



Generalization and application of the flux-conservative thermodynamic equations in the AROME model of the ALADIN system

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Abstract. General yet compact equations are presented to express the thermodynamic impact of physical parameterizations in a NWP or climate model. By expressing the equations in a flux-conservative formulation, the conservation of mass and energy is a built-in feature of the system. Moreover, the centralization of all thermodynamic calculations guarantees a consistent thermodynamical treatment of the different processes. The generality of this physics-dynamics interface is illustrated by applying it in the AROME NWP model. The physics-dynamics interface of this model currently makes some approximations, which typically consist of neglecting some terms in the total energy budget, such as the transport of heat by falling precipitation, or the effect of diffusive moisture transport. Although these terms are usually quite small, omitting them from the energy budget breaks the constraint of energy conservation. The presented set of equations allows to get rid of these approximations, in order to arrive at a consistent and energy-conservative model. A verification in an operational setting shows that the impact on monthly-averaged, domain-wide meteorological scores is quite neutral. However, under specific circumstances, the supposedly small terms may turn out not to be entirely negligible. A detailed study of a case with heavy precipitation shows that the heat transport by precipitation contributes to the formation of a region of relatively cold air near the surface, the so-called cold pool. Given the importance of this cold pool mechanism in the life-cycle of convective events, it is advisable not to neglect phenomena that may enhance it.

1 Introduction

The conservation of mass and energy are important characteristics of a numerical atmospheric model. Especially in view of the application in climate studies, even small violations of the conservation laws can accumulate over a long integration time, and lead to faulty results (Staniforth and Wood, 2008; Lucarini and Ragone, 2011). Atmospheric forecast models are usually constructed by combining a dynamical core with physical parameterizations. In general, the dynamical core describes the atmospheric behaviour up until the resolved scales, while the physical parameterizations estimate the effect of subgrid processes (Gassmann, 2013).



A lot of research has been spent in designing dynamical cores that conserve mass and energy (Thuburn, 2008). Common strategies include a careful selection of the prognostic variables (Ooyama, 1990, 2001; Klemp et al., 2007), the formulation of the equations in flux-form (Sato, 2003), or taking advantage of properties of the Hamiltonian character of the atmospheric equations (Salmon, 2004; Gassmann and Herzog, 2008; Zängl et al., 2014). In contrast with these efforts on the dynamical core, the energy conservation and consistent thermodynamics seem to be less of a priority in the development of the physical parameterizations, or in the way they are coupled to the dynamical core.

A possible explanation is that the thermodynamics of the dynamical core are less complicated than those of the physical parameterizations. First, the dynamics are usually considered adiabatic and reversible (except for numerical diffusion) (Gassmann, 2013). The physics, on the other hand, include mass and energy exchange with the surface, as well as radiative fluxes at the top of the atmosphere. They constitute an open thermodynamic system, for which the conservation laws are more difficult to enforce. Second, the dynamics are formulated assuming an atmosphere consisting only of a (perfect) gas, while the physics should take into account the effect of water condensates. Third, it is tempting to consider physics parameterizations as plug-compatible, i.e. they are considered as a black box which, given an atmospheric state, returns an effect on the dynamical prognostic variables. Unfortunately, this plug-compatibility seems to go at the expense of carefully investigating the thermodynamic consistency between the dynamical core and the physics parameterization.

There is, however, an increased interest in different aspects of the coupling of physical parameterizations to the dynamical core. One of the issues is the organization of the timestep. This problem has been studied with academic toy-models (see, e.g. Caya et al. (1998); Staniforth et al. (2002); Termonia and Hamdi (2007)), as well as in 3D models (Hortal, 2002; Williamson, 2002). The thermodynamic aspects of the physics-dynamics coupling is another topic that deserves some attention. Although some attempts have been made to rigorously formulate the equations for a multicomponent atmosphere (Ooyama, 2001; Bannon, 2002), it remains a fact that many operational models make several ad-hoc approximations (Bryan and Fritsch, 2002). Catry et al. (2007), hereafter CGTBT07, presented a set of equations that express the effects of physics parameterizations in a flux-conservative formulation. The advantage of this approach is that this system is inherently mass- and energy-conservative.

The current paper develops the proposal of CGTBT07 further by generalizing it for a system with an arbitrary number of hydrometeors with arbitrary interactions between them. The next section presents the equations of this generalized system. In section 3, this set of equations is applied in the AROME numerical weather prediction (NWP) model (Seity et al., 2011), thus allowing to get rid of some approximations that are currently made. Section 4 discusses the impact on the meteorological results, both by means of monthly scores and with an in-depth case study of a cold pool formation under heavy precipitation. Section 5 presents the conclusions.

2 Formulation of the generalized flux-conservative equations

2.1 Framework of hypotheses

Because the behaviour of the atmosphere is too complex to describe exactly, every numerical model needs to make simplifying hypotheses. This is no different for the work described in the current paper. It is not our aim to present a set of equations which



is exact in the sense that it is free of approximations. But a crucial aspect of the work presented in CGTBT07, is that the set of hypotheses is defined from the very beginning. This is important for two reasons. First, it ensures that the simplifications act consistently throughout the model. Second, it allows to set some non-negotiable constraints. For instance, the conservation of energy must be satisfied, no matter what other simplifications are made. This approach of setting the simplifying hypotheses from the beginning contrasts with the conventional approach of ignoring supposedly small terms along the way.

The framework of hypotheses is the following:

- A fully barycentric view of air parcels is adopted. This means that all hydrometeors (both suspended and precipitating) are considered as integral parts of the air, and contribute to the parcel's motion, density and heat capacity. This barycentric view has been studied and motivated by many researchers (Wacker and Herbert, 2003; Bott, 2008; Gassmann and Herzog, 2008).
- Water condensates are assumed to have zero volume. This is a common approximation in atmospheric modeling.
- Gases follow Boyle-Mariotte's and Dalton's laws.
- Temperature is homogeneous across all species, even falling hydrometeors. For small hydrometeors, this approximation is easily justified, given their short relaxation time Bott (2008). For larger hydrometeors, it is a more crude approximation, but it goes together with the barycentric view: since such hydrometeors are considered part of the parcel, they also take the parcel's temperature.
- The specific heats of all species are constant with temperature.
- The latent heats vary linearly with temperature:

$$L_{i|l}(T) = L_{i|l}(T_0) + (c_{pv} - c_{il})T \quad (1)$$

with $T_0 = 0\text{K}$.

