repeated long-term simulations.

We set the horizontal resolution at "glevel-5" (Fig. 1). The number 5 denotes the number of division of the icosahedron. NICAM adopts the finite volume method (Tomita and Satoh, 2004), whose control volume is a shaped pentagon at twelve vertices of the original icosahedron and hexagon at other grids (Fig. 1b). Those control volumes are constructed by connecting mass centers of the triangular elements that are produced by the recursive division of the icosahedron (Fig. 1a). The mean grid interval of glevel-5 is approximately 240 km. Although this horizontal resolution is much coarser than the high-resolutions that NICAM mainly targets, it is still comparable to ~~or finer than~~ the resolutions used in previous GHG inversion studies (e.g. Peylin et al., 2013; Le Quéré et al., 2016).

Because the dynamical core is constructed with the finite volume method, NICAM achieves the consistency with continuity (CWC: Gross et al., 2002) for the tracer transport (Satoh et al., 2008; Niwa et al., 2011b), which cannot be achieved in spectral AGCMs (Jöckel et al., 2001). Owing to this CWC property, tracer mass is perfectly conserved without any numerical mass fixer. Indeed, motivated by the CWC property, atmospheric transport studies have been conducted using NICAM-TM with glevel-5. The model reproduces reasonably well the synoptic scale and vertical variations of radon ($^{222}$Rn) and the inter-hemispheric gradients of sulfur hexafluoride ($SF_6$) at the surface and in the upper troposphere (Niwa et al., 2011b, 2012).

The model configuration in this study is basically the same as the one described in Niwa et al. (2012), except for the cumulus parameterization. The cumulus parameterization scheme is changed from Arakawa and Schubert (1974) to Chikira and Sugiyama (2010). The number of vertical model layers is 40, 12 layers of which exist below about 3 km. The top of the model domain is at about 45 km. The tracer advection process is calculated with the scheme of Miura (2007), and the vertical turbulent mixing is calculated with the MYNN Level 2 scheme (Mellor and Yamada, 1974; Nakanishi and Niino, 2004; Noda et al., 2010). The model time step of glevel-5 is 20 min., both for the on-line and off-line calculations. For the nudging used in the on-line calculation, we use the 6-hourly horizontal wind velocities of the Japan Meteorological Agency Climate Data Assimilation System (JCDAS) reanalysis data (Onogi et al., 2007).

## 2.2 Off-line NICAM-TM

As is the case in the on-line model, the off-line model integrates tracer mass $\rho q$ ($\rho$ is air mass density and $q$ is tracer mixing ratio) as

$$\frac{\partial \rho q}{\partial t} = \nabla \cdot (\rho \mathbf{v} q) + \frac{\partial}{\partial z}\left[\rho K_v \left(\frac{\partial q}{\partial z}\right)\right] + f_c(\rho, q_w, T, M_B, q), \tag{1}$$

where $\nabla$ and $\mathbf{v}$ are the three-dimensional divergence operator and wind vector, respectively, and $K_v$ is the vertical diffusion coefficient. On the right hand side of the equation, the first and second terms represent the grid scale tendency of advection and the sub-grid scale one of vertical diffusion, respectively. The third term $f_c$ denotes the sub-grid scale tendency of cumulus convection, determined by $\rho$ and the mixing ratios of water substances ($q_w$), temperature ($T$), and cumulus base mass flux ($M_B$).

Table 1 shows the archived meteorological parameters that drive the off-line model. Integrative time resolutions of these parameters are thinned out (i.e., reduced) from the model time step interval of 20 min. to several hours. In this study, we conduct sensitivity of the model results to changes in the time resolution of each of the driving meteorological transport variables (advection, vertical diffusion and cumulus convection) (Section 3.2).

In the archiving of the meteorological data, averaged values are saved for the air mass flux $\mathbf{V}$ ($= \rho \mathbf{v}$), while instantaneous values are saved for other meteorological parameters. The averaging of $\mathbf{V}$ is intended to preserve the CWC property. Originally, NICAM calculates the tracer advection using time-averaged air mass fluxes that are derived from air mass fluxes at a shorter time interval. The tracer advection is calculated with the Euler scheme, while momentums are calculated at a shorter time interval using the Runge-Kutta scheme. The time-averaged air mass flux retains the CWC property (Satoh et al., 2008; Niwa et al., 2011b). The off-line model uses air mass fluxes that are further averaged for the thinning interval as

$$\overline{\mathbf{V}}^t = \frac{1}{N}\sum_{i=1}^{N}\mathbf{V}^{t+i\Delta\tau}, \tag{2}$$

where $\Delta t$ is the thinning interval and $\Delta\tau$ is the model time step, and $N$ is the integer defined as $N = \Delta t/\Delta\tau$. The off-line calculation, whose time step is the same as that of the on-line, uses the above repeatedly during $N$ steps from $t$ to $t + \Delta t$ as

$$(\rho q)^{t+(i+1)\Delta\tau} = (\rho q)^{t+i\Delta\tau} + \Delta\tau\left(\nabla \cdot \overline{\mathbf{V}}^t q^{t+i\Delta\tau}\right). \tag{3}$$

In order to preserve CWC, $\rho$ is simultaneously integrated with the same time-averaged air mass flux as

$$\rho^{t+(i+1)\Delta\tau} = \rho^{t+i\Delta\tau} + \Delta\tau\left(\nabla \cdot \overline{\mathbf{V}}^t\right) + \alpha, \tag{4}$$

where $\alpha$ is the modification term. If $\alpha = 0$, the Lagrangian conservation ($dq/dt = \partial q/\partial t + \mathbf{v} \cdot \nabla q = 0$) is achieved, which can easily be shown by substituting (4) to (3). In practice, $\alpha$ is nonzero and the Lagrangian conservation is not strictly satisfied owing to evaporation and precipitation (note that $\rho$ includes not only dry air but also water substances). This $\alpha$ is calculated as

$$\alpha = \frac{1}{N}\left(\rho^{t+\Delta t} - \rho^t - \Delta t\left(\nabla \cdot \overline{\mathbf{V}}^t\right)\right), \tag{5}$$

so that the integrated $\rho$ with Eq. (4) after $N$ steps coincides with $\rho^{t+\Delta t}$ that is provided from the on-line calculation. The other meteorological parameters ($K_v$, $q_w$, $T$ and $M_B$) are linearly interpolated from the thinned interval steps to the model time steps.

## 2.3  Adjoint NICAM-TM

5  When $\mathbf{M}$ represents a forward model matrix and $\mathbf{a}$ and $\mathbf{b}$ are arbitrary vectors, an adjoint model matrix $\mathbf{M}^*$ satisfies $\langle \mathbf{a}, \mathbf{Mb} \rangle = \langle \mathbf{M}^* \mathbf{a}, \mathbf{b} \rangle$, where $\langle .,. \rangle$ is an inner product. In the usual case, the inner product is defined as $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^{\mathrm{T}} \mathbf{b}$; therefore, $\mathbf{M}^*$ is equivalent to $\mathbf{M}^{\mathrm{T}}$. In practice, $\mathbf{b}$ represents mixing ratio or surface flux and $\mathbf{a}$ is its adjoint variable. An adjoint model integrates adjoint variables backward in time to calculate sensitivities.

An adjoint model is constructed based on the above off-line forward model. The adjoint model reads the archived meteo-
10  rological data in the same way as the off-line model does, but in reverse. Furthermore, similar to Eq. (4), $\rho$ is simultaneously integrated with the reversed winds and $-\alpha$ in place of $\alpha$.

For the vertical diffusion and cumulus convection processes, we use the discrete adjoint approach in which linear program codes are ~~simply~~ transposed. For the advection process, we employ both the discrete and continuous approaches. In the discrete adjoint approach, we give up the monotonicity. In NICAM, the tracer monotonicity is achieved by the use of the flux limiter
15  of Thuburn (1996) (Miura, 2007; Niwa et al., 2011b). In fact, this flux limiter improves the model accuracy to some extent (Miura, 2007). All the transport calculations other than the flux limiter are linear. Therefore, we obtain a completely linear forward model by just switching off the flux limiter. From that linear forward model, we construct the adjoint model by ~~simply~~ transposing the linear codes. Because of the linearity, this adjoint model is expected to have the exact adjoint relationship with the forward model (with the flux limiter off), as proven in Section 3.4. The relationship is expressed as

20  $$ (\mathbf{Mx})^{\mathrm{T}} \mathbf{y} = \mathbf{x}^{\mathrm{T}} \left( \mathbf{M}^{\mathrm{T}} \mathbf{y} \right), \tag{6} $$

where $\mathbf{x}$ and $\mathbf{y}$ represent the model input parameter vector and the observation vector, respectively. By giving up the monotonicity, however, the discrete adjoint produces negative (or oscillatory) sensitivities ~~and reduces the model accuracy to some extent~~.

