Author's response to referees

The responses to the 4 referees have also been posted in the interactive discussion. An updated manuscript with changes marked has also been uploaded. For completeness sake, these documents are also included here.
The most important changes to the manuscript are the following:

- Page 1, line 13; Page 2, line 25; Page 5, line 8: Clear definition of the scope of the paper.
- Page 4, line 13: Extensive explanation and motivation of the concept of pseudofluxes.
- Page 5, line 19: Working out the relation between absolute and relative precipitation fluxes.
- Page 6, line 12: Explanation of why the latent heat values in the right-hand side of the thermodynamic equation are evaluated at the reference temperature $T_0 = 0\mathrm{K}$.
- Page 8, line 28; Page 12, line 19: Better explanation and motivation of the assumption of a compensentating fictitious flux of dry-air.
- Page 10, line 1: Extending the discretization of a two-process case to a time-split case.
- Page 12, line 32: Indicating the limitations of a standard validation.
- Page 18, line 4: More explicit indication of the value of this work.
- Page 18, line 14: Indication of topics for future research.

Author's response to Referee #1

Referee's comment: *A general comment is that the tone of the manuscript is somewhat apologising, defendig. The authors feel that the influence of the suggested formulations on weather forecast is small and try to convince the reader that in spite of this it should still be applied in the operational NWP model. They do not discuss systematically possible problems related to the introduction of the new formalism, but a feeling remains that such may exist as they try kind of convince invisible opponents?*

The authors admit that they have been very careful in their formulations. This general comment aside, we agree that the virtues of the approach presented in this manuscript could be accentuated more. (see also next comment)

Referee's comment: *Based on the information given in the manuscript it is evident that the system of equations if physically logical and well based, allows using less simplifying assumptions and a clear definition of the remaining ones, conserves energy and mass. If, in addition, the application of the new approach does not lead to worse weather forecast (neutral impact), computational cost does not increase significantly, the model code does not grow extremely complicated etc, then it seems natural that it should be applied.*

We obviously believe in the value of this work, and we are grateful that it is acknowledged by this referee. We share the referee's view that the advantages of the presented set of equations are sufficient to justify their application (given the neutral impact etc.). A sentence to emphasize this will be added to the conclusions of the manuscript.

Referee's comment: *In this context, it might be good to mention the limitations of the standard station verification applied to validate research results against surface pressure, screen-level temperature and relative humidity and anemometer-level wind when the verification is* (sentence truncated in Referee's document)

This is a good suggestion. Implicitly, the manuscript already points out the limitations of standard verification by the inclusion of a case-study, but we agree that it should be stated explicitly. A paragraph has been added to the manuscript to amend this.

Referee's comment: *In this sense, the key component seems to be formulation of the pseudofluxes, related to the phase changes between the chosen prognostic hydrometeor species, water vapour (and dry air). It would be good to discuss concepts related to these fluxes, perhaps using an example of some of the existing and/or not yet introduced R_j terms (conversion between cloud ice crystals and precipitating snow, cloud liquid droplets and cloud ice crystals, for example). There is most probably a whole system of microphysics parametrizations behind any of these terms. Not all details should be discussed but the principles discussed and references given, to allow understanding the concepts not only at the level of general formalism.*

The concept of pseudofluxes is indeed the key to the flux-conservative formulation. However, this does not mean that the physical parameterizations should use these pseudofluxes internally. In fact, the AROME microphysics are internally formulated in terms of local tendencies $\partial q_k/\partial t$. In order to use the flux-conservative equations, these tendencies are then converted to pseudofluxes with the formula

$$R_j = \int_0^p \frac{1}{g}\frac{\partial q_k}{\partial t} dp \tag{1}$$

The referee is entirely right that the manuscript is lacking a more detailed explanation of why pseudofluxes are introduced, and how they relate to (the more conventional) local tendencies. A section will be added to the manuscript to remediate this.

As the referee indicates, the internal machinery of the (microphysical) parameterizations that provide the pseudofluxes is quite complex and a topic in itself. References to such parameterizations are already given in the manuscript, e.g. Lascaux et al. (2007). The authors therefore believe that the discussion of the internals of such schemes falls outside the scope of the presented manuscript.

Referee's comment: *Another issue are the limitations of the approach. Three limitations are mentioned in the text but not discussed systematically: 1) interactions between microphysics related to the radiation transfer, on one hand, and that related to the evolution of clouds and precipitation, on the other hand, 2) interactions related to the surface, e.g. between moisture in air and in the soil, 3) the presented formulations being based on the hydrostatic assumption. In this case, the equations were evidently applied in the non-hydrostatic AROME. However, no details are given, but mentioned that it is safe to apply the hydrostatic equations also in the nonhydrostatic model (p.7,l.19). In the conclusions, the possibility to approach in the future these now missing interactions related to radiation and surface could be outlined.*

1. Interactions between the microphysics parameterization and the radiation parameterization are not a limitation of the set of equations itself. The scope of the presented manuscript is the interaction between the parameterizations and the dynamical core, but *not* the interaction between one parameterization and another. To indicate that such interactions exist, the example is given of cloud microphysics influencing radiation. The scope limitations of the present work will be mentioned more explicitly in the introduction section.