It should be mentioned that this same framework of assumptions has been used by Marquet (2011), Marquet and Geleyn (2013) and Marquet (2015) to cleanly develop moist atmospheric thermodynamic quantities such as moist entropy, moist potential temperature, and moist Brunt-Väisälä frequency.

2.2 The flux-conservative equations for a system with 5 water species

The system considered in CGTBT07 consists of dry air (mass fraction q_d) plus five prognostic water species: vapour (q_v), suspended water droplets (q_l), suspended ice crystals (q_i), rain (q_r) and snow (q_s). For this system, the following equations are



derived for the time evolution of the prognostic species:

$$\frac{\partial q_v}{\partial t} = g \frac{\partial}{\partial p} \left[R_{r,v} + R_{s,v} - R_{v,l} - R_{v,i} + \frac{q_v(P_r + P_s)}{1 - q_r - q_s} - D_v \right] \quad (2)$$

$$\frac{\partial q_l}{\partial t} = g \frac{\partial}{\partial p} \left[R_{v,l} - R_{l,r} + \frac{q_l(P_r + P_s)}{1 - q_r - q_s} - D_l \right] \quad (3)$$

$$\frac{\partial q_r}{\partial t} = g \frac{\partial}{\partial p} [R_{l,r} - R_{r,v} - P_r] \quad (4)$$

$$5 \quad \frac{\partial q_i}{\partial t} = g \frac{\partial}{\partial p} \left[R_{v,i} - R_{i,s} + \frac{q_i(P_r + P_s)}{1 - q_r - q_s} - D_i \right] \quad (5)$$

$$\frac{\partial q_s}{\partial t} = g \frac{\partial}{\partial p} [R_{i,s} - R_{s,v} - P_s] \quad (6)$$

$$\frac{\partial q_d}{\partial t} = g \frac{\partial}{\partial p} \left[\frac{q_d(P_r + P_s)}{1 - q_r - q_s} - D_d \right] \quad (7)$$

In these equations, P_k denotes precipitation fluxes, and D_k denotes diffusive fluxes. R_{k_1,k_2} denote pseudofluxes between two water species (i.e. the mass-weighted integral of the transfer due to phase changes). Although pseudo-fluxes are less intuitive to reason with than tendencies, they allow to write the equations in a flux-conservative way. Also note that it is necessary that $\sum_{k=d,v,i,l} D_k = 0$ to ensure that all terms on the right hand sides cancel out.

The thermodynamic equation for this system is as follows:

$$\begin{aligned} \frac{\partial}{\partial t}(c_p T) = & -g \frac{\partial}{\partial p} [(c_l - c_{pd})P_r T + (c_i - c_{pd})P_s T \\ & - (\hat{c} - c_{pd})(P_r + P_s)T + J_s + J_{rad} \\ & - L_l(T_0)(R_{v,l} - R_{r,v}) - L_i(T_0)(R_{v,i} - R_{s,v})] \end{aligned} \quad (8)$$

where $\hat{c} = \frac{c_{pd}q_d + c_{pv}q_v + c_iq_i + c_lq_l}{1 - q_r - q_s}$, and J_s and J_{rad} are the diffusive and radiative heat fluxes, respectively.

15 A full discussion of these equations is given in CGTBT07, but we would like to stress the following characteristics:

- All equations are flux-conservative, i.e. every right hand side is a divergence of a summation of fluxes. The importance of this property cannot be underestimated, because it means that this system intrinsically conserves mass and energy. Put somewhat simplistically, in a flux-conservative system, the only way energy or mass can leave one model layer, is by transporting it to an adjacent layer. Therefore, mass and energy are conserved by design of the system.

20 – The precipitation fluxes P_r and P_s are relative to the center of mass. They relate to the absolute precipitation fluxes P_r^* and P_s^* through

$$P_r = (1 - q_r)P_r^* - q_r P_s^* \quad (9)$$

$$P_s = -q_s P_r^* + (1 - q_s)P_s^* \quad (10)$$

– The latent heats of sublimation and condensation L_i and L_l that appear in the right hand side, are evaluated at $T_0 = 0\text{K}$. This does not mean that the temperature-dependency of these latent heats is neglected. What happens is that this dependency is accounted for by putting c_p inside the time derivative on the left hand side.



Table 1. Variables λ_{kj} for the system of CGTBT07

		process	$r \rightarrow v$	$v \rightarrow l$	$l \rightarrow r$	$s \rightarrow v$	$v \rightarrow i$	$i \rightarrow s$
		j	1	2	3	4	5	6
species	k							
v	1		-1	1	0	-1	1	0
l	2		0	-1	1	0	0	0
r	3		1	0	-1	0	0	0
i	4		0	0	0	0	-1	1
s	5		0	0	0	1	0	-1

2.3 The generalized flux-conservative equations

Despite the clear strength of the equations proposed by CGTBT07, their application is not straightforward because of the fixed number of water species, and because of the fixed set of interactions between them (six pseudofluxes). More advanced microphysics schemes often consider more water species, including graupel and/or hail (Lascaux et al., 2006), separating convective and nonconvective fractions of hydrometeors (Piriou et al., 2007), ... Also the fact that only six transfer mechanisms between the water species are possible is limiting. For instance, snow melting cannot be represented directly, but it should be written as a combination of snow sublimation ($R_{s,v}$) and rain condensation ($R_{r,v}$). Although thermodynamically fully correct, it would be better to have a system that digests all kinds of transfers between water species.

