1 r.avaflow v1, an advanced open source computational frame-

2 work for the propagation and interaction of two-phase mass

3 flows

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16 Abstract

17 r.avaflow represents an innovative open source computational tool for routing rapid mass flows, 18 avalanches or process chains from a defined release area down an arbitrary topography to a depo-19 sition area. In contrast to most existing computational tools, r.avaflow (i) employs a two-phase, 20 interacting solid and fluid mixture model (Pudasaini 2012); (ii) is suitable for modelling more or 21 less complex process chains and interactions; (iii) explicitly considers both entrainment and stop-22 ping with deposition, i.e. the change of the basal topography; (iv) allows for the definition of mul-23 tiple release masses and/or hydrographs; and (v) serves with built-in functionalities for validation, 24 parameter optimization and sensitivity analysis. r.avaflow is freely available as a raster module of 25 the GRASS GIS software, employing the programming languages Python and C along with the 26 statistical software R. We exemplify the functionalities of r.avaflow by means of two sets of com-27 putational experiments: (1) generic process chains consisting in bulk mass and hydrograph release 28 into a reservoir with entrainment of the dam and impact downstream; (2) the prehistoric Acheron 29 rock avalanche, New Zealand. The simulation results are generally plausible for (1) and, after the 30 optimization of two key parameters, reasonably in line with the corresponding observations for 31 (2). However, we identify some potential to enhance the analytic and numerical concepts. Fur-32 ther, thorough parameter studies are necessary in order to make r.avaflow fit for reliable forward 33 simulations of possible future mass flow events.

34 Keywords: GIS raster analysis, mass flows, open source, process chains, two-phase flow model

35 **1** Introduction

36 Rapid flows or avalanches of snow, debris, rock or ice, or processes, process chains or process in-37 teractions involving more than one type of movement or material frequently lead to loss of life, property and infrastructures in mountainous areas worldwide. All state-of-the-art methods for 38 39 anticipating the occurrence, characteristics, and dynamics of such events rely on computer simula-40 tions. On the one hand, models attempt to identify those areas where mass flows are likely to release (landslide susceptibility; Guzzetti, 2006; Van Westen et al., 2006). On the other hand, they 41 attempt to anticipate the motion of rapid mass flows once they are released (Hungr et al., 2005a). 42 43 Whilst conceptual models (Lied and Bakkehøi, 1980; Gamma, 2000; Wichmann and Becht, 2003; 44 Horton et al., 2013; Mergili et al., 2015) are employed to identify possible impact areas at broad 45 scales, physically-based dynamic models are used for the detailed back-analysis or prediction of

46 specific events.

47 Advanced fluid dynamics offers a broad array of physically-based dynamic modelling approaches for mass flows, mostly referred to as granular avalanches or debris flows. Such models often centre 48 49 on two-dimensional "shallow flow" equations, but they vary considerably among themselves in 50 terms of their concept, complexity and capacity to model specific types of phenomena. Voellmy 51 (1955) pioneered mass flow modelling, followed by the work of Grigoriyan et al. (1967); Savage 52 and Hutter (1989); Takahashi (1991); Iverson (1997); Pitman and Le (2005); and many others (see 53 Pudasaini and Hutter, 2007 for a review). Savage and Hutter (1989) introduced depth-averaged 54 mass and momentum conservation equations which were later utilized, modified and extended by 55 Mangeney et al. (2003, 2005); Denlinger and Iverson (2004); and McDougall and Hungr (2004, 2005). The Savage and Hutter (1989) model was further extended to include the effects of pore 56 57 fluid by Iverson and Denlinger (2001); Savage and Iverson (2003); Pitman and Le (2005); Puda-58 saini et al. (2005); Pastor al. (2009); and Hutter and Schneider (2010a, b). Still, these approaches 59 either represent effectively one-phase models, or do not fully consider the two-phase nature of most mass flows. More recently, the software GeoClaw and its extension D-Claw consider shallow 60 61 water and quasi two-phase flows (Berger et al., 2011; Iverson and George, 2016). Pudasaini (2012) 62 introduced a general two-phase mass flow model including several essentially new physical as-63 pects of two-phase solid-fluid mixture flows. In comparison to one phase models, this amongst few other (e.g. Kowalski and McElwaine, 2013) two-phase approaches appears suitable for the realistic 64 65 simulation of most types of process chains and interactions such as overtopping of a lake and a 66 subsequent flood or debris flow due to the impact of a landslide into the lake.

67 Entrainment of the basal material into the flow may substantially alter the dynamics and charac-68 teristics of mass flows, increasing their destructive potential (Hungr and Evans, 2004; Hungr et al., 69 2005b; Reid et al., 2011; Berger et al., 2011; Pirulli and Pastor, 2012). Empirical laws for entrain-70 ment were proposed by Rickenmann et al. (2003); McDougall and Hungr (2005); and Chen et al. 71 (2006), whereas mechanical concepts were introduced by Fraccarollo and Capart (2002); Pitman et 72 al. (2003a); Sovilla et al. (2006); Medina et al. (2008); and Iverson (2012). The available entrain-73 ment models are effectively single-phase, and developed for bulk debris (Armanini et al., 2009; 74 Crosta et al., 2009; Hungr and McDougall, 2009; Pirulli and Pastor, 2012). Whilst the importance

- of erosion, and the associated change of the basal topography (Fraccarollo and Capart, 2002; Hungr
- and Evans, 2004; Hungr et al., 2005b; Le and Pitman, 2009) has been recognized by the scientific
- community, attempts to simulate deposition of mass flow material are sparsely documented.

78 Various types of numerical schemes have been used to solve mass flow model equations in order to 79 redistribute mass and momentum (e.g. Davis, 1988; Toro, 1992; Nessyahu and Tadmor, 1990; 80 Tai et al., 2002; Wang et al., 2004). Previously, equations were commonly formulated and solved for pre-defined types of topographies (Pudasaini et al., 2005, 2008; Wang et al., 2004) whereas a 81 82 mathematically consistent application to arbitrary mountain topographies – and therefore to real-83 world conditions – still remains a challenge (Mergili et al., 2012). This issue is closely related to 84 the fact that the model equations are commonly expressed in topography-following coordinates 85 hardly compatible with global Cartesian coordinates, which usually appear in Geographic Infor-86 mation Systems (GIS) and are referred to as GIS coordinates in the following. Nevertheless, some 87 of the mass flow models mentioned have been implemented in computational tools used for haz-88 ard mapping and zoning, such as DAN (Hungr et al., 1995); TITAN2D (Pitman et al., 2003b; Pitman and Le, 2005); SamosAT (Sampl and Zwinger, 2004); or RAMMS (Christen et al., 2010a, b). 89 90 Hergarten and Robl (2015) developed a modelling tool relying on the open source flow solver 91 GERRIS (Popinet, 2009).

- 92 None of these models explicitly consider stopping and deposition, and they offer only basic func-
- 93 tionalities for simulating chains or interactions of two-phase mass flows. There is, however, a par-
- 94 ticular need to appropriately consider process chains and interactions in mass flow simulations:
- 95 some of the most destructive events in history have evolved from cascading effects, such as the
- 96 1970 Huascarán event in Peru (Evans et al., 2009) or the 2002 Kolka-Karmadon event in Russia
- 97 (Huggel et al., 2005).