In the second approach, a continuous adjoint model is developed by discretizing the continuous adjoint equation (Sandu
25  et al., 2005; Hakami et al., 2007; Gou and Sandu, 2011). In this approach, the flux limiter can be employed not only in the forward model, but also in the adjoint model, keeping the tracer concentrations or sensitivities positive (or non-oscillatory). However, due to the non-linearity of the flux limiter, the adjoint relationship is no longer exact. The continuous adjoint equation of advection is written as

$$ -\frac{\partial q^*}{\partial t} = \nabla \cdot \left( \mathbf{v} q^* \right), \tag{7} $$

30  where $q^*$ is the adjoint variable for $q$. Equation (7) can be derived by the Lagrange's method of undetermined multipliers and partial integral from the advection part of Eq. (1) (a detailed derivation can be found in Sandu et al. (2005)). Let $\tilde{q}^* = q^*/\rho$,

and we obtain

$$-\frac{\partial \rho \tilde{q}^*}{\partial t} = \nabla \cdot (\rho \mathbf{v} \tilde{q}^*). \tag{8}$$

By comparing with Eq. (1), we find that we can reuse the divergence operator of the forward code by reversing the wind direction and integrating it backward in time. Thus, we can employ the non-linear flux limiter to maintain the monotonicity of $\tilde{q}^*$.

All the adjoint codes are manually written, achieving numerical efficiency of the model. Some studies use an automatic differentiation tool to readily ~~make~~ create the adjoint model, but ~~it sometimes makes~~ this carries the risk of making the model numerically inefficient. Furthermore, we retain the parallel computational ability of NICAM, allowing significant savings in computational time.

## 2.4  CO$_2$ flux data

For the validation of the off-line forward model, we simulate atmospheric CO$_2$ for the year 2010. For the surface boundary CO$_2$ flux input to the model, we use the inversion flux of Niwa et al. (2012) that is optimized for atmospheric CO$_2$ concentrations for 2006–2008. The inversion (posterior) flux consists of prior flux datasets and monthly flux adjustments derived from the observations. In this study, we replace the prior flux datasets with those for 2010 other than the climatological air-sea exchange data from Takahashi et al. (2009); we use the monthly data of fossil-fuel emission from the Carbon Dioxide Information Analysis Center (CDIAC) (Andres et al., 2013), of biomass burning emission from the Global Fire Emissions Database ver. 3.1 (van der Werf et al., 2010) and of terrestrial biosphere net ecosystem production (NEP) from the Carnegie-Ames-Stanford Approach (CASA) model (Randerson et al., 1997). To represent the diurnal variation of the terrestrial biosphere flux, we redistribute the monthly CASA NEP into 3-hourly fluxes by the same method as Olsen and Randerson (2004) using 2 m height air temperature and downward shortwave radiation data of JCDAS for 2010. Although the integrated surface CO$_2$ flux input to the model does not necessarily represent the actual flux variations in 2010, the overall resulting atmospheric CO$_2$ concentration field is consistent with the actual observed CO$_2$ concentrations, permitting an effective evaluation of the model transport performance. The initial concentration field is also ~~made by~~ constructed by running the model with the inversion flux ~~through the simulation~~ for 2003–2009.

## 3  Results

### 3.1  Computational cost

All the simulations are performed on PRIMEHPC FX100 with MPI parallelization by 10 nodes (each node has 32 cores). For the one-year-long sensitivity test simulation discussed below, the off-line forward model requires only 7 min., while the on-line model requires about 70 min. Therefore, the off-line model is 10 times faster computationally than the on-line model. ~~With the corresponding adjoint calculation requiring about additional 7 min.,~~ The corresponding adjoint calculation also requires 7 min., therefore the 4D-Var calculation is demonstrated to be reasonably feasible. These computational costs are evaluated

using the highest temporal resolution of the input meteorological data in the following sensitivity runs (A3V1C3, see below). However, we found that the computational costs are not significantly affected by the data thinning interval.

## 3.2 Evaluation of the data thinning error

As described in Section 2.2, the off-line model can use a different data-thinning interval for each transport process. In order to determine an appropriate data-thinning interval, we perform five sensitivity runs (A6V6C6, A3V6C6, A3V6C3, A3V3C3, A3V1C3), changing the interval from 6 hours to 1 hour, as shown in Table 2. In addition, we test A3V1C3 with the flux limiter in the advection scheme switched off (which is the counterpart of the discrete adjoint).

Figure 2 shows zonal mean pressure-latitude cross-section of the root-mean-square difference (RMSD) in $CO_2$ concentration between the off-line and on-line models. The RMSD value is calculated from hourly model output. The RMSD value represents the error induced only by the data-thinning. Generally, the RMSD values are small even in the coarsest resolution case in which all the transport processes are calculated with 6-hourly data (A6V6C6). In most areas, the RMSDs are less than 1 ppm, indicating that the atmospheric transport is generally well simulated by the 6-hourly resolution. The relative error, which is defined as RMSD divided by the standard deviation of the concentration variation simulated by the on-line model, is 12.9 and 5.3 % on average at the surface and 300 hPa, respectively (Table 2).

A closer examination shows that the temporal resolution of each transport process affects the spatial distribution of RMSD. As shown in Figs. 2a and b, halving the interval of the advection data from A6V6C6 to A3V6C6 does not significantly reduce RMSDs, with the relative errors at the surface and 300 hPa decreasing slightly to 12.7 (from 12.9) and 4.8 (from 5.3) %, respectively. However, the RMSDs values are noticeably reduced in the mid to upper troposphere by halving the interval of the cumulus convection data from A3V6C6 to A3V6C3, with the relative error in 300 hPa reduced to 3.3 %. This indicates a significant role of cumulus convection on $CO_2$ concentration variations in the mid to upper troposphere. Furthermore, increasing the temporal resolution of the vertical diffusion coefficient from A3V6C3 to A3V3C3, and to A3V1C3, we find greater RMSD reductions near the surface (Figs. 2d and e). The relative errors at the surface are reduced to 6.3 and 2.6 % for A3V3C3 and A3V1C3, respectively. This is attributable to the fact that vertical diffusion has a much higher temporal ~~frequency~~ variability than the other transport processes, especially near the surface.

When the flux limiter is switched off in the A3V1C3 case, RMSDs are increased globally (Fig. 2f). The region where the RMSD has most pronouncedly increased is the stratosphere. This is probably because the flux limiter no longer suppresses the numerical oscillation near the top of the model domain, which is much larger than the $CO_2$ concentration variations in the stratosphere. But in the troposphere, the numerical oscillations are not so large compared to the $CO_2$ concentration variations. Consequently, the relative error is 7.5 % at the surface, which is larger than A3V1C3 but less than the 6-hourly vertical diffusion cases (A6V6C6, A3V6C6, A3V6C3), and 9.0 % at 300 hPa, which is the highest number in all the sensitivity tests (Table 2).