It is, however, possible to generalize the equations from CGTBT07, without touching the important characteristics. We introduce the following notation: n is the number of water species, $k = 1, \dots, n$ denotes a single water species, and by convention, $k = 0$ denotes the dry air component. The specific heat capacities at constant pressure of the different species are written generically as c_k , the latent heat of evaporation or sublimation at 0 K is written as L_k^0 . Now let R_j be a pseudoflux representing the conversion process between a source water species k_j^s and a target water species k_j^t . We consider an arbitrary number m of such conversion processes. We now define variables $\lambda_{kj} = \delta_{k,k_j^s} - \delta_{k,k_j^t}$ for $k = 1, \dots, n$ and for $j = 1, \dots, m$. This variable takes value 0 if a species k is not involved in the conversion process j ; it takes value -1 if it is the target species of this process, and it takes value 1 if it is the source species of this process. The variable λ_{kj} will allow to write the time tendency of a water species by summing over all conversion processes, regardless of the role this specific water species plays in each process. Furthermore, a variable $\Lambda_j^0 = L_{k_j^s}^0 - L_{k_j^t}^0$ is defined. This variable is the latent heat released at temperature T_0 under a conversion process with source species k_j^s and target process k_j^t . To clarify these notations, consider the original system of CGTBT07 with 5 water species and 6 conversion processes between them. By convention, we assign $k = 1, \dots, 5$ to water vapour, cloud water, rain, cloud ice crystals and snow, respectively. Tables 1 and 2 give the values of λ_{kj} and Λ_j^0 for the different conversion processes.

Next, a precipitation flux P_k is defined for each component, even for the non-precipitating species (dry air, vapour, cloud water and cloud ice crystals). Contradictory as this may sound, it should be stressed that in our barycentric system, these fluxes



Table 2. Variables Λ_j^0 for the system of CGTBT07

process	$r \rightarrow v$	$v \rightarrow l$	$l \rightarrow r$	$s \rightarrow v$	$v \rightarrow i$	$i \rightarrow s$
j	1	2	3	4	5	6
Λ_j^0	$L_l(T_0)$	$-L_l(T_0)$	0	$L_i(T_0)$	$-L_i(T_0)$	0

express the motion of the species with respect to the center of mass of the parcel. When precipitating species are present, the suspended species will move upward with respect to the mass center. The relative precipitation fluxes P_k are determined from the absolute fluxes P_k^* as

$$P_k = P_k^* - q_k \sum_{i=0}^n P_i^* \quad (11)$$

- 5 where the absolute precipitation fluxes of suspended species can be taken to be zero. It should be noted that the strict distinction in CGTBT07 between suspended and precipitating species is somewhat arbitrary and scale-dependent. Indeed, also the so-called suspended cloud water species can undergo a slow sedimentation. This arbitrary distinction is no longer necessary in the generalized set of equations that is presented here. Similarly to defining (relative) precipitation fluxes for all species, also diffusive fluxes D_k are defined for all species, where the diffusive fluxes of precipitating species can be taken equal to zero.
- 10 These notations make it possible to formulate the specific mass equations and the thermodynamic equation as follows:

$$\frac{\partial q_k}{\partial t} = -g \frac{\partial}{\partial p} \left[\sum_{j=1}^m \lambda_{kj} R_j + P_k + D_k \right], \quad \text{for } k = 0, \dots, n \quad (12)$$

$$\frac{\partial}{\partial t} (c_p T) = -g \frac{\partial}{\partial p} \left[\sum_{k=0}^n c_k P_k T + J_s + J_{rad} - \sum_{j=1}^m \Lambda_j^0 R_j \right] \quad (13)$$

- These equations generalize the ones from CGTBT07 in three ways: (i) an arbitrary number n of water species is considered; (ii) an arbitrary number m of inter-species conversion processes is considered; and (iii) the strict distinction between suspended and precipitating species can be abandoned. The fact that quite compact equations are obtained, which are valid for all components of the atmosphere, is an additional indication of the strength of the barycentric approach.
- 15

2.4 Remarks

Some comments should be given on the application area of the physics-dynamics interface presented in Eqs. (12–13).

- The fact that these equations are very general, opens the road for a ‘plug-compatible’ view of physics parameterizations. Indeed, the only output that is needed from a parameterization are diffusive and precipitative transport fluxes, pseudo-fluxes for phase changes and the radiative and diffusive energy fluxes. The physics-dynamics interface then receives these quantities and determines the effect on the prognostic variables of the model, thereby ensuring satisfaction of the conservation of mass and energy, as well as consistency in the thermodynamic assumptions.
- 20



However, it should be kept in mind that other conditions should be met before parameterizations can really be considered plug-compatible. A first aspect is that interactions exist between parameterizations. For instance, the parameterization of cloud processes will affect the radiation scheme. This kind of interactions should properly be accounted for when plugging a new parameterization into a model. It is interesting to see that the recommendations that were made in Kalnay et al. (1989) regarding the design of parameterizations and their interactions, are still relevant at present. A second aspect is that parameterizations should also obey the second law of thermodynamics (Gassmann and Herzog, 2014). This condition cannot be enforced at the higher level of the physics-dynamics interface, and should be taken care of at the level of the parameterization itself.

- A common assumption in atmospheric modeling (although it is often made implicitly) is that all vertical transport is compensated for by a flux of dry air (Courtier et al., 1991). This assumption ensures the conservation of total mass in the atmosphere. From a barycentric point of view, this means that the center of mass of an air parcel does not move vertically. The equations (12–13) remain valid under this assumption, if the relative flux of dry air is defined as $P_d = -\sum_{k=1}^n P_k$.
- The Eqs. (12–13) are theoretically only valid for a model using the hydrostatic primitive equations. In a fully compressible system, the diabatic heating from the physics parameterizations does not only affect the temperature equation, but also the continuity equation (Laprise, 1998). CGTBT07 present the extension of their flux-conservative system to the fully compressible case. An entirely equivalent development can be made for the generalized equations presented in this paper.