98 The present work addresses some of the needs and issues raised by introducing the multi-99 functional open source computational framework r.avaflow, employing an enhanced version of 100 the Pudasaini (2012) two-phase flow model for routing mass flows from a defined release area 101 down arbitrary topography to a deposition area. Next, we introduce the structure and functionali-102 ties of r.avaflow (Sect. 2). Then we perform two computational experiments in order to demon-103 strate the functionalities of the computational framework (Sect. 3). We discuss the implementa-104 tion of r.avaflow and the implications of our findings (Sect. 4), and finally conclude with the key 105 messages of the work and a brief outlook to the next steps (Sect. 5).

106 **2** The computational framework r.avaflow

107 2.1 Computational implementation

108 r.avaflow computes the propagation of mass flows from one or more given release areas over a giv-109 en basal topography until (i) all the material has stopped and deposited; (ii) all the material has left 110 the area of interest; or (iii) a user-defined maximum simulation time has been reached. r.avaflow is 111 developed along two lines with regard to its software environment and energies. FX

111 developed along two lines with regard to its software environment and operation, r.avaflow [EX-

- 112 PERT] and r.avaflow [PROFESSIONAL]. The latter represents a stand-alone GUI with still re-113 duced functionalities, suitable for practitioners. The present work, however, refers to r.avaflow 114 [EXPERT] which is implemented as a raster module of the open source software package GRASS GIS 7 (Neteler and Mitasova, 2007; GRASS Development Team, 2016). We use the Python pro-115 116 gramming language for data management, pre-processing and post-processing tasks (module 117 r.avaflow). The flow propagation procedure (see Sect. 2.3 and 2.4) is written in the C programming language (sub-module r.avaflow.main). Together with Python, the R software environment for 118 119 statistical computing and graphics (R Core Team, 2016) is employed for built-in validation and 120 visualization functions. Fig. 1 illustrates the logical framework of r.avaflow.
- 121 Multiple model runs may be executed in parallel, exploiting all computational cores available (see
- 122 Sect. 2.5). This speeds up the processing considerably, and allows the use of r.avaflow on computa-
- 123 tional clusters. Parallelization is implemented at the Python level (Mergili et al., 2014, 2015): for
- each model run a batch file is produced within the module r.avaflow. This batch file calls the Py-
- 125 thon-based sub-module r.avaflow.mult, launching r.avaflow.main which is then executed with the
- 126 specific parameters for the associated model run. Thereby, the Python library "Threading", a high-
- er-level threading interface is exploited. The Python class "Queue" is employed for handling the
- 128 queue of items to be processed.



130 Figure 1 Logical framework of r.avaflow. The transformations and re-transformations refer to the

conversion of heights and GIS coordinates to depths and topography-following coordinates, and
 vice versa (see Sect. 2.3).

r.avaflow was developed and tested with the operating systems (OS) Ubuntu 12.04 LTS and
16.04 LTS, and Scientific Linux 6.6 (Red Hat). It is expected to work on other UNIX systems, too.
A simple user interface is available. However, the tool may be started more efficiently through
command line parameters, enabling a straightforward batching on the shell script level. This feature facilitates model testing and the combination with other GRASS GIS modules.

All experiments where parallel processing is not applied are performed on an Intel® Core i7 975 with 3.33 GHz and 16 GB RAM (DDR3, PC3-1333 MHz), exploring a maximum of eight cores through hyperthreading and using the OS Ubuntu 12.04 LTS. All experiments with parallel processing are performed on the Vienna Scientific Cluster, serving with approx. 2020 nodes (Supermicro X9DRD-iF Board), each equipped with two Intel Xeon E5-2650v2 with 2.6 GHz und $8 \cdot 8$ GB RAM. The OS for these computations is Scientific Linux 6.6 (Red Hat).

144 **2.2 Input and output**

145 The key input parameters of r.avaflow are summarized in Table 1. Essentially, r.avaflow relies on 146 (i) a digital terrain model (DTM) representing the elevation of the basal surface before the event 147 under investigation; (ii) raster maps of the spatial distribution of the solid and fluid release heights 148 or hydrographs of solid and fluid release; (iii) a set of flow parameters (Table 2). Input raster maps 149 of the entrainable solid and fluid heights, and a raster map or value defining the empirical en-150 trainment coefficient (needed for entrainment) are optional. Instead of the solid and fluid release 151 and entrainable heights, the total heights and fixed values of the solid concentration may be de-152 fined.

153 There is no restriction imposed on the arrangement of the release cells. With the term 'cell' we 154 refer to a regular, square, equidistant, ground projected computational/numerical unit, i.e. an element of a GIS raster. Patches of cells where the release height is larger than zero may be defined 155 156 in various parts of the investigation area. An arbitrary number of release hydrographs - each asso-157 ciated to a given set of coordinates – can be defined alternatively or in addition to the different 158 release masses. This allows the simulation of complex interactions between different types of pro-159 cesses (see Sect. 3). Hydrographs are defined through their solid and fluid heights at the centre 160 point of the hydrograph profiles, and by the solid and fluid flow velocities. The flow height distri-161 bution along the hydrograph profile - which should be aligned perpendicular to the main flow 162 direction - is derived from the assumptions of a horizontal cross section of the flow table and a 163 maximum profile length (Fig. 2).



Figure 2 Sketch of a hydrograph profile. The flow surface of input hydrographs is defined by H_P and is extended in cross-profile direction either to the edge of the profile or until it intersects with

the basal topography.

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168 Table 1 Key input and output parameters of r.avaflow. s = solid; f = fluid; t = total. Remarks: 1 –

169 mandatory; 2 – one of the input data sets A, B or C+D is mandatory, C+D may also be provided in

170 addition to A or B; $n_D \ge n_C$, if $n_D > n_C$ the remaining sets of D are output hydrographs; 3 – either A

171 or B may be provided if entrainment is activated, otherwise all values of $H_{\text{Emax}} = \infty$; C is mandatory

172 with entrainment; 4 – at least one of the data sets A, B and C is mandatory for validation.

Parameter	Symbol	Unit	Format	Remarks
Input				
Initial elevation of basal surface	Z_0	m	Raster map	1
s, f release heights	$H_{0,s}, H_{0,f}$	m, m	Raster maps	2A
Total release height, s concentration of release mass	H_0, α_{s0}	m, –	Raster map, value	2B
s, f entrainable heights	H _{Emax,s} , H _{Emax,f}	m, m	Raster maps	3A
Entrainable total height, s concentration of entrainable mass	$H_{ m Emax},$ $lpha$ s,Emax	m, –	Raster map, value	3B
<i>n</i> ^C hydrograph tables: s and f flow heights and velocities at defined points of time (see Fig. 2)	HP,s, VP,s HP,f, VP,f	m, m s ⁻¹ m, m s ⁻¹	Tables	2C
$n_{\rm D}$ sets of centre coordinates, length and aspect of hydrograph	_	m, degree	Sets of four val- ues	2D
Flow parameters (see Table 2)	_	_	Set of 14 values	1
Entrainment coefficient (see Table 2)	<mark>C</mark> e	<mark>kg⁻¹</mark>	<mark>Value</mark>	<mark>3C</mark>
Time interval for output, max. time after which simulation terminates	Δt out, t term	S, S	Set of 2 values	1
Threshold flow height for visualization and validation	Ht	m	Value	1
Observed impact area, observed deposi- tion area	OIA, ODA	-, -	Raster maps	4A, B
Vertex coordinates of flow path	_	m	Even number of ≥4 values	4C