Figure 3 shows the annual mean difference of $CO_2$ concentration at the surface between the off-line and one-line models, for A6V6C6, A3V3C3, A3V1C3 and A3V1C3 without the flux limiter. In fact, these differences represent biases from the on-line calculation induced by the data-thinning. Figure 3a shows that even the lowest temporal resolution of 6-hourly data input (A6V6C6) reproduces ~~well~~ the $CO_2$ concentrations over the oceans well, where the bias is quite small (< 0.2 ppm). Meanwhile

over the terrestrial areas, we see significantly larger biases. Specifically, they are more than 4 ppm over the tropical regions of Amazon and Africa; these values are all negative due to ~~the systematic underestimation of $CO_2$ accumulation during nighttime~~ systematically smaller nighttime accumulation, i.e. larger mixing, than the on-line model. Furthermore, since the results of A3V6C6 and A3V6C3 are very similar to that of A6V6C6, the resolution of the vertical diffusion coefficient data is a major factor contributing to the data-thinning error, particularly over the terrestrial biosphere in the summertime when strong diurnal variations exist. These biases are reduced but still larger than 1 ppm even by halving the temporal interval (A3V3C3; Fig. 3b). However, by increasing the temporal resolution of the vertical diffusion coefficient data to hourly, the biases become nearly indiscernible (A3V1C3; Fig. 3c). Since we still use the moderate resolution of 3-hourly time step for the advection and cumulus convection processes, the necessary disk storage of A3V1C3 for 1 year is not ~~significantly~~ extremely large (approximately 50 GB, after partly performing a 2-byte data compression). Therefore, we set the A3V1C3 configuration to be the default for the glevel-5 simulations. When the flux limiter is switched off in A3V1C3, the bias increases slightly over the terrestrial areas but remains mostly less than 1 ppm (Fig. 3d). This bias is relatively small compared to the RMSD shown in Fig. 2f. Therefore, A3V1C3 without the flux limiter would be permissible, only if the focus is on the concentration in the troposphere. This model configuration should be used when the model linearity is stringently required, such as in the use with the discrete adjoint.

## 3.3 Comparison with observations

In order to assess the magnitude of the off-line model error, we compare the simulated $CO_2$ concentrations with the observed measurements at Minamitorishima, located in the western North Pacific (Wada et al., 2011), and at Karasevoe, located in West Siberia (Sasakawa et al., 2010), as well as with the continuous aircraft CONTRAIL (Comprehensive Observation Network for Trace gases by Airliner: Machida et al., 2008) measurements obtained at 8-10 km altitude over Narita, Japan (each observation location is shown in Fig. 3a), representing marine background, continental, and upper-troposphere conditions, respectively.

Figure 4 shows the observed and simulated $CO_2$ concentrations at each site. Generally, the on-line model reproduces ~~relatively well~~ the observed $CO_2$ concentration variations relatively well, partly due to the inversion flux we use. In the inversion, we used the Minamitorishima and CONTRAIL data to constrain the terrestrial biosphere and ocean fluxes ~~optimization~~ (Niwa et al., 2012). But the high reproducibility of the synoptic variations indicates a reasonable transport performance of NICAM-TM, given the fact that we used monthly mean observations in the inversion. Table 3 shows correlation coefficients of the synoptic variations between the simulated and observed concentrations at the three sites, all of which are found statistically significant. Here, the synoptic variations are defined by residual $CO_2$ concentrations from a smoothed curve represented by a linear trend and three harmonics, similarly to Patra et al. (2008).

At Minamitorishima and over Narita, the RMSD value between the observation and the model is quite small; 0.92 and 1.36 ppm~~at Minamitorishima and over Narita~~, respectively. ~~Meanwhile the RMSD at Karasevoe, whose observation is independent of the inversion, is 5.72 ppm.~~ Compared to those RMSDs, the RMSD between the off-line and on-line models is negligibly small, even for the lowest resolution of A6V6C6 ~~except at Karasevoe~~ (Figs. 4a and c). Furthermore, changes in the correlation coefficients of the synoptic variations are also quite small (Table 3). These negligible influences of the data thinning are accentuated by comparing with an additional on-line simulation in which different wind data from the Japanese 55-year

Reanalysis (JRA-55: Kobayashi et al., 2015; Harada et al., 2016), instead of JCDAS, are used for the nudging. The RMSD values between the two on-line models (JCDAS versus JRA-55) are 0.22 and 0.40 ppm, respectively for Minamitorishima and over Narita, which are larger than the RMSDs between the on-line and off-line models. Also, the correlation coefficient change from JCDAS to JRA-55 is larger than the changes from the on-line to the off-line (Table 3). In fact, these correlation

5 coefficients would change more distinctly by a different model, given the fact that Patra et al. (2008) showed a large range of the correlation coefficients ($\sim$ 0.4–0.7) for Minamitorishima among multiple models.

However, for A6V6C6 at the continental site Karasevoe, we found a significant influence of the data thinning. Here, the RMSD between the observation and the on-line model is 5.72 ppm, probably due to the fact that the observation is independent of the inversion and consequently the flux data have a large error for this area. ~~At Karasevoe,~~ Comparably, the meteorological

10 resolution of A6V6C6 results in ~~the~~ an RMSD value of 2.93 ppm. As shown in Fig. 4b, the off-line model produces lower $CO_2$ values compared to those produced by the on-line model during the summer. ~~, whose magnitude is comparable to the difference between the on-line model and the observation.~~ As stated earlier, this is likely due to ~~the underestimation of $CO_2$ accumulation during the nighttime~~ the systematically smaller nighttime accumulation and is the cause of the negative bias against the on-line model shown in Fig. 3a. This lower $CO_2$ of A6V6C6 does not necessarily cancel out the positive deviations

15 of the on-line model from the observation (Fig. 4b) and hence is not closer to the observation than the on-line model. In fact, the correlation coefficient of the synoptic variations reduces from the on-line model to A6V6C6 (from 0.610 to 0.579), whose magnitude is relatively large compared to the other changes (Table 3). By increasing the temporal resolution of the vertical diffusion coefficient to hourly (A3V1C3), we obtain a sufficiently small RMSD value of 0.24 ppm compared to the on-line model.

20 ~~Even~~ Without the flux limiter, the RMSD~~s~~ ~~is~~ are modestly small (at most 0.86 ppm for Karasevoe) and the difference does not have any distinct positive or negative tendency (Fig. 4~~b~~). Meanwhile, the correlation coefficients are reduced by switching off the flux limiter coherently at the three sites (Table 3). Although they are all minute changes, it suggests an improvement of the model accuracy by the flux limiter.

### 3.4 Validation of the adjoint model

25 We now validate the exactitude of the adjoint model using the reciprocity property with its corresponding forward models. Detail description of the reciprocity property can be found in the literature (Hourdin and Talagrand, 2006; Hourdin et al., 2006; Haines et al., 2014; Wilson et al., 2014). In Eq. (6), if $\mathbf{x}$ and $\mathbf{y}$ are the basis unit vectors having 1 for $i$th and $j$th elements respectively and 0 for all the others (i.e. $\mathbf{x} = (0 \cdots 0, 1, 0 \cdots 0)^{\mathrm{T}}$ and $\mathbf{y} = (0 \cdots 0, 1, 0 \cdots 0)^{\mathrm{T}}$), a value sampled at $j$, which is simulated from $\mathbf{x}$ with the forward model $((\mathbf{Mx})^{\mathrm{T}}\mathbf{y} = (\mathbf{Mx})_j = M_{j,i})$, should coincide with the value simulated from

30 ~~y~~ $\mathbf{y}$ with the adjoint model and subsequently sampled at ~~$i$ ($x^T(MTy)=(MTy)i=Mj,i$)~~ $i$ $(\mathbf{x}^{\mathrm{T}}(\mathbf{M}^{\mathrm{T}}\mathbf{y}) = (\mathbf{M}^{\mathrm{T}}\mathbf{y})_i = M_{j,i})$. Checking this reciprocity, we can verify the exactitude of the adjoint ~~method~~ code. To evaluate the reciprocity for both the discrete and continuous adjoint models, we use the forward model without and with the flux limiter, respectively. The former forward/adjoint model set is linear but does not ensure monotonicity, while the latter set is nonlinear but ensures monotonicity. Both in the forward and adjoint model simulations, we use the configuration of A3V1C3 for the meteorological input.