However, as shown by Malardel (2010), the impact of including the heat from parameterizations in the continuity equation is quite limited. It is therefore safe to apply the presented equations also in a non-hydrostatic model.

- The fact that Eq. (13) describes the evolution of enthalpy $h = c_p T$, does not mean that this variable should become the prognostic thermodynamic variable of the model. A model that uses temperature T as the prognostic thermodynamic variable, can also use the presented interface. After all, one can easily calculate the total heat capacity tendency as follows:

$$\frac{\partial c_p}{\partial t} = \sum_{k=0}^n c_k \frac{\partial q_k}{\partial t}, \quad (14)$$

which in turn can be used to determine the temperature tendency from the enthalpy tendency:

$$\frac{\partial T}{\partial t} = \frac{1}{c_p} \left(\frac{\partial}{\partial t} (c_p T) - T \frac{\partial c_p}{\partial t} \right) \quad (15)$$

The importance of writing Eq. (13) as a time evolution of enthalpy only becomes clear in the time-discretized case:

$$c_p^{t+\Delta t} = c_p^t + \Delta t \sum_{k=0}^n c_k \frac{\partial q_k}{\partial t} \quad (16)$$

$$T^{t+\Delta t} = \frac{1}{c_p^{t+\Delta t}} \left(c_p^t T^t + \Delta t \frac{\partial}{\partial t} (c_p T) \right) \quad (17)$$



Using an enthalpy-based formulation of the interface is reflected in the use of $c_p^{t+\Delta t}$ in the right-hand side of Eq. 17. Although this appears to be a small detail, it is crucial in ensuring the conservation of energy. The importance of appropriately discretizing a conserved nonlinear variable such as enthalpy is also indicated by Gassmann and Herzog (2008).

As a side remark, it can be noted that simply adding temperature tendencies from several parameterizations cannot lead to an energy-conserving atmospheric model. For example, consider a model containing two parameterizations (indicated with a and b), yielding a respective change in temperature of ΔT^a and ΔT^b , and a respective change in heat capacity of Δc_p^a and Δc_p^b . Suppose that each of these parameterizations is energy-conservative in itself. Then the joint effect of the parameterizations cannot be expressed as $\Delta T = \Delta T^a + \Delta T^b$, but it should be determined as

$$\Delta T = \frac{(c_p^t + \Delta c_p^a)\Delta T^a + (c_p^t + \Delta c_p^b)\Delta T^b}{c_p^t + \Delta c_p^a + \Delta c_p^b}$$

10 3 Application of the flux-conservative equations in the AROME model

AROME is a limited area model that was developed at Météo-France and is now a configuration inside the ALADIN system. It became operational in France in 2008, and it is currently used in many European countries of the ALADIN and HIRLAM consortia. AROME uses a nonhydrostatic, fully compressible dynamical core (Bubnová et al., 1995; Bénard et al., 2010), with the same spectral semi-implicit semi-Lagrangian space-time discretization as the ECMWF's IFS model. The height coordinate is terrain-following mass-based (Laprise, 1992). The physics parameterizations in AROME originate from the Méso-NH research model (Lafore et al., 1998). The Méso-NH model has a dynamical core which is explicit in time, with a staggered spatial grid and a height-based vertical coordinate, so it is substantially different from the AROME dynamical core. The plugging of the physics from this model to a different dynamical core was quite challenging, and several approximations were made during this process.

A first approximation that is made in the current AROME physics-dynamics interface concerns the heat transport by precipitation. From Eq. (13), it is clear that precipitation has two thermodynamic effects. Falling species modify the composition of the atmosphere, so they also change the specific heat capacity $c_p = \sum_{k=0}^n c_k q_k$. Secondly, if a vertical temperature gradient exists, falling species are heated, thus cooling down the surrounding air. The effect on the enthalpy due to a change in c_p is given by

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec, c_p} = T \sum_{k=0}^n c_k \frac{\partial q_k}{\partial t} = -gT \frac{\partial}{\partial p} \sum_{k=0}^n c_k P_k \quad (18)$$

while the second effect due to a vertical temperature gradient is given by

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec, heat} = -g \sum_{k=0}^n c_k P_k \frac{\partial T}{\partial p} \quad (19)$$

The combination of these two effects indeed corresponds to the effect of precipitation in the right-hand side of Eq. (13):

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec, c_p} + \left(\frac{\partial}{\partial t} c_p T\right)^{prec, heat} = -g \frac{\partial}{\partial p} \sum_{k=0}^n c_k P_k T \quad (20)$$



The approximation made by the current physics-dynamics interface in AROME is that it neglects the heat transport effect of precipitation, i.e. the term given in Eq. (19).

A second approximation concerns the effect of diffusive moisture transport (shallow convection and turbulence) in the energy budget. Similar to the effect of precipitation, diffusive moisture transport modifies the total specific heat capacity c_p , and this effect should be accounted for in the energy budget. However, this effect is neglected in the current AROME physics-dynamics interface.

A third approximation is that the values of specific heat capacity c_p and latent heat $L_{il}(T)$ are not consistent between the different parameterizations. For instance, the heat capacity in the radiation scheme only accounts for water vapour and neglects the other hydrometeors ($c_p^{rad} = (1 - q_v)c_{pd} + q_v c_{pv}$). This situation stems from the fact that the different physics parameterizations are developed by different teams, each using their own conventions.

A final approximation by the current physics-dynamics interface in AROME is that the total temperature tendency is obtained by summing the temperature tendencies from the individual parameterizations. As indicated in the previous section, such an approach cannot lead to an energy-conserving system.

Although it can be expected that the net effect of these approximations and inconsistencies is quite limited, the generalized physics-dynamics interface as presented in the previous section offers the possibility to get rid of them in order to take a (admittedly small) step towards a more accurate model. A second motivation to equip the AROME model with the generalized flux-conservative physics dynamics interface is that this opens the route towards importing physics parameterizations from other NWP models, thus allowing a fair comparison of different parameterizations and stimulating scientific progress.