Output (excluding validation and visualization output; see Sect. 2.6)						
Maximum flow height, kinetic energy,	<i>Н</i> Мах,	<mark>m, J, Pa</mark>	Raster maps	Always		
and pressure (each for s, f, t)	$T_{\mathrm{Max}},p_{\mathrm{Max}}$					
Flow height, flow kinetic energy, and	$H_{ m tout}, T_{ m tout},$	<mark>m, J, Pa</mark>	Raster maps	Always		
flow pressure at each output time step	$p_{ m tout}$					
t_{out} (each for s, f, t)						
Flow velocities in x and y direction, and	<mark>Vx, Vy, V</mark>	$m s^{-1}$	Raster maps	Always		
in absolute values (each for s, f)						
Change of basal topography (s, f, t)	<mark>H</mark> c	m	Raster maps	<mark>Always</mark>		
Impact indicator index, deposition indi-	III, DII	-, -	Raster maps	Multiple		
cator index				runs		
<i>n</i> D– <i>n</i> C output hydrograph tables: flow	<u>H₽, v₽, Q</u>	<mark>m, m s⁻¹,</mark>	Tables	If $n_D > n_C$		
heights, velocities and discharges at de-		<mark>m³ s⁻¹</mark>				
fined points of time (s, f)						

174 Mandatory parameters further include the time interval at which output maps are written Δt_{out} (s), 175 the maximum time after which the simulation terminates, and the threshold flow height for visu-176 alization and validation H_t (m; see Table 1). Optional parameters further include raster maps of the 177 observed impact area and deposition height as well as a set of flow path coordinates (for validation 178 and visualization; see Fig. 1 and Sect. 2.6). An exhaustive list of input parameters is provided in the 179 user manual of r.avaflow, available at http://www.avaflow.org/software.html.

180 If a single model run is executed (see Fig. 1), the output of r.avaflow consists in raster maps of sol-181 id, fluid and total flow heights, flow velocities in x and y direction and in absolute terms, pressures 182 and kinetic energies, and the change of the basal topography (only relevant with entrainment or 183 stopping; see Sect. 2.4). All raster maps are produced for each output time step (defined by Δt_{out}) 184 and for the maximum over all time steps. Further, a table summarizing the maximum solid and 185 fluid flow heights and velocities as well as flow volumes and kinetic energies for all output time 186 steps is produced. Optionally, solid and fluid output hydrographs are generated for an arbitrary 187 number of given output hydrograph profiles (see Table 1 and Fig. 2). With multiple model runs, the results of each single run are aggregated to impact or deposition indicator indices (see 188 189 Sect. 2.5). In the present work we focus on the output heights, hydrographs and indices when ana-190 lyzing the results, rather than on velocities or deduced results such as pressures or kinetic energies 191 (see Sect. 3).

192 2.3 Mass and momentum evolution

193 The core functionality of r.avaflow consists in the redistribution of mass and momentum, employ-194 ing a dynamic flow model and a numerical scheme. Thereby the tool offers implementations (i) of 195 a single-phase shallow water model with Voellmy friction relation (Christen et al., 2010a, b; 196 Fischer et al., 2012) and (ii) essentially the Pudasaini (2012) two-phase flow model with ambient 197 drag (Kattel et al., 2016) and a set of additional numerical treatments (complementary functions)

198 outlined in Sect. 2.4. In the present work we only consider the implementation (ii). It builds on

199 the conservation of mass and momentum, computed separately but simultaneously for the solid

and fluid components of the flow. A system of six differential equations (expressed in locally topography-following coordinates) represents the basis for a set of six flux and source terms, regarding solid and fluid flow depths (D_s , D_t), solid momentum M_s and fluid momentum M_t in x direction ($M_{sx} = D_s \cdot v_{sx}$, $M_{fx} = D_t \cdot v_{fx}$), and M_s and M_t in y direction ($M_{sy} = D_s \cdot v_{sy}$, $M_{fy} = D_t \cdot v_{fy}$), where v is flow velocity.

205 The Pudasaini (2012) model employs the Mohr-Coulomb plasticity for the solid stress. The fluid 206 stress is modelled as a solid-volume-fraction-gradient-enhanced non-Newtonian viscous stress. 207 The generalized interfacial momentum transfer includes viscous drag, buoyancy, and virtual mass 208 induced by relative acceleration between the phases. A new, generalized drag force is proposed 209 that covers both solid-like and fluid-like contributions. Strong coupling between the solid- and 210 the fluid-momentum transfer leads to simultaneous deformation, mixing, and separation of the 211 phases. Inclusion of the non-Newtonian viscous stresses is important in several aspects. The advec-212 tion and diffusion of the solid volume fraction play an important role. The model includes a num-213 ber of innovative, fundamentally new, and dominant physical aspects. Please consult Pudasaini 214 (2012) for the full details of the model, including the corresponding equations. The flow parame-215 ters required are summarized in Table 2.

216 Solving the differential equations and propagating the flow from one cell to the next requires the 217 implementation of a numerical scheme. For this purpose r.avaflow employs a high resolution Total 218 Variation Diminishing Non-Oscillatory Central Differencing (TVD-NOC) Scheme, a numerical 219 scheme useful to avoid unphysical numerical oscillations (Nessyahu and Tadmor, 1990). Cell aver-220 ages of all six state variables are computed using a staggered grid: the system is moved half of the 221 cell size with every time step, the values at the corners of the cells and in the middle of the cells 222 are computed alternatively at half and full time steps, respectively. The TVD-NOC scheme with 223 the Minmod limiter has successfully been applied to a large number of mass flow problems (Tai et 224 al., 2002; Wang et al., 2004; Mergili et al., 2012; Pudasaini and Krautblatter, 2014; Kafle et al., 225 2016; Kattel et al., 2016).

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Table 2 Flow parameters and entrainment coefficient required with the enhanced version of the Pudasaini (2012) two-phase flow model. Exp. 1 and 2 refer to the values used for the computational experiments introduced in Sect. 3.

Symbol	Parameter	Unit	Exp. 1A, B, C	Exp. 2A, B
$ ho_{ m S}$	Solid material density	kg m⁻³	2700	2700
$ ho_{ extsf{F}}$	Fluid material density	kg m⁻³	1000	1000
φ	Internal friction angle	Degree	35	35
δ	Basal friction angle ¹⁾	Degree	20	15–25, 17
$C_{\rm VM}$	Virtual mass	_	0.5	0.5
V T	Terminal velocity	$m s^{-1}$	1	1
Р	Parameter for combination of solid- and	_	0.5	0.5
	fluid-like contributions to drag resistance			
$Re_{\mathbb{P}}$	Particle Reynolds number	_	1	1
J	Exponent for drag (1 = linear,	_	1	1
	2 = quadratic)			
$N_{\rm R}$	Quasi Reynolds number	_	30,000	30,000

$N_{ m RA}$	Mobility number	_	1,000	1,000	
X	Viscous shearing coefficient for fluid	_	0	0	
ξ	Solid concentration distribution with	_	0	0	
	depth				
$\mathcal{C}_{ ext{AD}}$	Ambient drag coefficient ²)	_	0	0	
$C_{\mathbb{E}}$	Entrainment coefficient ¹⁾	kg^{-1}	-, 10 ^{-5.3} , 10 ^{-6.0}	_	

229 ¹⁾ Alternatively, these parameters may be provided as raster maps instead of global values; ²⁾ Refer 230 to Kattel et al (2016) for ambient drag

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232 The input and output of r.avaflow (see Sect. 2.2) is discretized on the basis of GIS coordinates, i.e. 233 in cells which are rectangular in shape in the ground projection. For the numerical solution the 234 cell lengths in x and y directions, and the area, are corrected for the local slope in order to main-235 tain consistency with the state variables expressed in the local topography-following coordinates. 236 Gravitational acceleration in the topography- following x, y, and z directions – representing a fun-237 damental input to the Pudasaini (2012) model equations – is computed from the DTM, employing 238 a finite central difference scheme. All input heights H (m) are expressed in vertical direction, and 239 are converted into depths D (m) expressed in direction normal to the local topography as in the model equation formulation. The resulting depths are converted into heights for output. The time 240 241 step length Δt (s) is dynamically updated according to the CFL condition (Courant et al., 1967; 242 Tai et al., 2002; Wang et al., 2004).