2. The referee points out correctly that the interaction with the surface is somewhat underexposed in the manuscript. Regarding this topic, we would like to make the following remarks:

   - The presented set of equations describes the impact of physical parameterizations on atmospheric prognostic variables. In this sense, the evolution of the (sub)surface prognostic variables falls outside the scope of this document.
   - The presented flux-based equations do not pose any direct limitations regarding the interaction with the surface. Moreover, they are directly compatible with the work of Best et al. (2004), who present a generalized flux-based coupling between the surface scheme and the atmospheric model. The AROME model uses the interface from Best et al. (2004) to couple its externalized surface scheme SURFEX to the upper-air parameterizations.

   These points are added to the manuscript.

3. The issue of applying the presented set of equations in a non-hydrostatic model like AROME is discussed in section 2.4. The following 2 points are made there:

   - The extension of the flux-based equations to the non-hydrostatic case is pretty straightforward, as is shown in Catry et al. (2007). The main consequence of this extension is that heat appears as a source not only in the temperature equation, but also in the continuity equation.
   - Evidence is found in literature that the impact of the source term in the pressure equation is quite limited. This is not just 'mentioned' in the manuscript, but a reference is given.

   Given these points, we have chosen to use the 'hydrostatic formulation' in the nonhydrostatic AROME model. Maybe this was not sufficiently clear from the manuscript, so it has been rephrased somewhat in section 2.4, and it is repeated in section 4. A sentence is also added in the conclusions to mark an in-depth assessment of using the nonhydrostatic formulation of the flux-based thermodynamic equations as an interesting future research topic.

Referee's comment: *My specific comments are given as comments in the manuscript pdf*

We agree with most of these valuable suggestions. The manuscript will be adapted accordingly.

Author's response to Referee #2

Referee's comment: *It is written in a clear way, and I recommend publication upon a few minor edits.*
Page 3: Line 14: relaxation time Bott (2008) → relaxation time, Bott (2008).
Page 3: Line 17: heats → heat
Page 3: Line 18: heats → heat
Page 4: Line 24: heats → heat, appear in → appear on
Page 4: Line 25: these latent heats → the latent heat
Page 5: Line 5:(Piriou et al., 2007), ... Also → (Piriou et al., 2007). Also
Page 5: Line 7: rain condensation → rain evaporation (?)
Page 5: Line 8: all kinds of transfers → all kind of transfers
Page 5: Line 11: It would be helpful to define j on this line before it is used, e.g k=0 denotes the dry air component, and j denotes the the conversion process.
Page 5: Line 11: heat capacities → heat capacity
Page 5: Line 15: if you define j further up, remove in the conversion process j from this line.
Page 8: Section 3. It would be clearer if you changed the word current to previous or something alike everywhere in this section. The first time I read through the section I thought it explained the new interface in the current study, which was confusing.
Page 10: Line 2: Meteorologic → Meteorological

The authors agree with most of these comments. The manuscript is adopted accordingly.

Author's response to Referee #3

Referee's comment: *The equation set (2-8) is only the equation set for the physics, not for the whole model. This should be made clear.*

The scope of the presented manuscript is indeed restricted to the equation set of the physics, something that was not sufficiently clear in the original manuscript.

The introduction is modified in order to set this scope more clearly. Also at the point where the equations are introduced, it is now stated explicitly that these equations only describe the physics part. The abstract and conclusions section also have been modified to emphasize this important restriction of the scope.

Referee's comment: *It is not clear why pseudo-fluxes are employed to describe source terms. I think that this makes the issue unecessarily unintuitive. Why the flux-conservative form is enforced here? Is there a coding style advantage?*

Writing equations in a flux-conservative formulation is a way to ensure that conservation laws are obeyed after spatial and temporal discretization. This technique is applied commonly in the dynamics (literature references are given in the introduction section), and in this manuscript it is applied in the physics.

In order to write the physics equations, and more specifically the contributions of phase changes, in a flux-conservative way, the concept of pseudofluxes is introduced in Catry *et al.*, (2007) . We agree that this concept deserves more explanation, and the manuscript is adopted in this sense. Also, it is noted that this does not mean that the internals of the physics parameterizations should be formulated in terms of pseudofluxes. Instead, it is quite easy to calculate pseudofluxes from local species tendencies by integrating from the top of the atmosphere:

$$R_j = \int_0^p \frac{1}{g} \frac{\partial q_k}{\partial t} dp \tag{1}$$

(same notations as in the manuscript).