4 Operational impact

The impact of the presented flux-conservative formulation of the physics-dynamics interface is investigated with the AROME operational high-resolution LAM model running at Météo-France. Before April 2015, this model ran on a 739×709 grid with a resolution of 2.5 km. Figure 1 shows the model domain. The timestep is 60 s. The model is provided with lateral boundary conditions by the operational global model 'ARPEGE' from Météo-France. The initial conditions are generated with a 3D-Var data-assimilation (Fischer et al., 2005; Brousseau et al., 2011). At the surface level, precipitation and evapotranspiration imply a net mass-flux across the surface. However, the surface boundary condition of the nonhydrostatic dynamical core of AROME currently does not allow such mass exchange between the soil and the atmosphere. For the experiments described in this section, we are therefore forced to make the above-mentioned approximation that all vertical transport is compensated by a fictitious flux of dry air. All these settings are identical for the operational run (denoted REF) with the temperature-tendency based interface and for the run with the flux-conservative interface (denoted FCI).

4.1 Monthly scores

The daily forecasts during two periods are considered in this section: 1–30 November 2014 and 6 January – 6 February 2015. The first month is characterized by exceptionally mild weather, with numerous episodes of heavy precipitation in the South-

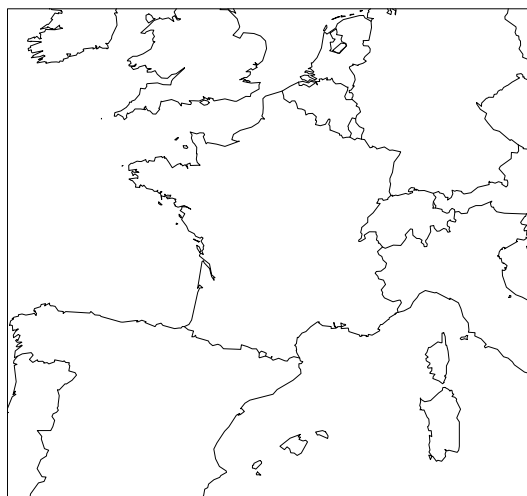


Figure 1. Operational AROME domain with a resolution of 2.5 km.

West of France. The second month was characterized by strong winds and episodes of heavy snowfall. Figures 2 and 3 show bias and rmse for several meteorologic variables for the two periods, respectively. These scores are calculated by comparing the AROME forecasts with observations throughout the French territory. Figures 4 and 5 compare the forecasted precipitation over the two periods. To avoid the problem of the double penalty, the precipitation is verified with the neighbourhood observation
5 Brier skill score (Amodei and Stein, 2009). This score is determined by calculating the probability that a precipitation threshold is exceeded in the vicinity of an observation. By choosing the threshold, one focuses the verification more on light or on heavy precipitation.

The scores indicate that the impact of using the flux-conservative set of equations appears quite limited when considering time- and space-averaged scores as the ones considered here. It should be stressed that no retuning has been done for the
10 experiments with the flux-conservative equations. As a result, no significant improvement of the scores could be expected anyway. The fact that the scores do not change substantially, merely indicates that the approximations that are made in the current temperature-tendency based interface are indeed small on a domain-wide scale. However, as is shown in the next section, some significant differences are observed under specific circumstances.

4.2 Case study of a cold pool originating from heavy precipitation

15 When precipitation evaporates while falling through unsaturated air, it cools its environment. As such, a region of relatively cool air, the so-called cold pool, originates when heavy, localized precipitation occurs, for instance in precipitating convective systems (Fujita, 1959). It has been shown that the cold pool is in fact a key element in the lifecycle of a such systems. On the