243 2.4 Complementary functions

Table 3 summarizes some additional functions of r.avaflow. The functions with ID 1–3 have been 244 245 introduced to compensate for deficiencies of the numerical scheme and its implementation experi-246 enced with complex real-world flows (see Sect. 6). Entrainment and stopping, in contrast, repre-247 sent dynamic functions not covered by the Pudasaini (2012) model and are executed at the end of 248 each time step (see Fig. 1). Even though the separation of the complementary functions from the 249 TVD-NOC scheme, and their treatment in a simple forward Euler manner, can be questioned 250 physically and mathematically, we consider the current implementation a reasonable first approx-251 imation (see Sect. 4). We now elaborate the concepts employed for entrainment and stopping in 252 more detail.

253 Table 3 Functionalities of r.avaflow introduced for numerical purposes (ID 1–3) or complementing 254 the Pudasaini (2012) model (ID 4,5). Exp. 1 and 2 refer to the computational experiments intro-

255

duced in Sect. 3; Y = activated; N = deactivated.

ID	Function	Description	Exp.	Exp.
			1ABC	2AB
1	Diffusion control	Propagation of the flow from one <mark>cell</mark> to	YYY	YY
		the next is suppressed if the velocity is		
		not high enough, reducing numerical		
		diffusion		
2	Conservation of volume	Flow volume lost due to numerical reasons is replaced through an increase of D	YYY	YY

		of all <mark>cell</mark> s by the fraction of lost volume		
		after each time step		
3	Surface control	Numerical oscillations of undisturbed	YYY	NN
		flat surfaces (such as reservoirs) are		
		avoided		
4	Entrainment	Empirical approach to compute en-	NYY	NN
		trainment of basal material		
5	Stopping and deposition	Energy balance approach for stopping	NNN	YY
		and deposition of flow material		

257 Full handling of the evolution of the basal topography within the TVD-NOC scheme is not 258 straightforward and could also produce some diffusion. Therefore, and as entrainment is not in-259 cluded in the original Pudasaini (2012) model, entrainment is treated as a complementary function 260 in a first step. We note, however, that the time steps at which entrainment and the change of the basal topography are updated are identical to the time steps of the numerical scheme. The poten-261 262 tial solid and fluid entrainment rates $q_{E,s}$ and $q_{E,f}$ (m s⁻¹; expressed perpendicular to the basal topography) build on the user-defined empirical entrainment coefficient C_E (kg⁻¹) (see Table 2) and the 263 264 solid and fluid momenta. We assume a vertically homogeneous solid fraction $\alpha_{s,Emax}$ within the entrainable material, which is reflected in the ratio between $q_{E,s}$ and $q_{E,f}$. 265

266
$$q_{\rm E,s} = C_{\rm E} |M_{\rm s} + M_{\rm f}| \alpha_{\rm s,Emax}, \ q_{\rm E,f} = C_{\rm E} |M_{\rm s} + M_{\rm f}| (1 - \alpha_{\rm s,Emax}).$$
(1)

The fact that the basal velocities, which are relevant for entrainment, are lower than the depthaveraged velocities is not explicitly considered, but has to be reflected in the value of $C_{\text{E.}}$ $q_{\text{E,s}}$ and $q_{\text{E,f}}$ are always positive. The solid and fluid changes of the basal topography $H_{\text{E,s}}$ and $H_{\text{E,f}}$ (m) due to entrainment are:

271
$$H_{\mathrm{E,s,t}} = \min\left(H_{\mathrm{E,s(t-\Delta t)}} + \frac{q_{\mathrm{E,s}}\Delta t}{\cos\beta}, H_{\mathrm{Emax,s}}\right), \qquad (2)$$

272
$$H_{\rm E,f,t} = \min\left(H_{\rm E,f(t-\Delta t)} + \frac{q_{\rm E,f}\Delta t}{\cos\beta}, H_{\rm Emax,f}\right),\tag{3}$$

273 where $H_{E,s(t-\Delta t)}$ and $H_{E,f(t-\Delta t)}$ (m) are the change of the basal topography at the start of the time step, 274 $H_{\text{Emax},\text{s}}$ and $H_{\text{Emax},\text{f}}$ (m) are the maximum entrainable depths at the given cell, t (s) is the time passed 275 at the end of the time step, Δt (s) is the time step length, and β is the local slope of the basal sur-276 face. The division by $\cos\beta$ accounts for the conversion from depths to heights. The solid and fluid 277 entrained depths $D_{\text{E,s}} = (H_{\text{E,t}} - H_{\text{Es}(t-\Delta t)}) \cos \beta$ and $D_{\text{E,f}} = (H_{\text{Ef},t} - H_{\text{Ef}(t-\Delta t)}) \cos \beta$ are added to the solid 278 and fluid flow depths. We further assume that entrainment increases the solid and fluid momen-279 tum of the flow in each direction by the product of the entrained solid and fluid depth and the 280 total velocity in the given direction ($M_{\rm E}$; Fig. 3a). The basal topography and, consequently, the x 281 and y cell sizes, cell areas, and gravitational acceleration components in x, y, and z direction are 282 updated after each time step.

283 The changes in gravitational acceleration also influence the magnitude of the frictional terms 284 (Pudasaini and Hutter, 2003), which are important for stopping processes. In the literature few 285 approaches explicitly consider stopping processes directly in their numerical scheme by operator 286 splitting methods coupled with the determination of admissible stresses (e.g. Mangeney et al., 287 2003; Zhai et al., 2015). Here, in order to consider stopping which occurs at a spatial scale that is 288 not numerically resolved, we choose a different approach by proposing the dimensionless factor of 289 mobility *FoM*, relating the distance required for stopping stop to the numerical spatial resolution 290 Δs in the direction of movement. The flow stops if $s_{\text{stop}} \leq \Delta s$, i.e. $FoM \leq 1$ (see Fig. 3b):

291
$$FoM = \frac{S_{stop}}{\Delta s}$$
. (4)

To estimate s_{stop} we formulate the energy balance considering that the initial kinetic energy at an initial velocity v_0 (m s⁻¹) and the change of potential energy while travelling the distance s_{stop} have transformed in dissipative energy due to Coulomb friction, which dominates close to stopping. With this the energy balance estimate yields:

296
$$\frac{v_0^2}{2} + s_{\text{stop}} \sin \beta_v g = s_{\text{stop}} \tan \delta \cos \beta_v g .$$
 (5)

297 Consequently,

298
$$s_{\text{stop}} = \frac{v_0^2}{2g\cos\beta_v(\tan\delta - \tan\beta_v)},$$
 (6)

where δ is the basal friction angle, β_v is the slope angle in the direction of movement, and g (m s⁻²) is gravitational acceleration (see Table 2). According to Eq. 6 the stopping distance s_{stop} is positive for $\delta > \beta_v$, meaning that stopping is possible when the friction angle is higher than the slope angle, i.e. in particular at flat or even counter slopes. We note that, by a simple transformation of Eq. 6, *FoM* can alternatively be derived by relating the stopping time to the time step length. The stopping criterion is only relevant for w > 0. With w = 0, movement will be initiated as long as the local slope is larger than the friction angle.