For a case study, we examine an Asian outflow event, which is a typical transport phenomenon in East Asia during the winter–spring season (e.g. Sawa et al., 2007). We prescribe a surface flux at the model grid (X) located on the coast of East Asia representing the basis unit vector **x** (its location is denoted by the cyan open circle in Fig. 5b). Meanwhile, we prepare 160 observational basis unit vectors (**y**), whose sampling points are regularly located at 3 km altitude over an area enclosed by 14–32°N and 111–159°E (denoted by cyan dots in Fig. 5a). The simulation period is 7 days starting on 1 January 2010. The flux is time invariant; that is, the vector **x** is a function of space only. Figure 5a shows the concentration field at the end of the period, as simulated by the forward model with the flux limiter. On the other hand, Fig. 5b shows the sensitivities of the observation Y that is located at the eastern edge of the range (denoted by the cyan triangle in Fig. 5a) against the surface fluxes (i.e. footprint). This sensitivity is calculated by the continuous adjoint. We find that, using the discrete adjoint, the calculated footprint pattern is quite similar to that shown in Fig. 5b; this is not surprising since the forward simulation without the flux limiter does not introduce substantial errors in the troposphere, as previously shown. As the concentration field shown Fig. 5a is simulated from the unit flux, it also represents the degree of spatial sensitivity between the flux and the observation. According to Eq. (6), the concentration value sampled at the observation point Y should coincide with the footprint value located at the flux point X. Performing adjoint simulations for the remaining 159 observation points, we can evaluate the overall reciprocity of the adjoint model.

Figure 6 shows a scatter diagram of the 160 pairs of forward concentration values at the observation points with their corresponding adjoint footprint values at the flux point X. It can be seen in the figure that the footprint values simulated by the discrete adjoint show a near complete one-to-one correspondence with the corresponding concentration values to within the computer machine accuracy. This demonstrates that the discrete adjoint has the exact reciprocity against the forward model without the flux limiter. On the other hand, the continuous adjoint does not have the exact reciprocity but it is reasonably approximated. Figure 6 also demonstrates that the continuous adjoint successfully avoids negative sensitivities because of the flux limiter, while the discrete adjoint calculation does produce negative sensitivities.

### 3.5 Adjoint trajectory analysis

Finally, we apply the adjoint sensitivities to a transport trajectory analysis. Generally, the adjoint model provides sensitivities of a specified scalar value with respect to concentrations and surface fluxes (Appendix A). When the scalar value is set to an observed concentration, the cost functional is defined a

$$J = \int\limits_{t}^{t_o} \int\limits_{\Omega} g(q) d\Omega dt', \tag{9}$$

$$g(q) = q(x,t')\delta(x-x_o)\delta(t'-t_o), \tag{10}$$

where $\delta$ is the delta function and $x_o$ and $t_o$ represent the observed location and time respectively; therefore, the value of the cost functional corresponds to the observed concentration $q(x_o, t_o)$. According to Appendix A, the change of the cost functional is

given by

$$\Delta J = \Delta q(x_o, t_o) = \int\limits_t^{t_o} \int\limits_\Omega \left( \mathcal{F}^* q^*(x, t') \right) \Delta s(x, t') d\omega dt' + \int\limits_\Omega q^*(x, t) \Delta q(x, t) d\omega, \tag{11}$$

where $\mathcal{F}^*$ represents the adjoint of the transferring operator from flux to concentration, and $\Delta s$ denotes the flux perturbation. $\mathcal{F}^* q^*$ and $q^*$ denote the sensitivities of the observed concentration with respect to the surface flux and concentration, respectively. If considered processes are all linear (which is the case in this study since we consider only atmospheric transport), we can omit $\Delta$. Then, the first and second terms represent respectively the actual contributions of the surface flux from $t$ to $t_o$ and the concentration at $t$ to the observed concentration. For our analysis, we investigate the spatial structures of these sensitivity quantities normalized by $q(x_o, t_o)$, i.e. $\int_t^{t_o} \left( \mathcal{F}^* q^*(x, t') \right) s(x, t') dt'/q(x_o, t_o)$ and $q^*(x, t) q(x, t)/q(x_o, t_o)$. These quantities derived by the adjoint model have been utilized in previous studies for diagnosing trace gas transport in the atmosphere (Vukićević and Hess, 2000; Hess and Vukicevic, 2003) and a pathway of a ~~waterer~~ water mass in the ocean (Fujii et al., 2013).

Using such adjoint-derived quantities, we analyze three high $CO_2$ concentration events; they were observed at Minamitorishima on 24 January, at Karasevoe on 27 December, and at 8 km over Narita on 12 January in 2010 (denoted by the cyan arrows in Fig. 4). These high-concentration events are chosen ~~for relatively good model reproducibility~~ because NICAM-TM captures this event well (see Fig. 4). Figure 7 shows the normalized flux contribution and "adjoint trajectory volumes" (Hess and Vukicevic, 2003) against each event. The adjoint trajectory volume is derived by averaging $q^*(x, t) q(x, t)/q(x_o, t_o)$ for each day previous to the observation time. Overlaying the averaged $q^*(x, t) q(x, t)/q(x_o, t_o)$ shows the pathway of the air mass that caused each of the high $CO_2$ events at the observation location and time. This analysis approach resembles the one taken by Hess and Vukicevic (2003). The forward simulation that calculates $q(x, t)$ and the adjoint simulation that calculates $q^*(x, t)$ and $\mathcal{F}^* q^*(x, t)$ are performed for one week period previous to each observation time. Here, we show the results calculated by the continuous adjoint model, taking the advantage of its monotonicity property. However, the discrete adjoint model ~~also~~ produces similar sensitivity features (not shown).

Interestingly, the analysis indicates that three high concentration events were produced ~~differently from each other~~ by three distinctly different transport phenomena. The flux contribution shows that the event observed at Minamitorishima originated from the Korean Peninsula and eastern China (Fig. 7a). Furthermore, the sharp adjoint trajectory volume indicates that the transport of the high $CO_2$ plume was characterized by slow diffusion. For the event observed at Karasevoe, the analysis indicates that the air mass that produced the high $CO_2$ concentration was advected from the west, but fluxes in the vicinity of the observation site also contributed to the observed concentration (Fig. 7b). This local flux contribution is a result of a very ~~thin~~ shallow mixed layer, as indicated by the vertical structure of the trajectory volume that is concentrated below 1km. Figure 7c shows that the high concentration event observed over Narita originated from southeast China. The adjoint trajectory volume indicates that the horizontal transport of the air mass from China to Japan was fast (taking only about 2 days) compared to the other cases because of the strong westerlies in the free troposphere. Before this fast eastward transport, the analysis also indicates the possibility of an air mass propagation westward along the slope of the topography. Therefore, this result suggests

that the topographical uplifting may ~~have~~ play a significant role in high $CO_2$ concentration events frequently observed over Narita (Shirai et al., 2012).

## 4  Conclusions

We have developed forward and adjoint models based on NICAM-TM, as part of the 4D-Var system for atmospheric GHGs inversions. Both of these models are off-line. Therefore, the models are computationally efficient enough to make the 4D-Var iterative calculation feasible. The computational cost of the off-line forward model is about ten times less than that of the corresponding on-line model calculation, which is irrespective of the temporal resolution of the input meteorological data. Furthermore, the adjoint model computational cost is nearly the same as that of the forward model.

The archived meteorological data used in the forward and adjoint models are prepared by the on-line AGCM calculation of NICAM in advance. In this study, we have developed the capability of variable temporal resolution of individual meteorological transport data, to minimize the off-line model errors owing to the data thinning. Through sensitivity tests using $CO_2$ as a tracer, we have determined that the temporal resolution of the vertical diffusion coefficient should be high, otherwise a significantly large systematic bias is introduced near the surface due to the ~~underestimation of~~ smaller $CO_2$ accumulation during the nighttime. For the spatial resolution used in this study (the horizontal grid interval is approximately 240 km), the use of 1-hour interval for the vertical diffusion coefficient ~~, while~~ and 3-hour interval for the ~~others~~ other meteorological fields (A3V1C3), is enough to simulate $CO_2$ concentrations that are reasonably consistent with ~~those produced by~~ the on-line calculation. By comparing with ~~the~~ observations, we have found that the error from the data thinning in A3V1C3 is negligible compared with the intrinsic model performance. In a case without using the flux limiter, we have found ~~significantly large~~ significant errors in the stratosphere, while the errors in the troposphere were ~~less so~~ smaller and tolerable. Therefore, simulations without the flux limiter can be carried out in studies focused only on the troposphere.

For the adjoint model, we have explored the relative impact of using discrete adjoint or continuous adjoint on the advective transport process. Using an Asian outflow case, we have demonstrated perfect adjoint relationship of the discrete adjoint with its corresponding forward model in which the flux limiter is turned off. In the same analysis, the continuous adjoint has also shown reasonable adjoint exactitude against the forward model with the flux limiter turned on. Furthermore, we have found that the adjoint model can be used in attribution studies in which surface flux contributions are diagnosed as a function of air mass pathway when interpreting observed high $CO_2$ concentration events.