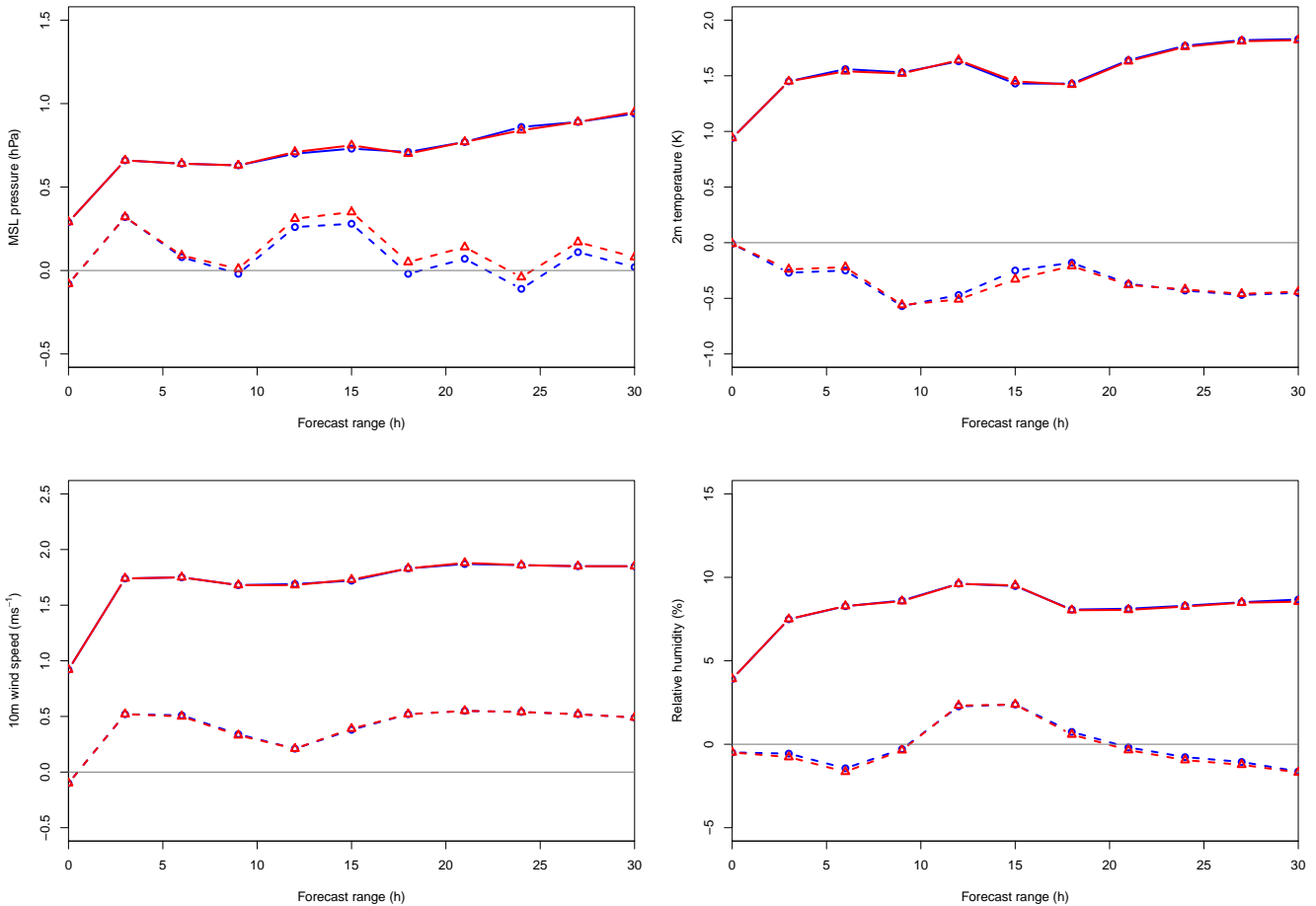


Figure 2. RMSE (solid line) and bias (dashed line) over the period 1 November 2014 – 30 November 2014, for REF (blue circles) and FCI (red triangles).

one side, new convective cells originate at the border of the cold pool and its warmer surroundings, but on the other hand, if the cold pool becomes too strong, it may cut off the supply of warm air to the updraft (Engerer et al., 2008). The cold pool is also accompanied by a meso-scale high pressure area (Fujita, 1959) which plays a crucial role in the wind gusts that go with heavy precipitation. For these reasons, it is no surprise that the representation of the cold pool is also crucial in a NWP model (Engerer et al., 2008; De Meutter et al., 2014).

Although evaporative cooling is the main cause for a cold pool, a second mechanism may enhance it. As precipitation falls from colder layers aloft to hotter layers below, it will be heated by the surrounding air, which in response will cool down (Johnson and Hamilton, 1988). As explained in Sect. 3, this secondary thermodynamic effect (the advection of sensible heat) of precipitation is neglected in the current AROME physics-dynamics interface, while it is correctly accounted for with the