This equation is also added to the manuscript.

Referee's comment: *Regarding the sedimentation fluxes in equations (9) and (10), I cant see at a glance why the rain flux Pr shoud depend on both absolute Pr* and Ps*. Could you explain this?*

The relative precipitation flux P_r describes the movement of rain with respect to the mass center of the parcel, including *all* components. It may be easier to understand when writing the fluxes explicitly as a product of (vertical) velocity and specific humidity:

$$P_r^* = \rho_r w_r^* = \rho_{tot} q_r w_r^* \tag{2}$$

$$P_s^* = \rho_s w_s^* = \rho_{tot} q_s w_s^* \tag{3}$$

where ρ_r, ρ_s and ρ_{tot} are the rain density, snow density and total density, and w_r^* and w_s^* are the absolute velocities of rain and snow. If we now consider a parcel only consisting of dry air, rain and snow, the absolute velocity w^* of the center of mass of this parcel is equal to

$$w^* = w_r^* q_r + w_s^* q_s \tag{4}$$

The relative velocities of rain and snow then become

$$w_r = w_r^* - w^* \tag{5}$$

$$w_s = w_s^* - w^* \tag{6}$$

and the relative precipitation fluxes become

$$P_r = \rho_{tot} q_r w_r = \rho_{tot} q_r (w_r^* - w^*) = (1 - q_r) P_r^* - q_r P_s^* \tag{7}$$
$$P_s = \rho_{tot} q_s w_s = \rho_{tot} q_r (w_s^* - w^*) = (1 - q_s) P_s^* - q_s P_r^* \tag{8}$$

as given in the manuscript.

Some explanation is added to the manuscript to clarify the relationship between absolute and relative precipitation fluxes.

Referee's comment: *You also mention the relative flux of dry air to be defined as Pd=-sum(Pk) (Page 7 about lines 10). This is correct. What is about the flux of water vapour or other nonsedimenting species? It should have the same compensating velocity as dry air.*

The mentioned equation ($P_d = -\sum_k P_k$) only holds when the approximation is made that all mass transport by physical processes is compensated for by a fictitious flux of dry air. The fact that this is a rather artificial approximation is indeed insufficiently clear from the manuscript. The text has been modified to amend this.

Regarding water vapour and nonsedimenting species: under this approximation, they do not have the same vertical velocity as dry air; in fact, their vertical velocity is zero. In other words, the behavior of dry air is rather artificial (relatively large vertical velocity), but the description of the transport of the suspended water species is close to what happens in the atmosphere. It should be noted that this artificial behavior of dry air is restricted to the physics-dynamics interface: it does not affect the dynamical equations, nor does it affect the internals of the physics parameterizations.

Referee's comment: *In Section 4 it is mentioned that the surface boundary condition of AROME does not allow for mass exchange between soil and atmosphere. A consequence is then, that energy exchange associated with moisture and precip is also not possible? Do I understand this correctly? Then, with regard to the cold pool example you give later on in Figure 7, which consequences would this imply? Could you try to implement this boundary condition? An why should it not be possible to implement this boundary condition?*

This is a correct remark by the referee. The statement that mass exchange is not allowed is not entirely accurate. It should rather be that there is no *net* mass exchange. In other words, all mass exchange of water at the surface (evapotranspiration and precipitation) is compensated for by a fictitious exchange of dry air.

Therefore, water mass exchange is possible –and accounted for– in AROME, as are the thermodynamic effects of these processes.

The reason that it is quite difficult to implement this boundary condition in AROME is that the vertical coordinate of the AROME model is mass-based. Correctly accounting for a net mass exchange between atmosphere and surface would have far-reaching consequences, especially in the solution of the dynamical equations. We agree with the referee that this was not explained sufficiently in the manuscript, and some sentences are added to amend this. This issue is also mentioned in the conclusions section as an interesting future research topic.

Author's response to Referee #4

1 General comments

Referee's comment: *The authors claim that, thanks to the equations used in this interface "the conservation of mass and energy is a built-in feature of the system". I don't agree with this affirmation. I think the problem is much more complicated, and the best solution for the coupling depends a lot of the method which has been chosen inside the physics driver and inside the parametrisations themselves. I don't think this interface replaces a global solver which would be the only clean way to solve consistently together all the processes represented by the system of equations (2)-(8).*

Writing equations in a flux-based formulation is in fact a fairly common way to enforce conservation laws. Several references are given in the manuscript to show this. Could the referee please explain why this technique would not lead to a conserving system when applied in the field of physics-dynamics coupling?