306 FoM can relate to various spatial units: (i) a single cell, i.e. FoM is computed separately for each 307 cell. It may happen that stopping of the flow occurs at a certain cell, but not at its neighbour cells. 308 (ii) v_0 and β_v are averaged over a certain cell neighbourhood to compute FoM, so that stopping oc-309 curs at patches of adjacent cells. (iii) β_v and the associated component of v are averaged over the 310 entire area of interest. This means that the entire flow stops at once.





Figure 3 Interactions of the flow with the basal topography: (a) Entrainment, assuming that $H_{\text{Emax},s}$ 312 and $H_{\text{Emax},\text{f}}$ are not limiting. D_{i} = total initial flow depth (s+f); M_{i} = total initial momentum (s+f); 313 314 $D_{\rm E}$ = entrained depth; $M_{\rm E}$ = total increase in momentum due to entrainment (s+f). (b) Stopping. Both panels represent sections along the steepest slope of the basal topography. Note that stopping 315 316 usually occurs on less inclined slopes than drawn in (b) which represents upslope movement.

317 The third possibility is currently implemented with r.avaflow as an optional function. If activated, 318 the simulation terminates as soon as stopping occurs and the entire flow material is deposited. 319 Note that, in the current implementation, stopping always considers the total mass, without dif-320 ferentiating between the solid and the fluid components. This simplification is reasonable for 321 flows characterized by a relatively small fluid volume fraction. The change of basal topography 322 due to entrainment $H_{\rm E}$ after the last time step is subtracted from the height of the deposited mate-323 rial H_0 in order to derive the change of basal topography H_0 at the end of the simulation (all for

324 solid, fluid, and total; positive for an increase, negative for a decrease of terrain elevation).

Multiple model runs 325 2.5

326 r.avaflow includes a built-in function to perform multiple model runs at a time with controlled or 327 random variation of uncertain input parameters between given lower and upper thresholds. Es-328 sentially, this concerns the flow parameters (see Table 2), but also the solid concentration of the 329 release mass α_{s0} . Multiple parameters can be varied at a time. This procedure serves for two pur-330 poses:

331

It facilitates multi-parameter sensitivity analysis and optimization efforts;

• The results of all model runs are aggregated to an impact indicator index (*III*) and a deposition indicator index (*DII*), each in the range 0–1. *III* represents the fraction of model runs where $H_{Max} \ge H_t$ at a given cell whilst *DII* represents the fraction of model runs where $H_{D} \ge H_t$ at a given cell. *III* and *DII* can be used to demonstrate the impact of uncertain input parameters on the simulation result.

The model runs can be split among multiple computational cores (parallel processing), enablingthe exploitation of high-performance computational environments (see Sect. 2.1).

339 **2.6 Validation and visualization**

340 r.avaflow can be used to produce map layouts and animations of the key results (see Fig. 1). It further includes built-in functions to validate the model results against observations. Validation relies 341 342 (i) on the availability of a raster map of the observed impact or deposition area of the event under 343 investigation, (ii) on a user-defined profile along the main flow path (see Table 1), or (iii) on 344 measurements H or v at selected coordinates and time steps. Those cells with observed impact or deposition are referred to as observed positives (*OP*), those without observed impact or deposition 345 346 as observed negatives (ON). When using the observed impact area (OIA) as reference, all cells 347 with $H_{Max} \ge H_t$ are considered as predicted positives (*PP*), all cells with $H_{Max} < H_t$ are considered as 348 predicted negatives (*PN*). When using the observed deposition area (ODA) as reference, all cells 349 with $H_D \ge H_t$ are considered as PP, all cells with $H_D < H_t$ are considered as PN. Intersecting ON 350 and *OP* with *PP* and *PN* results in four validation scores: true positive (*TP*), true negative (*TN*), 351 false positive (FP) and false negative (FN) predictions (Fig. 4). TN strongly depends on the size of the area of interest. It is normalized to $5 \cdot (TP + FN) - FP$ in order to allow a meaningful comparison 352 353 of model performance among different case studies. These scores build the basis for most of the 354 validation parameters described in Table 4. Only the excess travel distance ΔL relies on the ob-355 served and simulated terminal points of the flow, based on a user-defined longitudinal profile. We note that this profile is only needed for validation, but is not used for the mass flow simulation 356 357 itself.

358 Values of $\Delta L > 0$ and FoC > 1 indicate conservative results (simulated impact or deposition area is 359 larger than observed impact or deposition area) whilst values of $\Delta L < 0$ and FoC < 1 indicate non-360 conservative results. CSI, D2PC, and AUROC do not allow to conclude on the conservativeness of 361 the results. ΔL , FoC, CSI, and D2PC as defined in Table 4 target at the validation of H_{Max} or H_D de-362 rived with one single model run. With multiple model runs (see Sect. 2.5) the validation parameters are computed separately for each run, allowing to conclude on the sensitivity of the model 363 364 performance to given input parameters, or to optimize input parameter values. In this sense, opti-365 mum parameters always refer to one particular criterion, and different criteria may suggest differ-

- 366 ent optimum parameter values.
- 367 In contrast, ROC (Receiver Operating Characteristics) curves are used to test the performance of
- the overall output of multiple model runs. Such curves are produced for *III* (OIA as reference)and/or *DII* (ODA as reference): the true positive rate is plotted against the false positive rate for
- 370 various levels of *III* or *DII*. The area under the curve connecting the resulting points, *AUROC*, is

- 371 used as an indicator for model performance ($AUROC \approx 1$ indicates an excellent performance; see
- 372 Fig. 4 and Table 4).

Further, the difference between observed and simulated values of H and v at selected sets of coordinates and points of time can be analyzed. This function is mainly useful for very welldocumented case studies such as laboratory experiments and is not further used in the present work.



Figure 4 Validation of r.avaflow results. (a) Validation scores for single model run; (b) multiple
model runs: threshold levels of *III* or *DII*, employed to produce (c) ROC curves.

380Table 4 Validation criteriaused in r.avaflow (see also Fig. 4). S = single model run, binary simula-381tion result; M = multiple model runs, simulation result in the range 0–1. The concepts of *CSI* and382D2PC are taken from Formetta et al. (2015). All validation parameters are computed for H_{Max} (OIA383as reference) and/or H_D (ODA as reference), depending on which of the reference data are availa-384ble.