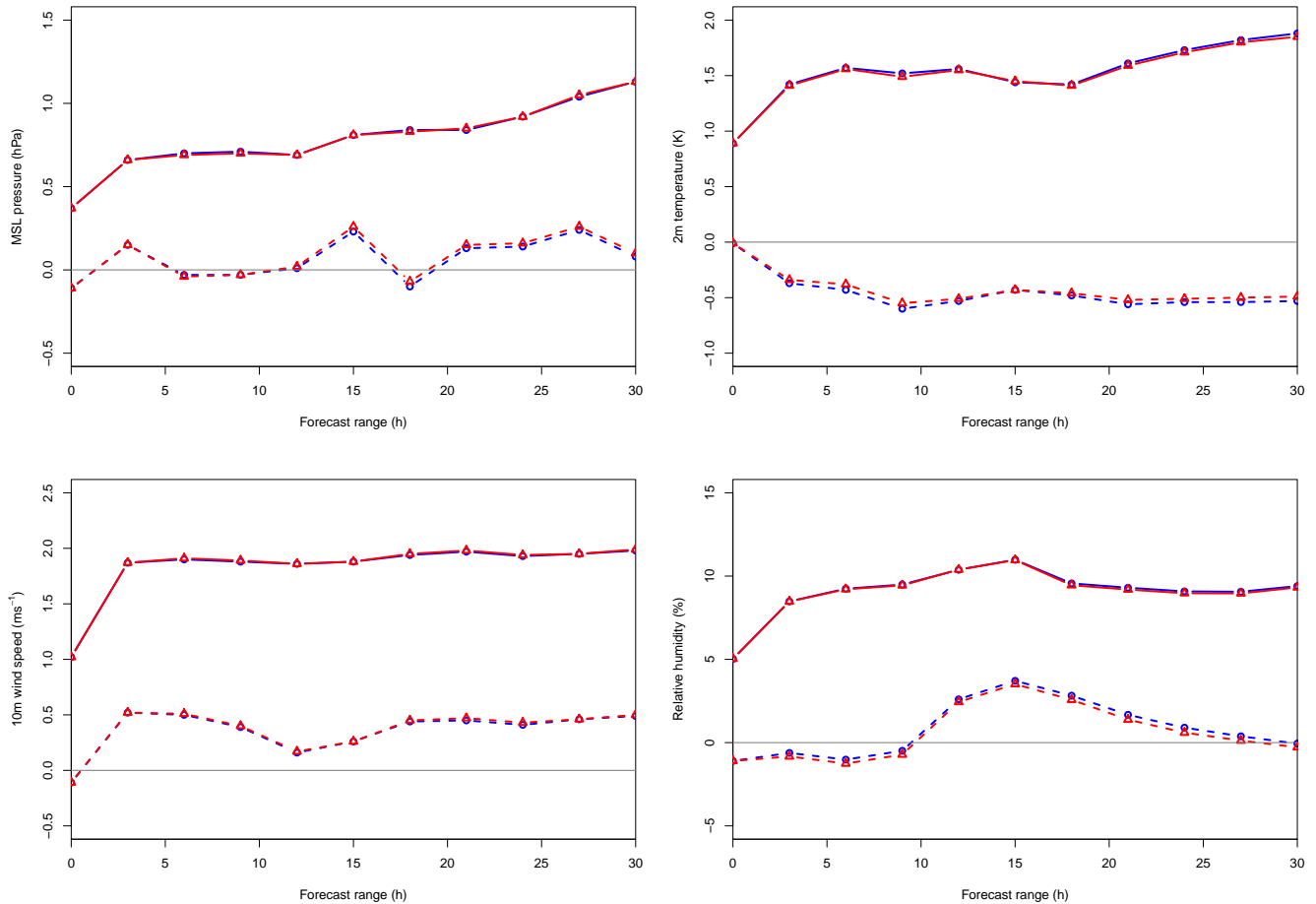


Figure 3. RMSE (solid line) and bias (dashed line) over the period 6 January 2015 – 6 February 2015, for REF (blue circles) and FCI (red triangles).

presented set of flux-conservative equations. One can thus expect that the intensity of a forecasted cold pool depends on which set of equations is used.

This is confirmed when looking at the AROME forecasts over the Balearic islands on 19 January 2015. This case is characterized by convection developing ahead of an active cold front coming from the south. Figures 6a and 6b show the forecasted 1200UTC–1800UTC accumulated precipitation with the current AROME interface (REF) and with the flux-conservative interface (FCI). It is observed that the overall structure of the precipitation is quite similar. However, when comparing the coldpool characteristics of both experiments, important differences appear. Figures 6c and 6d show the differences between both experiments for the 2m temperature and the surface pressure. The temperature is significantly lower with FCI (up to 5 K cooler), and the surface pressure is higher (up to 1.4 hPa).

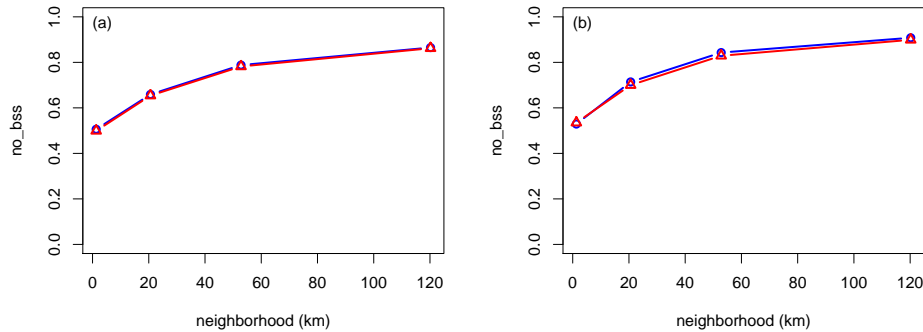


Figure 4. Neighbourhood observation Brier skill score for precipitation between 1200UTC and 1800UTC over the period 1 November 2014 – 30 November 2014, for REF (blue circles) and FCI (red triangles): (a) threshold 2 mm; (b) threshold 10 mm

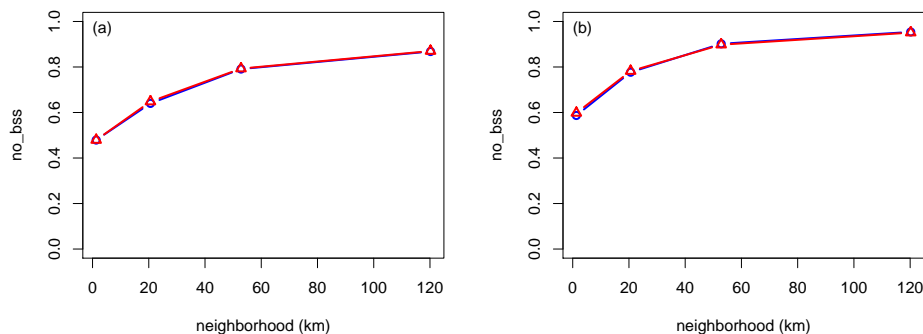


Figure 5. Neighbourhood observation Brier skill score for precipitation between 1200UTC and 1800UTC over the period 6 January 2015 – 6 February 2015, for REF (blue circles) and FCI (red triangles): (a) threshold 2 mm; (b) threshold 10 mm

To further illustrate the impact of the heat transport by precipitation on the cold pool, the vertical profiles in the point as marked in figure 6b are studied for the experiment with the flux-conservative interface. The vertical profile of the precipitation fluxes (figure 7a) shows how snow and graupel originate aloft, they melt to form rain at around 850 hPa, and the rain starts to evaporate below 930 hPa. Figure 7b shows the vertical profile of the two phenomena that are responsible for the development of the cold pool, averaged between 1200UTC and 1800UTC: the latent heat effects from phase changes (solid line), and the falling of cold hydrometeors into warmer air layers (dashed line). It is clear that the second effect is orders of magnitude smaller than the first effect, at least when considering the full vertical extent of the model. However, as shown in figure 7c, the heat transport by hydrometeors is not entirely negligible in the range between the surface and 900 hPa, and thus contributes to the intensity of the cold pool.