Referee's comment: *I think that what is really missing in this paper is a proper discussion about the discretization of the complex system given by equations (2)-(8). The only example which is given in page 8, line 12 is actually not valid for any physics package. In section 3 below, I illustrate with a simple example extracted from the paper why I don't think that the conservation of mass and energy is necessarily a built-in feature of the system of equations (2)-(8).*

The referee is entirely correct to point out that the resulting equation of the example given in page 8 is not valid for a time-split (sequential) coupling case. The fact that this example concerns the case of a process-split (parallel) coupling should definitely be stated explicitly in the manuscript. In the case of a time-split coupling, the equations (2)-(8) [or (12)-(13)] are still valid, but the discretization looks somewhat different.

Let's resume the same example of two parameterizations (denoted (a) and (b)), but this time with a time-split coupling. Assuming that each process is energy-conserving in itself means that the change in enthalpy due the first process is given by

$$\Delta h^a = (c_p^t + \Delta c_p^a)(T^t + \Delta T^a) - c_p^t T^t \tag{1}$$

To determine the enthalpy change due the second process, we should account for the fact that in a time-split coupling, it doesn't start from c_p^t and T^t, but from $\tilde{c_p} = c_p^t + \Delta c_p^a$ and $\tilde{T} = T^t + \Delta T^a$. Therefore, the change in enthalpy due to the second process is given by

$$\Delta h^b = (\tilde{c_p} + \Delta c_p^b)(\tilde{T} + \Delta T^b) - \tilde{c_p}\tilde{T} \tag{2}$$

The combined effect of both parameterizations is then written as

$$\Delta h = \Delta h^a + \Delta h^b = c_p^{t+\Delta t} T^{t+\Delta t} - c_p^t T^t \tag{3}$$

from which the temperature at the next timestep is solved as

$$T^{t+\Delta t} = \frac{c_p^t T^t + \Delta h}{c_p^{t+\Delta t}} = T^t + \Delta T^a + \Delta T^b \tag{4}$$

i.e. the same result as given by the referee in the Annexe. This shows that, when applied correctly, our set of equations is also valid for time-split coupling.

We agree with the referee that some explanation about the application of the presented equations in the case of a time-split coupling is missing in the manuscript, and a section has been added on this issue. However, we do not agree on the point that the conserving property of the equations does not hold in the case of a time-split coupling.

The remark of the referee about the atmospheric state not being adjusted after a process-split coupling is correct. However, this is a problem inherent to process-split coupling, not to the presented set of equations.

Referee's comment: *I don't think either that for local processes such as autoconversion or condensation, pseudofluxes are necessary to ensure conservation. The pseudofluxes are not necessary to express the system in a barycentric form. If a parametrisation has not been written in term of pseudofluxes but the parametrisation gives tendencies for the qx, how should the pseudofluxes be computed ? Should for example the method take into account the fact that the latent heat used in a parametrisation is* $L(T)$ *instead of* $L(T_0)$ *?*

The concept of pseudofluxes is indeed insufficiently explained in the manuscript. The referee is correct when saying that it is not necessary to ensure conservation: if a physics parameterization is constructed properly, it should conserve mass and energy in itself. However, the pseudofluxes are a way to enforce conservation at a higher level, for example to make sure that energy is also conserved when several parameterizations are put together (see previous comment). Writing the combined effect of all parameterizations in a single flux-conservative equation is only possible through the concept of pseudofluxes.

The microphysical parameterizations of AROME are internally formulated in terms of specific humidity tendencies. The transformation of these tendencies to pseudofluxes is simply done by taking a vertical integral:

$$R_j = \int_0^p \frac{1}{g} \frac{\partial q_k}{\partial t} dp \tag{5}$$

This equation and method are added to the manuscript.

The transformation of tendencies into fluxes does not depend on whether $L(T)$ or $L(T_0)$ is used. It is important to keep in mind that the pseudofluxes only describe a mass exchange between different species. The parameterizations should not determine the thermodynamic effect of phase changes themselves (well, they can do it for internal purposes, but they should not pass this information to the dynamical core). Instead, they only determine the effect of the phase changes on the specific humidities (expressed through pseudofluxes), and equation (13) of the manuscript determines its thermodynamic effect.

Referee's comment: *The test case of section 4 shows that the main impact of the new interface has been to include a term which was missing (had been neglected ?) in the parametrisation of the precipitation sedimentation. From the text, it is not clear if adding this term in the parametrisation, but still using the old interface, would produce the same effect. Could the author try to separate more clearly the impact of the new design of the interface from the impact of the missing term ?*

It has been tested carefully what the origin is of the differences between results with the temperature-based interface and with the flux-based interface. This has led to the identification of the list of approximations that is given in section 3. The referee is correct when saying that the effect of heat transportation by precipitation could also be included in the temperature-based interface.

However, this is not the point we wish to make with this test case. The purpose of this case is merely to show that small terms in the energy budget, which can safely be neglected on a large scale, still can have a significant impact under specific circumstances. The use of the eqs. (12)-(13) avoids such approximations. This is stated clearly in the last paragraph of section 4.