Scope	Name	Definition	Possible range	Optimum
S	Excess travel distance ΔL	Ls – Lo	[<i>−L</i> ₀ ,∞]	0.0
S	Factor of conservativeness <i>FoC</i>	$FoC = \frac{PP}{OP} = \frac{TP + FP}{TP + FN}$	[0,∞]	1.0
S	Critical success index CSI	$CSI = \frac{TP}{TP + FP + FN}$	[0,1]	1.0
S	Distance to perfect classi- fication <i>D2PC</i>	$D2PC = \sqrt{\left(1 - r_{\rm TP}\right)^2 + r_{\rm FP}^2}$ $r_{\rm TP} = \frac{TP}{OP}, \ r_{\rm FP} = \frac{FP}{ON}$	[0,1]	0.0
М	Area under ROC curve <i>AUROC</i>	Function of <i>r</i> _{TP} and <i>r</i> _{FP} for different levels of <i>DII</i> or <i>III</i> (see Fig. 4)	[0.1]	1.0

385



Figure 5 Generic landscape used for Experiment 1A–C. (a) Oblique view illustrating the topography and elements of the landscape. (b) Input hydrograph I1 employed for Experiment 1C.

389 3 Computational experiments

390 **3.1 Experiment 1: Generic process chain**

391 3.1.1 Topographic setup

In a first step, the potential of r.avaflow for simulating process chains is demonstrated, considering 392 393 the interaction between one or more landslides, a reservoir, and the dam impounding the reser-394 voir. This experiment represents a follow-up to the work of Pudasaini (2014); Kafle et al. (2016); 395 and Kattel et al. (2016). We construct a generic landscape of size $3200 \text{ m} \cdot 2000 \text{ m}$, illustrated in 396 Fig. 5a. This landscape consists of the following elements: (i) W–E stretching trough-shaped valley with an amphitheatre-shaped head, inclined towards E in its lower part; (ii) dam with a trapezoi-397 398 dal cross section running across the valley, consisting of 100% solid material; (iii) reservoir im-399 pounded by the dam; (iv) landslide release mass near the NW corner of the area of interest (Land-400 slide 1); (v) landslide release mass directly N of the dam (Landslide 2); (vi) hydrograph release of landslide near the SW corner of the area of interest; (vii) measurement profile for output hydro-401

graph downstream from the dam. Both landslide release masses assume the shape of a hemiellipsoid imposed on the basal topography (see Fig. 5a). The algorithm for exactly reproducing the
generic landscape in GRASS GIS is available at http://www.avaflow.org/casestudies.html.

405 **3.1.2 Modelling strategy and parameterization**

- The landslides 1 and 2 consist of 75% solid and 25% fluid by volume (uniformly mixed), the input
 hydrograph I1 (see Fig. 5b) consists of 50% each solid and fluid per volume. The parameters and
 settings applied are summarized in the Tables 2 and 3.
- 409 Three computational experiments are performed, with increasing complexity from A–C:
- Experiment 1A: Landslide 1 is released and interacts with the reservoir. The dam is assumed stable and may therefore not be entrained.
- 412 Experiment 1B: Again, Landslide 1 is released and interacts with the reservoir. However,
 413 dam material is allowed to be entrained in this experiment.
- Experiment 1C: Landslide 2 is released and interacts with the dam and the reservoir. The release from the input hydrograph I1 starts after 10 s and continues for a period of 130 s (see Fig. 5). Dam material is allowed to be entrained at all stages of the computational experiment.
- 418 All experiments are performed at a cell size of 10 m and for a duration of $t_{\text{term}} = 300 \text{ s}$; $\Delta t_{\text{out}} = 5 \text{ s}$.
- 419 The solid and fluid discharges are continuously recorded at the output hydrograph profile O1 420 downstream. The stopping function is deactivated (see Table 3).

421 3.1.3 Results

Animations illustrating the time evolution of the flow heights in all three experiments are en-closed in Supp. 1A, 1B, and 1C.

- 424 Fig. 6a-f illustrates the flow heights at selected points of time during Experiment 1A. The Land-425 slide 1 (see Fig. 5a) impacts the backward portion of the reservoir after few seconds and generates 426 a water wave – oblique and perpendicular to the impact – that overtops the dam from t = 50-55 s 427 onwards. The output hydrograph O1 starts recording discharge at t = 65 s, with the peak of the 428 first, major flood wave passing at t = 75 s ($Q_t = 8 \cdot 10^4$ m³ s⁻¹; Fig. 6g). We note that the discharge and 429 the flow height recorded by the hydrograph do not strictly follow the same pattern, as the dis-430 charge relates to a profile and the flow height relates to a point (see Fig. 2). Meanwhile the impact 431 wave is deflected at the dam and alleviates slowly. Further overtopping events caused by multiple 432 deflections of the alleviating wave occur mainly at the marginal parts of the dam at t = 110, 150,433 160, 200 and 270 s, leading to smaller peaks in the output hydrograph ($Q_{\rm f} = 1.5 \cdot 10^4 \, {\rm m}^3 \, {\rm s}^{-1}$ at 434 t = 175 s; $Q_{\rm f} = 2.2 \cdot 10^3$ m³ s⁻¹ at t = 285 s). The solid content passing the hydrograph profile is almost 435 negligible as all solid landslide material remains in the reservoir basin. At t = 300 s, the impact
- 436 wave in the lake has almost alleviated (see Supp. 1A).
- 437 Experiment 1B (Fig. 7) is identical to the Experiment 1A until the point when the impact wave 438 reaches the dam at t = 50 s. Entrainment of the dam starts with overtopping which sets on at the