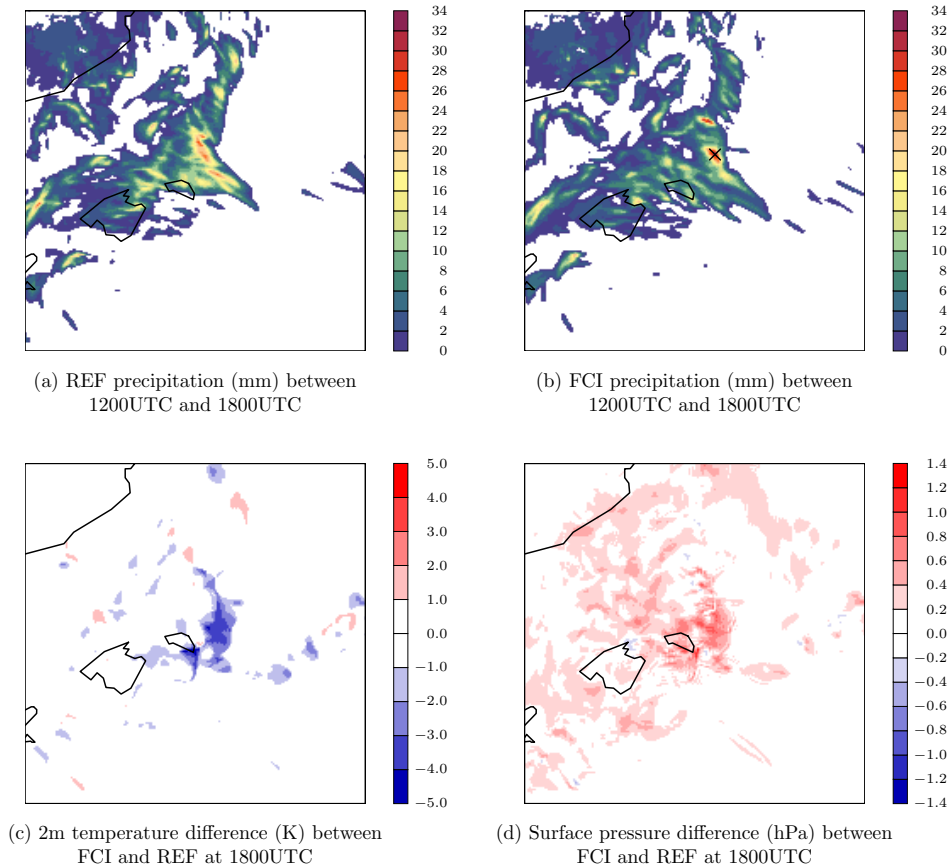


Figure 6. Case of heavy precipitation on 19 January 2015.

No comparison with observations is done for this case, because the purpose of this case study is merely to illustrate that even small terms can have a significant impact under certain conditions. The observations made for this case are in line with the results from Bryan and Fritsch (2002), where neglecting a supposedly small term in the energy budget unexpectedly leads to the worst results.

5 5 Conclusions

This paper starts from the equations presented in Catry et al. (2007) that describe how the effect of physical parameterizations on the dynamical core of an NWP model can be expressed in a flux-conservative way. The main advantage of these equations is that they impose the constraints of energy- and mass-conservation at a higher level in the model than at the level of the individual physical parameterizations. Secondly, centralizing the thermodynamic calculations also guarantees that one consistently respects a predefined framework of hypotheses.

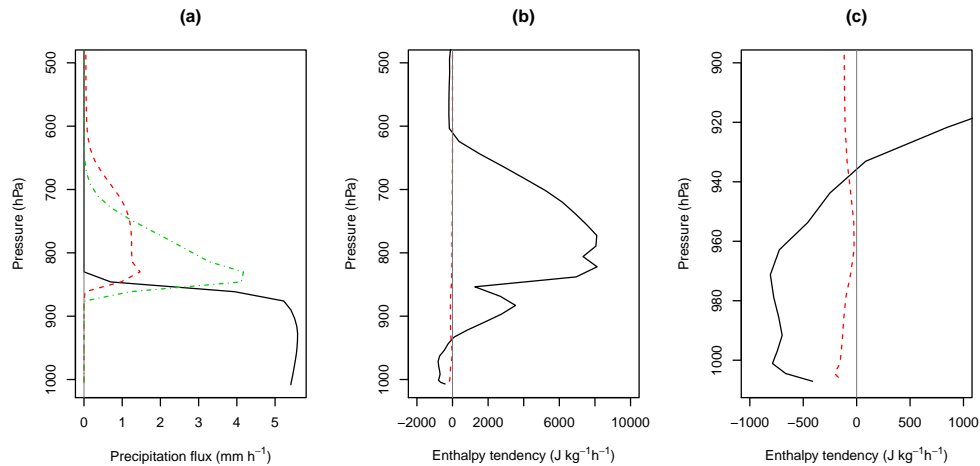


Figure 7. Vertical profiles at 1800UTC in the point indicated in figure 6b for the run with the flux-conservative interface. (a) precipitation fluxes: rain (solid line), snow (dashed line) and graupel (dash-dotted line); (b) cold pool-generating phenomena: latent heat effects due to phase changes (solid line) and sensible heat advection (dashed line); (c) same as (b) but focused on near-surface.

Notwithstanding these clear advantages, the equation set in the mentioned paper also faces limitations that hinder its application in existing NWP models. This paper presents a generalized set of thermodynamic equations that overcomes these restrictions without touching the sound theoretical foundations. More specifically, the presented equations are valid for an arbitrary number of hydrometeors, and can be applied in a model with an arbitrary number of conversion processes between these water species. This has allowed to use this set of equations in the AROME NWP model, which currently uses a physics-dynamics interface that makes some ad-hoc approximations. By moving to the generalized flux-conservative equations, the effect of these approximations can be studied.

Monthly verification scores show that the overall effect of introducing the flux-conservative equations in AROME is quite limited. There is no significant improvement or degradation of these scores. However, it appears that substantial differences may exist in specific cases. A detailed study of a heavy-precipitation case gives the example of the formation of a cold-pool, which is an essential mechanism in the life-cycle of a convective event. As it appears, one mechanism that contributes to the formation of this cold pool is the heat transport by precipitation. This effect is neglected in the current AROME physics-dynamics interface, while it is correctly accounted for in the presented flux-conservative set of equations. In this specific case, this leads to a different surface temperature and surface pressure within the cold pool. This case illustrates the importance of correctly accounting for supposedly small terms in the energy budget, something that is achieved with the presented set of thermodynamic equations.



Code availability

The used ALADIN Codes, along with all related intellectual property rights, are owned by the Members of the ALADIN consortium. Access to the ALADIN System, or elements thereof, can be granted upon request and for research purposes only.

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