Referee's comment: *I also think that the title of the paper is very misleading. The formulation of the Aladin dynamical core is not based on conservative flux-form equations. The new interface will surely not improve the non-conservative aspects of the Aladin semi-Lagrangian advection scheme for example. In the paper, conservative flux-form equations are used only to compute the tendencies from information provided by the physics parametrisations. This should be made clearer in the title and in the abstract.*

The first sentence of the abstract already defines the scope of this work as the "thermodynamic impact of physical parameterizations". However, we agree that it is quite important that the reader keeps this in mind. Therefore, the fact that only the effect physical parameterizations is considered, is added explicitly at several places in the manuscript: (i) in the abstract, (ii) in the introduction section, (iii) at the point where the equations are introduced, and (iv) in the conclusions.

Referee's comment: *I don't think there is a lot new information in the current state of the manuscript about physics/dynamics interface compared to Catry et al (2007) (the generalization to more water species and processes is quite straightforward). A more careful and general analysis of the physics/dynamics interfacing problem should be added to the manuscript to make it useful to the community. The results concerning the impact of the missing term in the sedimentation of precipitation are interesting but only more systematic sensitivity tests and comparison to observation would prove the importance of this effect versus many other sources of model errors for the simulation of organized convection in convection permitting NWP.*

We agree that the generalization is straightforward (from a mathematical point of view). However, this generalization is essential for the application in the AROME model, which is used for operations and/or research in 26 European and North-African countries. The application in AROME has led to the identification of some approximations that are present in the existing temperature-based interface of AROME. We believe that this is very valuable information for the users of this model. Moreover, our work provides a solution to get rid of these approximations, which is also of direct interest to these people.

A more complete investigation of the phenomenon discussed in the case study indeed would be interesting. This remark has been added to the manuscript. But as indicated in the manuscript and in reply to another comment, the phenomenon of heat transportation by precipitation is not the topic of the presented manuscript, but serves as an example of a case where the aforementioned approximations are not harmless.

2 More detailed comments

Referee's comment: *1. p 2, l3 : It is not correct to say that the dynamical core equations are equations written for a perfect gas. The equations of the Aladin dynamical core are written for a "barycentric" multiphase system which may contain condensed water phases, even if the physics is switched off. The water vapor but also the condensed species are taken into account to compute the mass of air in a given volume (e.g. liquid and solid species are "loading" the air parcel) and are then changing the "inertia" of the air parcel in the momentum equation. The gas law also knows about the composition of the air parcel, and only the "gas" part of the total mass is used in the gas law. But the full weight is used for the hydrostatic equilibrium. The information about the composition of the air parcels is known thanks to the definition of the specific water contents which are defined as the ratio between the mass of a given species and the total mass (including condensates). The specific quantities are used to compute the virtual temperature and the moist c_p (thermal inertia) and the moist gas constant R which are also used inside the dynamics.*

The statement that the dynamical core only considers a perfect gas has been removed from the manuscript.

Referee's comment: *2. p 2, l1-2 : Why only mass and enthalpy budgets but no momentum budget in this interface ?*

Exactly the same flux-conservative interface indeed could be used for momentum, as well as for other prognostic variables like TKE, passive tracers, etc. A sentence has been added to the manuscript to point this out. An important difference with water species, is that the evolution of water species affects the evolution of temperature in a complicated way, e.g. through latent heat effects or through the specific heat capacity. Therefore, the application of the presented methodology is much more important and interesting for the case of water species.

Referee's comment: *3. p2 l25 : I don't really understand this sentence. Shouldn't it be "to be described" instead of "to described"?*

Indeed, this has been adapted.

Referee's comment: *4. in section 2.1, the authors list the main hypotheses made for the design of Catry et al (2007) interface. However, nothing is said about the "Eulerian" and "vertical column" hypotheses used to write the system of equation (2)-(8) and how such a system should be interfaced with the dynamics. For example, in the case of a semi-Lagrangian dynamics, should the tendencies be computed at the beginning/middle/end of the semi-Lagrangian trajectories or along the trajectories?*

The hypothesis framework is indeed restricted to the hypotheses that relate to the thermodynamics of the model. This indeed should be stated more clearly in the manuscript.

The other hypotheses mentioned by the referee are more related to the general design of an atmospheric model. The use of the presented equations is rather independent of these hypotheses. For instance, the Green-Ostrogradsky theorem that forms the basis of the choice for a flux-conservative formulation, is also valid in 3 dimensions, so our work is not only relevant for physics parameterizations that are organized in vertical columns.