Based on the results of this study, we have developed a new 4D-Var system for performing $CO_2$ inversions. Application of the 4D-Var system and its results are described in the accompanying paper by Niwa et al. (2016). In the accompanying paper, the A3V1C3 configuration is used to judge which of the adjoint calculation methods, discrete or continuous, is better ~~relative to the quality of produced inversion results~~ suited for global inversion studies.

The icosahedral grid model such as NICAM is a new type of model and is becoming popular in dynamical meteorology research fields as remarkable innovations in supercomputers are made. However, there are still only a few studies of its applications in atmospheric chemistry and inversion/assimilation calculations (e.g. Elbern et al., 2010). One prominent feature of

the NICAM-TM 4D-Var system is the perfect mass conservation, as described in Section 2.1. Another advantage of the system is its computational efficiency when applied to linear GHG inversion problems. If we limit the analysis period to a short time, a global high-resolution inversion would be feasible as long as sufficient data storage capacity is available. Furthermore, regional high-resolution inversions would also be possible with the grid stretching technique (Tomita, 2008; Goto et al., 2015). It is

5    expected that the system developed in this study and in the accompanying paper ~~would~~ can exploit new observations and opens up new avenues for GHG inversions.

## Code availability

Development of NICAM-TM is being continued by the authors. The source codes of NICAM-TM are available for those who are interested. The source codes of NICAM-TM are included in the package of the parent model NICAM, which can be

10    obtained upon request under the general terms and conditions (http://nicam.jp/hiki/?Research+Collaborations).

## Appendix A:  Adjoint sensitivity

Here we explain the theory of the adjoint sensitivity following the description of Vukićević and Hess (2000). The tracer transport equation can be written in the following compact form of

$$\mathcal{L}q = \mathcal{F}s, \tag{A1}$$

15    where $\mathcal{L}$ is the transport operator,

$$\mathcal{L} \equiv \frac{\partial}{\partial t} + \mathcal{A} + \mathcal{V} + \mathcal{C}. \tag{A2}$$

The operator $\mathcal{F}$ is transferring surface flux $s$ to concentration, and the operators for advection, vertical diffusion and cumulus convection are represented as $\mathcal{A}$, $\mathcal{V}$, and $\mathcal{C}$, respectively. Then, the adjoint operator $\mathcal{L}^*$ is defined as

$$\mathcal{L}^* \equiv -\frac{\partial}{\partial t} + (\mathcal{A}^* + \mathcal{V}^* + \mathcal{C}^*), \tag{A3}$$

20    where $\mathcal{A}^*$, $\mathcal{V}^*$, and $\mathcal{C}^*$ are the adjoint operators for $\mathcal{A}$, $\mathcal{V}$, and $\mathcal{C}$.

We characterize the solution of the tracer transport equation (A1) by the cost functional $J$ as

$$J = \int_{t_1}^{t_2} \int_{\Omega} g(q) d\omega dt, \tag{A4}$$

where $[t_1, t_2]$ is the time interval examined, $\Omega$ is the spatial domain, $d\omega$ is the area differential, and $g(q)$ is the diagnostic operator of $q$. Then the change by the perturbation of $q$ is described as

25    $$\Delta J = \int_{t_1}^{t_2} \int_{\Omega} \frac{\partial g}{\partial q} \Delta q \, d\omega dt. \tag{A5}$$

The adjoint operator $\mathcal{L}^*$ defines an adjoint equation for the adjoint variable $q^*$ as

$$\mathcal{L}^* q^* = \frac{\partial g}{\partial q}. \tag{A6}$$

Multiplying Eq. (A6) by $\Delta q$, and integrating over time and over the spatial domain, we obtain

$$\Delta J = \int_{t_1}^{t_2} \int_{\Omega} (\mathcal{L}^* q^*) \Delta q \, d\omega dt. \tag{A7}$$

5   By integration by parts, Eq. (A7) transforms to

$$\Delta J = \int_{t_1}^{t_2} \int_{\Omega} q^* (\mathcal{L} \Delta q) \, d\omega dt - \int_{\Omega} [q^* \Delta q]_{t_1}^{t_2} \, d\omega - \int_{t_1}^{t_2} [q^* \Delta q]_{O(\omega)} \, dt, \tag{A8}$$

where $O(\omega)$ denotes the boundary of the integration domain. Using Eq. (A1) and the adjoint operator $\mathcal{F}^*$ for $\mathcal{F}$, and specifying $q^* = 0$ at $t = t_2$ and $O(\omega)$ (no inflow/outflow of $q^*$ at the boundary), we finally obtain

$$\Delta J = \int_{t_1}^{t_2} \int_{\Omega} (\mathcal{F}^* q^*(x,t)) \Delta s(x,t) d\omega dt - \int_{\Omega} q^*(x,t_1) \Delta q(x,t_1) d\omega. \tag{A9}$$

10   In Eq. (A9), $\mathcal{F}^* q^*(x,t)$ and $q^*(x,t_1)$ represent the sensitivities of $\Delta J$ against the flux change ($\Delta s(x,t)$) and the change of the concentration at $t_1$ ($\Delta q(x,t_1)$), which are generally called adjoint sensitivities. If the cost functional $J$ represents an observed quantity, then $\int_{t_1}^{t_2} \mathcal{F}^* q^*(x,t) dt$ expresses its footprint as shown in Fig. 5b.

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20   by using post K computer of the FLAGSHIP2020 Project. The calculations of this study were performed on the super computer system, FUJITSU PRIMEHPC FX100, of Meteorological Research Institute.

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**Table 1.** Meteorological parameters used for the off-line forward and adjoint models

| Parameter | Symbol | Time type | Related process |
|---|---|---|---|
| Air mass density | $\rho$ | Snapshot | Advection |
| Air mass flux | $\mathbf{V} (= \rho \mathbf{v})$ | Averaged | Advection |
| Vertical diffusion coefficient | $K_v$ | Snapshot | Vertical diffusion |
| Water substances | $q_w$ | Snapshot | Cumulus convection |
| Temperature | $T$ | Snapshot | Cumulus convection |
| Cumulus base mass flux | $M_B$ | Snapshot | Cumulus convection |

**Table 2.** Temporal intervals for advection, vertical diffusion, and cumulus convection processes in each sensitivity test and relative errors globally averaged at the surface and 300 hPa. The relative error is calculated at each model grid by dividing RMSD by the standard deviation of the $CO_2$ concentration variation simulated by the on-line model for 2010.

| Notation | Temporal interval [hour] | | | Relative error [%] | |
|---|---|---|---|---|---|
| | Advection | Vertical diffusion | Cumulus convection | Surface | 300 hPa |
| A6V6C6 | 6 | 6 | 6 | 12.9 | 5.3 |
| A3V6C6 | 3 | 6 | 6 | 12.7 | 4.8 |
| A3V6C3 | 3 | 6 | 3 | 12.6 | 3.3 |
| A3V3C3 | 3 | 3 | 3 | 6.3 | 2.2 |
| A3V1C3 | 3 | 1 | 3 | 2.6 | 2.0 |
| A3V1C3 w/o flux limiter | 3 | 1 | 3 | 7.5 | 9.0 |

**Table 3.** Temporal correlation coefficients of simulated residual $CO_2$ concentrations (see the text for details) with the observations at Minamitorishima, Karasevoe, and 8-10 km altitude over Narita for the on-line and off-line (A6V6C6 and A3V1C3) simulations. The results of A3V1C3 without the flux limiter and the on-line model nudged by JRA-55 are also shown.