- 439 lateral portions. Part of the dam is entrained during overtopping by the initial impact wave.
- 440 Whilst massive outflow from the reservoir occurs due to the decreased level of the dam crest, part
- 441 of the wave is deflected at the dam and pushed back towards the backward part of the reservoir,
- 442 inducing a system of secondary waves. The remaining dam material is entrained when hit by those
- 443 secondary waves. At t = 200 s the entire dam has disappeared and the reservoir starts emptying
- 444 completely. In contrast to Experiment 1A, due to the emptying process the system does not ap-
- 445 proach a static equilibrium after t = 300 s (see Supp. 1B).
- 446 The temporal patterns of the simulated entrainment and wave propagation are clearly reflected in 447 the discharge recorded at the output hydrograph O1 (see Fig. 7g). As a consequence of dam over-448 topping, fluid discharge at O1 starts increasing at t = 65 s and reaches a first peak at t = 80 s 449 $(Q_{\rm f} = 5.1 \cdot 10^4 \, {\rm m^3})$. Solid discharge – a consequence of entrainment of the dam – starts slightly delayed, reaching a first peak roughly ten seconds later ($Q_s = 2.1 \cdot 10^4 \text{ m}^3 \text{ s}^{-1}$). A depression in both of 450 451 the discharge curves at t = 155-160 s indicates that the initial impact wave has passed through. A second, larger peak of fluid discharge is simulated at t = 195 s ($Q_i = 1.0 \cdot 10^5$ m³ s⁻¹). It occurs syn-452 chronously with a second, smaller peak of solid discharge ($Q_{\rm s} = 2.1 \cdot 10^4 \, {\rm m}^3 \, {\rm s}^{-1}$), indicating a high 453 454 degree of mixing of the solid and fluid components of the flow. The pronounced second peak of Q 455 is a consequence of the secondary waves in combination with the lowered level of the dam. After 456 the peak, Qs slowly and unsteadily decreases (the entire dam has been entrained and the material has passed through) whilst Q^t remains high. Due to the entrainment of the dam, the simulated dis-457 458 charges are much higher than those computed in the Experiment 1A (see Fig. 6g).
- 459 In Experiment 1C (Fig. 8) Landslide 2 impacts the dam and the frontal part of the reservoir less 460 than 10 s after release. The proximal portion of the dam is entrained rapidly. The right part of the 461 landslide moves outside of the reservoir in downstream direction. Consequently, the solid dis-462 charge at the output hydrograph O1 starts at t = 30 s, reaching a peak of $Q_s = 2.9 \cdot 10^4$ m³ s⁻¹ ten seconds later (see Fig. 8g). Due to the high (75%) solid fraction of the landslide, the fluid discharge is 463 lower at that time ($Q_{\rm f} = 1.0 \cdot 10^4 \, {\rm m}^3 \, {\rm s}^{-1}$). The left part of the landslide interacts with the reservoir, 464 causing overtopping at the distal portion of the dam. This results in the increase of fluid discharge 465 466 recorded at O1, culminating at t = 60 s when the solid discharge is already decreasing 467 $(Q_{\rm f} = 2.9 \cdot 10^4 \, {\rm m}^3 \, {\rm s}^{-1})$. The immediate impact of the initial landslide and the resulting impact wave on O1 has largely alleviated after t = 100 s in terms of discharge, even though the total flow height 468 remains at H > 2 m. This means that the flow material largely remains in place at O1. 469
- 470 From t = 30 s onwards the flow released through the input hydrograph I1 (see Fig. 5b) pushes the 471 reservoir water towards NE. The remnants of the dam are overtopped by the resulting inhomoge-472 neous solid-fluid mixture (including material originating from Landslide 2), leading to substantial 473 further entrainment. In contrast to Experiment 1B, however, the dam is not completely entrained. The wave starts influencing the discharge recorded at O1 at t = 135 s. A subsequent steady increase 474 475 of solid and fluid discharge leads to a broad peak recorded at t = 230-250 s ($Q_s = 1.3 \cdot 10^4$ m³ s⁻¹; 476 $Q_{\rm f} = 3.7 \cdot 10^4 \, {\rm m}^3 \, {\rm s}^{-1}$). At that time the hydrograph indicates a well-mixed flow with $\alpha_{\rm s} \approx 0.25$, com-477 posed of fluid from the reservoir, solid-fluid mixtures from the landslide and the hydrograph re-478 lease, and solid material from the dam (see Fig. 5a). The solid and fluid discharge steadily decrease

- 479 after t = 250 s, reflecting the termination of the hydrograph release and the emptying of the reser-
- 480 voir. However, emptying of the reservoir operates much more slowly than in Experiment 1B due
- 481 to the comparatively high solid content of the system which is still far away from a static equilib-
- 482 rium after t = 300 s (see Supp. 1C).





484 Figure 6 Key results of Experiment 1A. (a)–(f) Sequence of simulated flow heights and solid ratios
485 at selected points of time; see Supp. 1A for animations of flow height and kinetic energy sequenc486 es; (g) output hydrograph O1 (see Fig. 5a).





Figure 7 Key results of Experiment 1B. (a)–(f) Sequence of simulated flow heights and solid ratios
at selected points of time; see Supp. 1B for animations of flow height and kinetic energy sequences; (g) output hydrograph O1 (see Fig. 5a).





492 Figure 8 Key results of Experiment 1C. (a)–(f) Sequence of simulated flow heights and solid ratios
493 at selected points of time; see Supp. 1C for animations of flow height and kinetic energy sequenc494 es; (g) output hydrograph O1 (see Fig. 5a).

496 **3.2 Experiment 2: Acheron rock avalanche, New Zealand**

497 **3.2.1 Event description**

510

498 The Acheron rock avalanche in Canterbury, New Zealand (Fig. 9), occurred approx. 1,100 years 499 BP (Smith et al., 2006). It is characterized by sharp bending of the flow path, a limited degree of 500 spreading into the lateral valleys and a high mobility (travel distance: 3,550 m; measured angle of 501 reach: 11.62°). It was used as a test event for the computational tool r.randomwalk (Mergili et al., 502 2015).

We use a 10 m resolution DEM derived by stereo-matching of aerial photographs. ODA and OIA are derived from field and imagery interpretation as well as from data published by Smith et al. (2006). The OIA possibly underrepresents the real impact area as it might exclude some lateral and run-up areas of the rock avalanche not any more recognizable as such in the field. The distribution of release and deposition heights and an estimated release volume of $6.4 \cdot 10^6$ m³ are deduced from the reconstruction of the pre-event topography. According to this reconstruction, the maximum release height is 78.7 m whilst the maximum deposition height is 25.9 m.







513 3.2.2 Modelling strategy and parameterization

514 Preliminary tests have shown that the simulation results of r.avaflow are potentially sensitive to 515 variations in the initial solid fraction α_{s0} and the basal friction angle δ , parameters which are un-516 certain in many real-world applications. We perform two computational experiments for the Ach-517 eron rock avalanche:

- 5181. Experiment 2A: *III* and *DII* are computed from a set of 121 model runs. Thereby, α_{60} is519varied from 0.5–0.9, and δ is varied from 15–25° (see Table 2). The variation is done in a520controlled way assuming a uniform probability density function, i.e. a regular grid with 11521grid points in each dimension is laid over the two-dimensional parameter space. *III* is then522evaluated against the OIA, and *DII* is evaluated against the ODA. α_{60} and δ are optimized523in terms of ΔL , *FoC*, *CSI*, and *D2PC* derived from H_0 and the ODA.
- 524 2. Experiment 2B: r.avaflow simulation with the optimized values of α_{s0} and δ .
- 525 Both experiments are conducted at a cell size of 20 m. Entrainment is not considered whilst stop-526 ping is included (see Table 3). All flow parameters except for δ are kept constant (see Table 2).

527 3.2.3 Results

528 Fig. 10 illustrates III and DII derived with the parameter settings shown in the Tables 2 and 3 (Ex-529 periment 2A). AUROC is 0.830 with regard to III and 0.838 with regard to DII. In general, those 530 areas with high values of III coincide with the OIA, whilst those areas with lower values of III lie 531 close to the margins or outside of the OIA. The performance of III suffers from the motion of small portions of the simulated avalanche in the wrong (N) direction and from excessive lateral 532 533 spreading and run-up in the upper part, observed for all tested combinations of α_{s0} and δ (high 534 values of *III*; see Fig. 10a). However, one has to consider that the event occurred hundreds of years 535 ago and run-up may have occurred even though it is not any more recognizable in the field and 536 therefore excluded from the OIA. High values of *DII* are fairly constrained to those cells within 537 the ODA (see Fig. 10b) which is most probably better defined than the OIA. Those areas with lower, but non-zero values of *III* or *DII* both reach well beyond the reference areas. Particularly 538 539 the travel distance appears highly sensitive to the choice of α_{s0} and δ .