Referee's comment: *5. p3 l7-10 : It is not clear if the precipitation are supposed to immediately get the temperature of the layers they are crossing during a time step, or only the temperature of the layer where they "seat" at the end of the time step (in other word, is there an exchange of energy with all layers which are crossed by the condensed phases or only inside the layer where they stop at the end of the time step).*

Precipitation also takes the temperature of the layers in-between, and the heat exchange between precipitation and these layers is accounted for. This can be seen from the relevant term in the equation (13) of the original manuscript:

$$\frac{\partial}{\partial t}(c_p T) = -g\frac{\partial}{\partial p}\left[\ldots + \sum_k (c_k - c_p)P_k T\right] \tag{6}$$

The fact that the product of the precipitation fluxes (P_k) with temperature (T) occurs here, means that there's an energy transfer, even when precipitation just 'falls through' a layer: in such case the divergence of the precipitation flux is zero ($\partial P_k/\partial p = 0$), but the divergence of the heat flux is not necessarily zero ($\partial(P_k T)/\partial p \neq 0$).

Referee's comment: *6. p3 l8 : Bott (2008) → (Bott, 2008)*

This has been adapted.

Referee's comment: *7. p3 l 21 : it should be said more clearly that the q_x are specific fractions (i.e. ratios with respect to total mass of the multiphase system).*

This has been adapted.

Referee's comment: *8. p4 l8 : I understand that in a barycentric system, the total mass should be conserved, i.e the sum of eq 2-7 gives 0 = 0. Does it mean that the surface scheme should produce diffusive fluxes of condensates to compensate the diffusive flux of water vapor?*

Barycentrism doesn't mean that the total mass should be conserved. It means that the motions are considered relative to the (moving) center of mass of the parcel. That the right-hand sides of eqs. (2)-(7) cancel out, is due to the definition of specific humidities: $q_k = \rho_k/\rho_{tot}$. From this, it follows that $\sum_k q_k = 1$, so $\sum_k \partial q_k/\partial t = 0$.

As indicated in Catry *et al.* (2007), a change in total mass due to exchange with the surface, is reflected in a change of the surface pressure.

Referee's comment: *9. p4 l20 : center of mass of what?*

The center of mass of the air parcel. This has been adapted in the manuscript, and a more elaborate explanation of the relation between relative and absolute precipitation fluxes has been added.

Referee's comment: *10. p7 l12-l15 : "all vertical transport is compensated by a flux of dry air" : I don't think it applies to "all vertical transport" (in particular, it does not applied to resolved vertical transport), but only to precipitation and subgrid mass transports (top of page 5 of Courtier et al, 1991).*

Indeed, this has been adapted.

Referee's comment: *11. p8 l7-12 : Formulae p8, l12 ensure conservation only if the parametrisations are called in parallel (process split in Williamson, 2002). It also supposes that the final specific ratios are given by process (a) and (b) (and not by a common resolution of both process (a) and (b)). If the parametrisations are called sequentially (time split), parametrisation (b) will already know about the evolution in time of both T and c_p after process (a). In this case, the conservation is ensure if $\Delta T = \Delta T^a + \Delta T^b$. See annexe for details.*

A correct remark. The fact that the mentioned formula only holds for process-split coupling has been added to the manuscript. The example also has been extended for the time-split case. (cfr. General comment)

Referee's comment: *12. p9, l31-32 and p10 l1-2 : The authors say that, in Arome, it is not possible to take into account the correct mass budget, therefore all vertical transport is compensated by a flux of dry air. Does it mean that equation 43 of Catry et al is used instead of equation (8) in the new interface ? Why the correct mass budget could not be taken into account in Arome ? Is the problem only at the surface or at every level ?*

In the system with 4 hydrometeors of Catry *et al.* (2007), it would indeed be their equation (43) that is used. In the generalized system presented in our manuscript, equation (13) is still valid if a modified relative mass flux of dry air $P_d = -\sum_{k=1}^{n} P_k$ is used.

The reason that this approximation is necessary is that the vertical coordinate of AROME is mass-based. Hence, correctly accounting for a net mass exchange between atmosphere and surface would have far-reaching consequences, especially for the surface boundary condition of the dynamical core. We agree with the referee that this was not explained sufficiently in the manuscript, and some sentences are added to amend this.

Referee's comment: *13. p10 l17 : I don't clearly understand why no significant improvement could be expected without a new tuning. Are the author thinking of model error compensation? If it is the case, it should be explained more clearly.*

Indeed, compensating errors are what we have in mind. This has been added to the manuscript.

Referee's comment: *14. section 4.2 : the discussion for this case study mainly consider the missing term (heat transport by the precipitation). If this term is the main problem in Arome, couldn't it be added to the old interface ? Would it give similar results ?*

It could be added to the old interface, and it has been verified that this gives similar results. But this is besides the point of the presented work. The strength of the presented framework lies exactly in the fact that one doesn't have to worry whether one term or another is accounted for in the physics-dynamics interface.