| Site | on-line | A6V6C6 | A3V1C3 | A3V1C3 w/o flux limiter | on-line nudged by JRA-55 |
|---|---|---|---|---|---|
| Minamitorishima | 0.577 | 0.572 | 0.575 | 0.573 | 0.587 |
| Karasevoe | 0.610 | 0.579 | 0.613 | 0.607 | 0.609 |
| Narita 8-10 km | 0.323 | 0.318 | 0.324 | 0.311 | 0.302 |
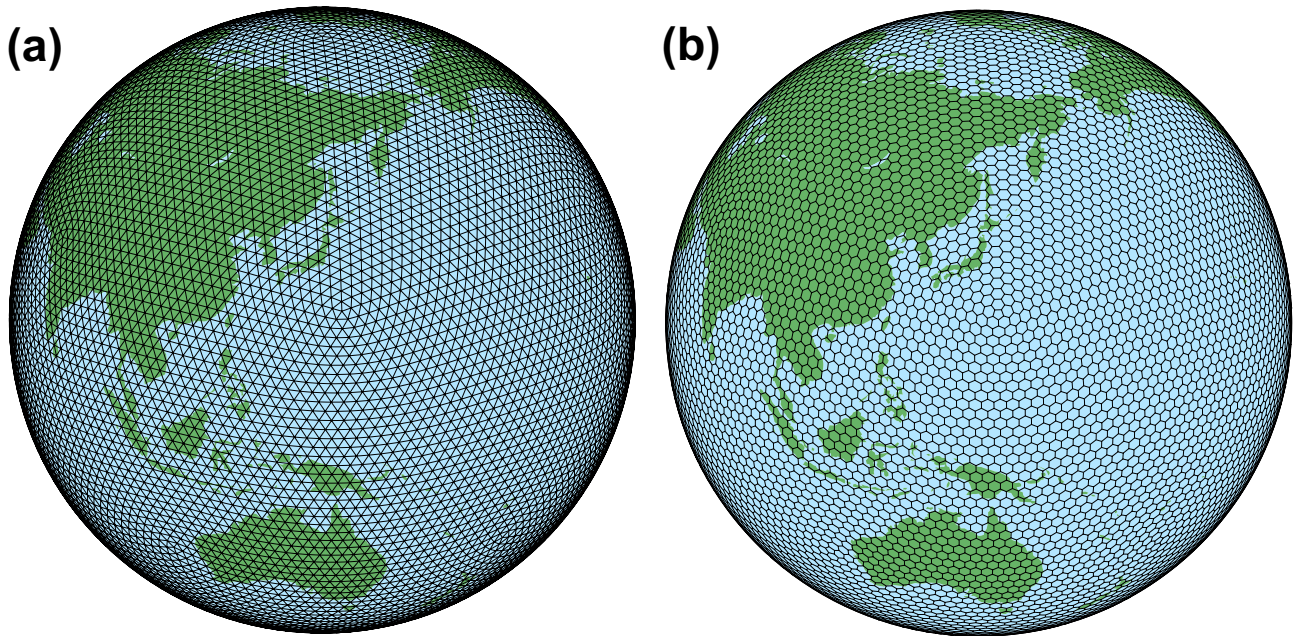
**Figure 1.** The grid distribution of NICAM glevel-5. Triangular elements produced from an icosahedron by five-times division (a) and control volumes constructed by connecting the mass centers of the triangular elements (b).
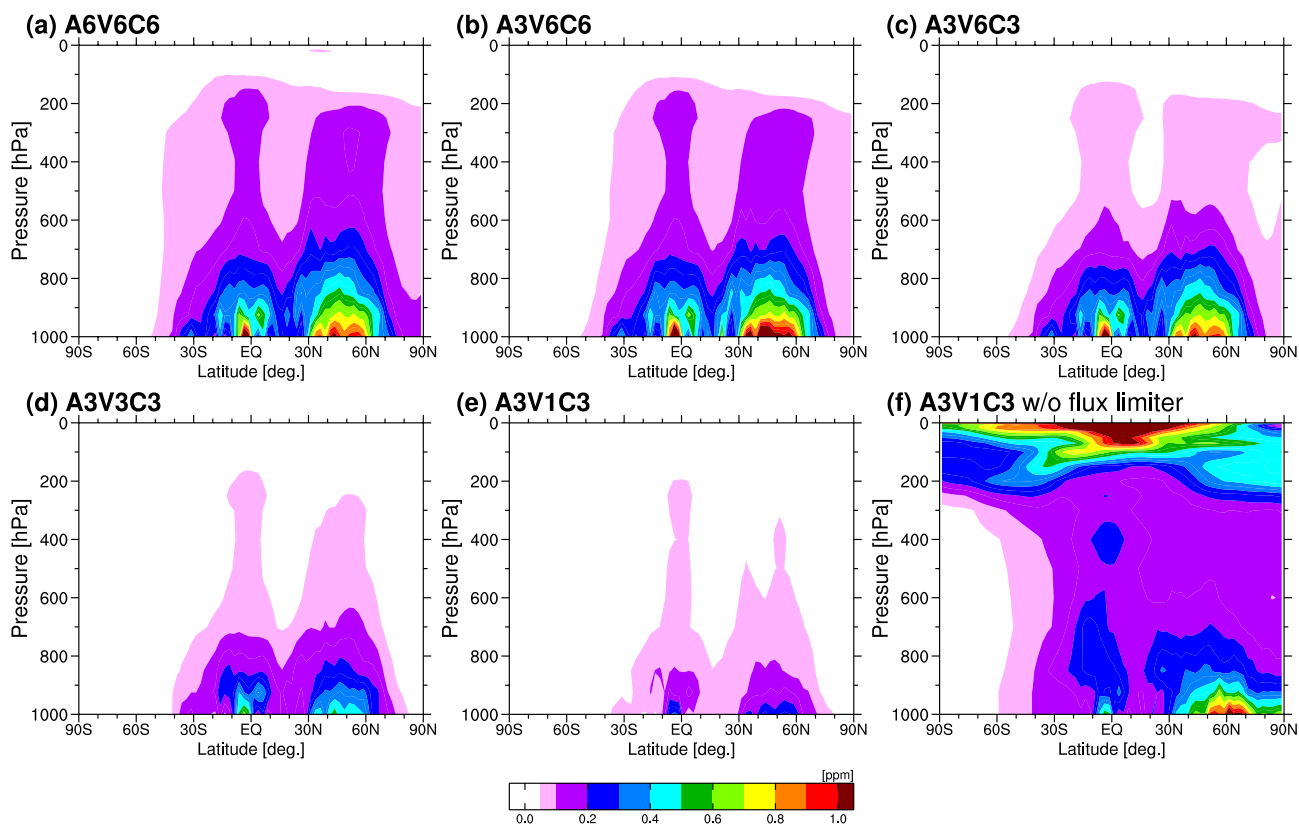
**Figure 2.** Zonal-mean latitude-pressure cross-section of annual root-mean-square difference (RMSD) of $CO_2$ concentration simulated by the off-line model against the on-line model. Time interval of the meteorological driver data is changed for each transport process as shown in Table 2 (a-e) and the same time resolutions are used as (e) but with the flux limiter switched off (f).
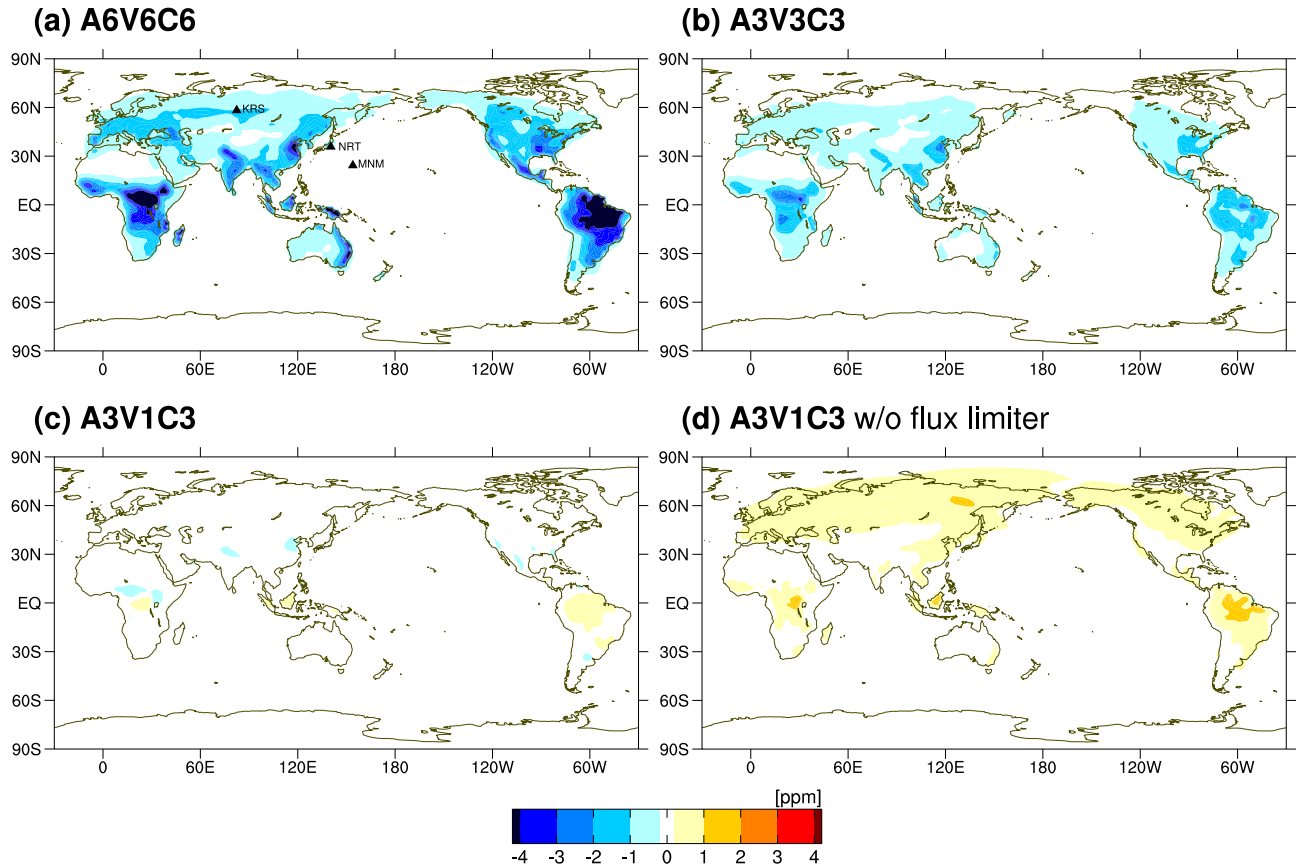
**Figure 3.** Annual mean difference of $CO_2$ concentration at the surface model layer between the off-line and on-line models (off-line minus on-line) for each sensitivity test: A6V6C6 (a), A3V3C3 (b), A3V1C3 (c) and A3V1C3 without the flux limiter (d). White colored areas signify absolute values less than 0.2 ppm. The geographical locations of Minamitorishima (MNM), Karasevoe (KRS), and Narita (NRT) are also indicated in (a).
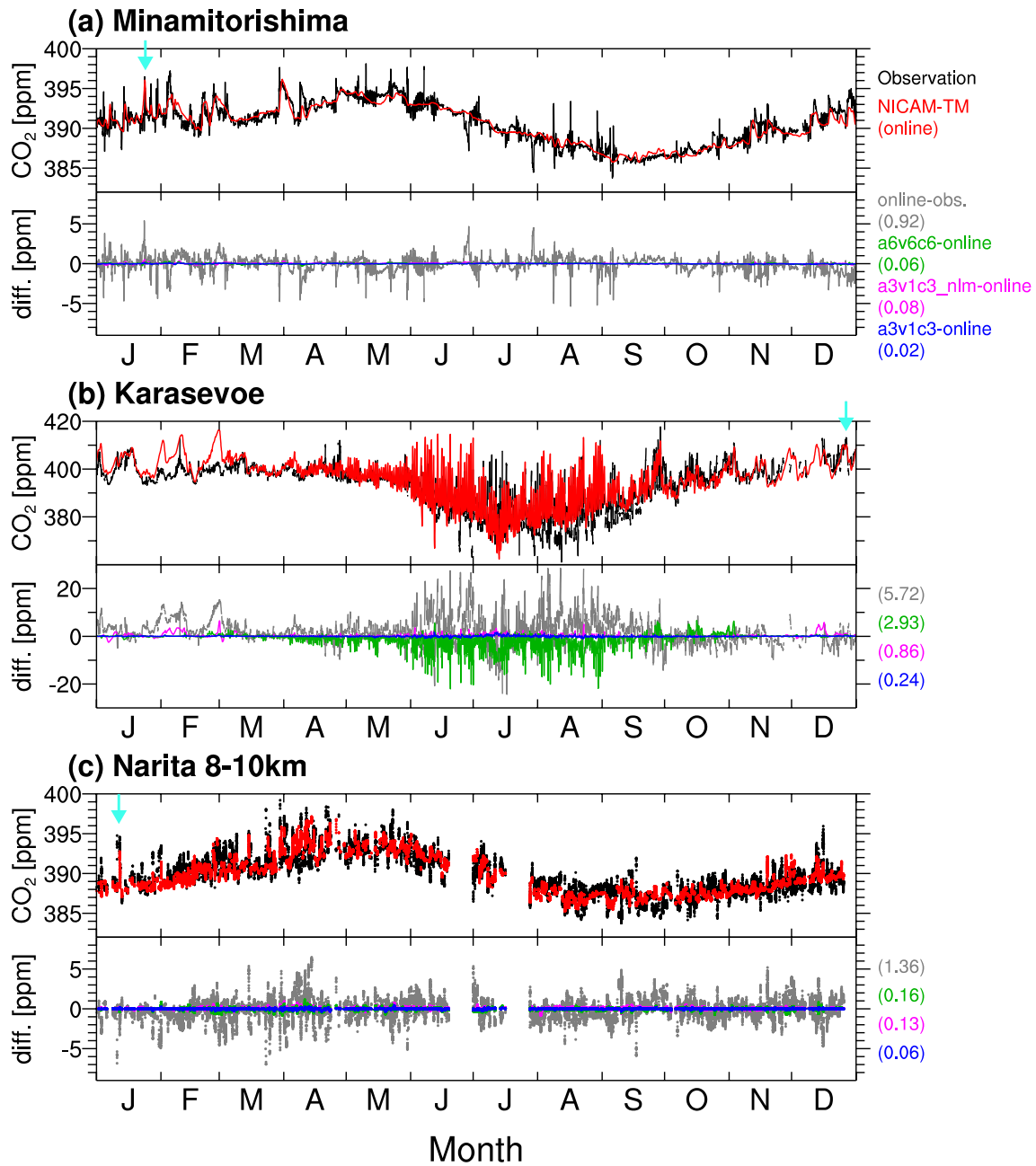