540 We now focus on the components of the *DII* map and evaluate the performance of the deposition 541 maps simulated with the various combinations of α_{so} and δ against the ODA. Fig. 11 illustrates the 542 dependency of the model performance (defined by the parameters summarized in Table 4) on the combination of α_{s0} and δ employed for a given model run. All four parameters clearly indicate 543 544 that, within the ranges tested, the model results are sensitive to both δ and α_{s0} . ΔL , *CSI*, and *D*2*PC* 545 display their optima near to $\delta = 17^{\circ}$ as long as $\alpha_{\rm s} \ge 0.7$. With higher fluid content, the optimum value of δ increases, arriving at 20° with $\alpha_{s0} = 0.5$ (see Fig. 11a, b and d). This pattern appears plau-546 547 sible as far as a higher fluid content is supposed to increase the mobility of the flow, compensating 548 for higher values of δ . However, values of $\alpha_{s0} < 0.7$ are not plausible for a rock avalanche of this 549 type. For $\alpha_{\rm s} \ge 0.7$ FoC displays its optimum of 1.0 at $\delta \ge 21^\circ$, depending on $\alpha_{\rm s0}$. FoC ≈ 1.25 for the 550 value of δ where the other parameters reach their optimum (see Fig. 11c). This would be fine for 551 many applications in practice where slightly conservative results are desirable.





Figure 10 Results of Experiment 2A: (a) Impact indicator index III and (b) deposition indicator index DII derived for the Acheron rock avalanche.





559 Consequently, we consider $\delta = 17^{\circ}$ and $\alpha_{s0} = 0.8$ – in addition to the parameter values given in Ta-560 ble 2 - useful for back-calculating the Acheron rock avalanche. The simulation is repeated with 561 exactly this combination (Experiment 2B). Fig. 12 shows the maps of HMax and HD, both corresponding reasonably well to the OIA and the ODA, respectively. The slightly larger simulated 562 than observed deposit (see Fig. 12b) corresponds to $FoC \approx 1.25$, the almost perfect correspondence 563 of the observed and simulated termini corresponds to $\Delta L \approx 0$. This means that the fact that the re-564 565 sult is rather conservative than non-conservative (FoC > 1) relates to lateral spreading rather than to the travel distance of the rock avalanche. Supp. 2 illustrates the time evolution of the flow 566 567 height in Experiment 2B.





572 **4 Discussion**

573 The key purpose of the present article is to provide a general introduction to the key functionali-574 ties of the computational tool r.avaflow. Thereby, the simulated patterns of flow height in Experiment 1 (see Sect. 3.1) appear plausible, and the correspondence of the observed and simulated 575 576 deposition areas in Experiment 2B (see Sect. 3.2) appears reasonable. Yet, these experiments can 577 neither replace model validation efforts with observed process chains or interactions, nor thor-578 ough multi-parameter sensitivity analysis and optimization efforts, which will both be the subjects 579 of future research. Fully documented two-phase process chains with readily available pre- and 580 post-event DTMs are scarce. Preliminary r.avaflow results for the 2012 Santa Cruz multi-lake out-581 burst flood in the Cordillera Blanca, Peru (Emmer et al., 2016) are, however, promising.

582 Experiment 2 serves for the demonstration of the parameter sensitivity analysis and optimization 583 functions of r.avaflow. The outcomes may be different when changing the cell size or any of the 584 flow parameter values (see Table 2). Making r.avaflow fit for forward predictions will require a 585 thorough multi-parameter sensitivity analysis and optimization campaign involving a large num-586 ber and variety of well-documented events. Thereby we aim at obtaining guiding parameter val-587 ues - or, more appropriately, guiding parameter ranges - for mass flow processes of different types 588 and magnitudes. Approaches to perform such analyses are readily available, and some of them can 589 be directly coupled to r.avaflow (Fischer, 2013; Fischer et al., 2015; Aaron et al., 2016; Krenn et al., 590 2016). However, due to the complex nature of two-phase mixture flows, r.avaflow depends on a 591 relatively large number of flow parameters, a fact that represents a particular challenge in terms of 592 the computational resources as well as in terms of visualization and interpretation of the results of 593 multi-parameter studies.

594 r.avaflow represents a modular framework, allowing for the future enhancement of its particular 595 components. One issue concerns the numerical implementation of the two-phase model equa-596 tions, combining topography-following coordinates with the quadratic cells of the raster data giv-597 en in GIS coordinates (see Sect. 2.3). As in comparable simulation tools (e.g. Christen et al., 2010a, b; Hergarten and Robl, 2015), approximations are currently used for coordinate transformation in 598 599 r.avaflow. This issue is closely related to the fact that the model equations that are commonly ex-600 pressed in topography-following coordinates are hardly compatible with the data given in GIS co-601 ordinates.

A detailed and fully discrete description of the TVD–NOC Scheme exists in the literature
(Wang et al., 2004), and the scheme served well for various theoretical test cases (e.g., Pudasaini
et al., 2014; Kafle et al.; 2016; Kattel et al., 2016). However, we also identify two major drawbacks:

Although the numerical scheme itself should be shock capturing and volume preserving
 (Tai et al., 2002; Wang et al., 2004) these properties may not fully hold in practical appli cations (i.e. bounded gravitational mass flows with well-defined margins over complex to pography). The complementary functions with ID 1–3 introduced in Sect. 2.4 partly com pensate for the issues raised.

 For real flow applications, full handling of the evolution of the basal topography is not straightforward: the TVD-NOC scheme may introduce diffusion even though the evolution of the basal topography is not a standard transport equation. Entrainment is therefore, as a first step, included as a complementary function.
 The numerical scheme employed will have to be enhanced to directly and effectively incorporate the complementary functions outlined in Sect. 2.4 in a fully consistent way. Extensions of similar

- 617 schemes have been tested for generic examples (e.g. Zhai et al., 2015) and could serve as a valuable
- 618 basis also to implement a mechanical model for erosion, entrainment and deposition (Pudasaini
- 619 and Fischer, 2016). On the one hand such an erosion model may build on existing concepts (e.g.,
- Fraccarollo and Capart, 2002; Sovilla et al., 2006; Medina et al., 2008; Armanini et al., 2009; Crosta
 et al., 2009; Hungr and McDougall, 2009; Le and Pitman, 2009; Iverson, 2012; Pirulli and Pastor,
- 622 2012). On the other hand, it may further require some fundamentally new ideas with regard to
- 623 deposition.

624 **5** Conclusions and outlook

625 We have introduced r.avaflow, a multi-functional open source GIS application for simulating twophase mass flows, process chains and interactions. The outcomes of two computational experi-626 627 ments have revealed that r.avaflow (i) has the capacity to simulate complex solid-fluid process in-628 teractions in a plausible way; and (ii) after the optimization of the basal friction angle and the solid 629 content of the release mass, reasonably reproduces the observed deposition area of a documented 630 rock avalanche. However, it was out of scope of the present work to validate the results obtained 631 for complex process interactions against observed real-world data, or even to conduct a compre-632 hensive multi-parameter optimization campaign. Such efforts will be the next step towards making r.avaflow ready for the forward prediction of possible future mass flow events. Thereby we 633 634 will attempt to establish guiding parameter values for different types of processes and process 635 magnitudes.

At the same time we have identified a certain potential for the future enhancement of some the
components of r.avaflow. The key challenges will consist in (i) integrating the model equations in
an up-to-date numerical scheme, allowing to directly include the complementary functions; and
(ii) replacing the empirical entrainment model with a mechanical model for entrainment and deposition.

641 **Code availability**

642 The model codes, a user manual, the scripts used for starting the computational experiments pre-643 sented in Sect. 3, and the GRASS locations with the spatial data necessary for reproducing the ex-644 periments are available at http://www.avaflow.org.

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