Referee's comment: *15. It would also be interesting to see the impact of the new interface independently from the addition of the missing term for the case in section 4.2 (i.e. also neglect the missing term in the new interface).*

As mentioned in response to the previous comment, the value of our work doesn't lie in discussing the impact of individual terms of the energy budget of the AROME model. This is interesting research in itself, but it is not the focus of this manuscript. The focus of this paper rather lies in the presentation of an overall framework that takes care of all terms in a consistent and conserving way.

In this sense, the case of the cold-pool formation under heavy precipitation is just an example where a clear impact of this framework can be noticed. A systematic study of the effect of heat transport of precipitation on the life-cycle of a cold-pool would be interesting in itself, but falls outside the scope of the present work. A sentence has been added to the conclusions to indicate this.

Referee's comment: *16. The author should be more modest in their conclusion. The simulation of such convective systems are very sensitive to many other source of model errors (time step, horizontal or vertical resolution, level of complexity in microphysics etc). A more systematic study would be necessary to really conclude about the importance of the heat transport by precipitation in Arome.*

We agree entirely that such a systematic study would be very interesting. But again, it is besides the point we try to make in this manuscript.

Generalization and application of the flux-conservative thermodynamic equations in the AROME model of the ALADIN system

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Abstract. General yet compact equations are presented to express the thermodynamic impact of physical parameterizations in a NWP or climate model. By expressing the equations in a flux-conservative formulation, the conservation of mass and energy by the physics parameterizations is a built-in feature of the system. Moreover, the centralization of all thermodynamic calculations guarantees a consistent thermodynamical treatment of the different processes. The generality of this physics-dynamics interface is illustrated by applying it in the AROME NWP model. The physics-dynamics interface of this model currently makes some approximations, which typically consist of neglecting some terms in the total energy budget, such as the transport of heat by falling precipitation, or the effect of diffusive moisture transport. Although these terms are usually quite small, omitting them from the energy budget breaks the constraint of energy conservation. The presented set of equations allows to get rid of these approximations, in order to arrive at a consistent and energy-conservative model. A verification in an operational setting shows that the impact on monthly-averaged, domain-wide meteorological scores is quite neutral. However, under specific circumstances, the supposedly small terms may turn out not to be entirely negligible. A detailed study of a case with heavy precipitation shows that the heat transport by precipitation contributes to the formation of a region of relatively cold air near the surface, the so-called cold pool. Given the importance of this cold pool mechanism in the life-cycle of convective events, it is advisable not to neglect phenomena that may enhance it.

1 Introduction

The conservation of mass and energy are important characteristics of a numerical atmospheric model. Especially in view of the application in climate studies, even small violations of the conservation laws can accumulate over a long integration time, and lead to faulty results (Staniforth and Wood, 2008; Lucarini and Ragone, 2011). Atmospheric forecast models are usually constructed by combining a dynamical core with physical parameterizations. In general, the dynamical core describes the atmospheric behaviour up until the resolved scales, while the physical parameterizations estimate the effect of subgrid processes (Gassmann, 2013).

A lot of research has been spent in designing dynamical cores that conserve mass and energy (Thuburn, 2008). Common strategies include a careful selection of the prognostic variables (Ooyama, 1990, 2001; Klemp et al., 2007), the formulation of the equations in flux-form (Satoh, 2003), or taking advantage of properties of the Hamiltonian character of the atmospheric equations (Salmon, 2004; Gassmann and Herzog, 2008; Zängl et al., 2014). In contrast with these efforts on the dynamical core, the energy conservation and consistent thermodynamics seem to be less of a priority in the development of the physical parameterizations, or in the way they are coupled to the dynamical core.

A possible explanation is that the thermodynamics of the dynamical core are less complicated than those of the physical parameterizations. More specifically, the dynamics are usually considered adiabatic and reversible (except for numerical diffusion) (Gassmann, 2013). The physics parameterizations, on the other hand, include mass and energy exchange with the surface, as well as radiative fluxes at the top of the atmosphere. They constitute an open thermodynamic system, for which the conservation laws are more difficult to enforce. Moreover, it is tempting to consider physics parameterizations as plug-compatible, i.e. they are considered as a black box which, given an atmospheric state, returns an effect on the dynamical prognostic variables. Unfortunately, this plug-compatibility seems to go at the expense of carefully investigating the thermodynamic consistency between the dynamical core and the physics parameterizations, and inserting a new parameterization in a model comes with implicit assumptions and ad-hoc approximations.