**Figure 4.** Time series of $CO_2$ concentration for 2010 at Minamitorishima (a), Karasevoe (b), and 8-10 km altitude over Narita (c). Each upper panel shows the time series observed (black) and simulated by the on-line model (red). Each lower panel shows differences of $CO_2$ concentrations between the on-line model and the observation (gray) and between the off-line model (green for A6V6C6, magenta for A3V1C3, blue for A3V1C3 without the flux limiter) and the on-line model. The number in the parenthesis gives the RMSD value for each sensitivity case. Note that only tropospheric data (determined by the dynamical tropopause (Sawa et al., 2008)) are used for the comparison over Narita.
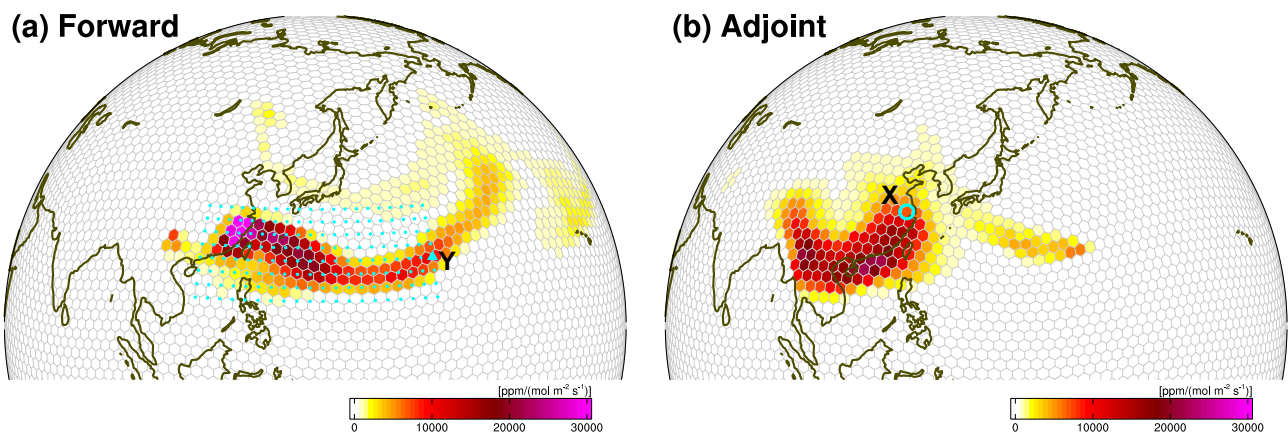
**Figure 5.** The concentration field at 3 km altitude on 0 UTC 8 January 2010 simulated by the forward model (with the flux limiter) from the basis unit flux X (a) and the sensitivities of the observation Y against the surface fluxes (footprint) simulated by the continuous adjoint model (b). The observation points, from which the adjoint sensitivities are calculated, are denoted as cyan dots and the location of observation Y is indicated by the cyan triangle (a). The location of the basis unit flux X is indicated by the cyan open circle (b).
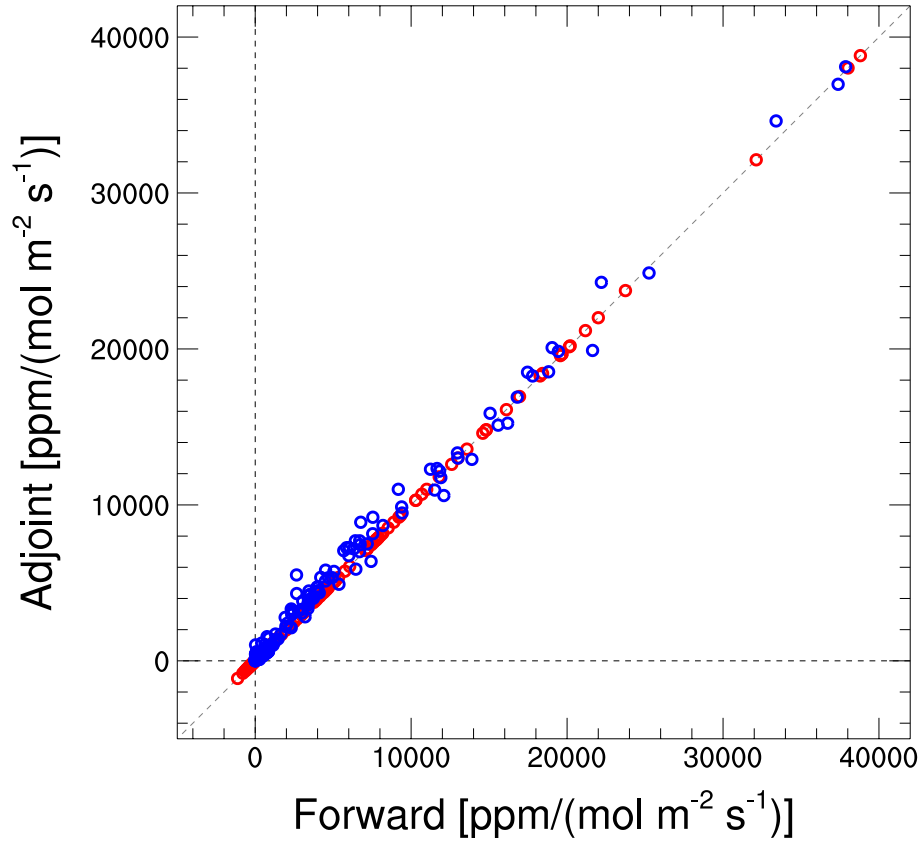
**Figure 6.** The scatter diagram showing 160 concentration values simulated by the forward model at the observation points versus their corresponding adjoint footprint values at the flux point X. The red open circles denote values from the linear model setup (the forward model without the flux limiter and the discrete adjoint model), while the blue open circles denote values from the nonlinear model setup (the forward model with the flux limiter and the continuous adjoint model).
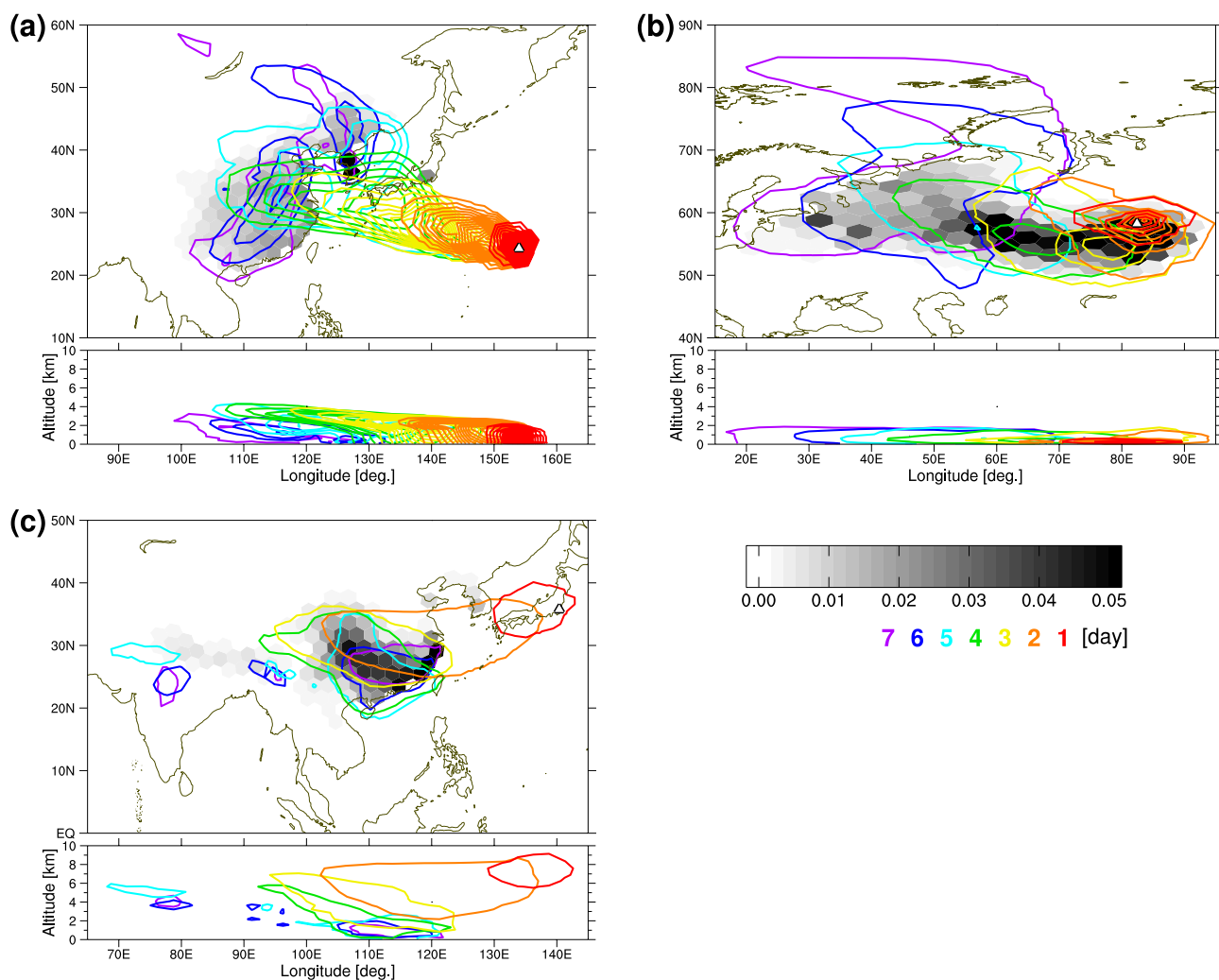
**Figure 7.** The normalized flux contributions (gray shades) and the adjoint trajectory volumes (color contours) for the high $CO_2$ concentration events observed at Minamitorishima on 24 January (a), at Karasevoe on 27 December (b), and at 8 km over Narita on 12 January in 2010 (c), which are denoted by the cyan arrows in Fig. 4. See the text for the definitions of the flux contribution and the adjoint trajectory volume. Each upper and lower panels present the horizontal and vertical structures of the adjoint trajectory volume, whose maximum values are projected onto the horizontal and vertical planes, respectively. The adjoint trajectory volumes are drawn by contours starting from a minimum value 0.02 at an interval of 0.6. The color of the contour represents how many days previous to the observation time to which the adjoint trajectory volume is associated.