There is, however, an increased interest in different aspects of the coupling of physical parameterizations to the dynamical core. One of the issues is the organization of the timestep. This problem has been studied with academic toy-models (see, e.g. Caya et al. (1998); Staniforth et al. (2002); Termonia and Hamdi (2007)), as well as in 3D models (Hortal, 2002; Williamson, 2002). The thermodynamic aspects of the physics-dynamics coupling is another topic that deserves some attention. Although some attempts have been made to rigorously formulate the equations for a multicomponent atmosphere (Ooyama, 2001; Bannon, 2002), it remains a fact that many operational models make several ad-hoc approximations (Bryan and Fritsch, 2002). Catry et al. (2007), hereafter CGTBT07, presented a set of equations that express the effects of physics parameterizations in a flux-conservative formulation. The advantage of this approach is that this system is inherently mass- and energy-conservative.

The current paper develops the proposal of CGTBT07 further by generalizing it for a system with an arbitrary number of hydrometeors with arbitrary interactions between them. It should be emphasized that the scope of this work is limited to the coupling of the atmospheric physics parameterizations to the dynamical core. For instance, when energy-conserving equations are presented, this property does not necessarily hold for the atmospheric model as a whole, but only regarding the influence of the physical parameterizations. Other aspects of the model, most notably its dynamical core, may not be energy-conserving. Also the mutual interactions between different parameterizations are not considered in this paper, as they relate only indirectly to the time evolution of the prognostic atmospheric variables. The next section presents the equations of this generalized system. In section 3, this set of equations is applied in the AROME numerical weather prediction (NWP) model (Seity et al., 2011), thus allowing to get rid of some approximations that are currently made. Section 4 discusses the impact on the meteorological results, both by means of monthly scores and with an in-depth case study of a cold pool formation under heavy precipitation. Section 5 presents the conclusions.

2 Formulation of the generalized flux-conservative equations

2.1 Framework of hypotheses

Because the behaviour of the atmosphere is too complex to be described exactly, every numerical model needs to make simplifying hypotheses. This is no different for the work described in the current paper. It is not our aim to present a set of equations which is exact in the sense that it is free of approximations. But a crucial aspect of the work presented in CGTBT07, is that the set of hypotheses that relate to the thermodynamics, is defined from the very beginning. This is important for two reasons. First, it ensures that the simplifications act consistently throughout the model. Second, it allows to set some non-negociable constraints. For instance, the conservation of energy must be satisfied, no matter what other simplifications are made. This approach of setting the simplifying hypotheses from the beginning contrasts with the conventional approach of ignoring supposedly small terms along the way.

The framework of hypotheses is the following:

– A fully barycentric view of air parcels is adopted. This means that all hydrometeors (both suspended and precipitating) are considered as integral parts of the air, and contribute to the parcel's motion, density and heat capacity. This barycentric view has been studied and motivated by many researchers (Wacker and Herbert, 2003; Bott, 2008; Gassmann and Herzog, 2008).

– Water condensates are assumed to have zero volume. This is a common approximation in atmospheric modeling.

– Gases follow Boyle-Mariotte's and Dalton's laws.

– Temperature is homogeneous across all species, even falling hydrometeors. For small hydrometeors, this approximation is easily justified, given their short relaxation time (Bott, 2008). For larger hydrometeors, it is a cruder approximation, but it goes together with the barycentric view: since such hydrometeors are considered part of the parcel, they also take the parcel's temperature.

– The specific heat values of all species are constant with temperature.

– The latent heat values of sublimation and evaporation, L_i and L_l respectively, vary linearly with temperature T:

$$L_{i|l}(T) = L_{i|l}(T_0) + (c_{pv} - c_{i|l})T \tag{1}$$

with $T_0 = 0\mathrm{K}$, c_{pv} is the specific heat capacity at constant pressure of water vapour, and c_i and c_l are the specific heat capacity values of ice and liquid water, respectively.

It should be mentioned that this same framework of assumptions has been used by Marquet (2011), Marquet and Geleyn (2013) and Marquet (2015) to cleanly develop moist atmospheric thermodynamic quantities such as moist entropy, moist potential temperature, and moist Brunt-Väisälä frequency.

2.2 The flux-conservative equations for a system with 5 water species

The system considered in CGTBT07 consists of dry air (specific mass fraction $q_d = \rho_d/\rho_{tot}$) plus five prognostic water species: vapour (specific mass fraction q_v), suspended liquid water droplets (q_l), suspended ice crystals (q_i), precipitating rain (q_r) and precipitating snow (q_s). For this system, the following equations are derived for the time evolution of the prognostic species due to physical parameterizations:

$$\frac{\partial q_v}{\partial t} = g\frac{\partial}{\partial p}\left[R_{r,v} + R_{s,v} - R_{v,l} - R_{v,i} + \frac{q_v(P_r + P_s)}{1 - q_r - q_s} - D_v\right] \tag{2}$$

$$\frac{\partial q_l}{\partial t} = g\frac{\partial}{\partial p}\left[R_{v,l} - R_{l,r} + \frac{q_l(P_r + P_s)}{1 - q_r - q_s} - D_l\right] \tag{3}$$

$$\frac{\partial q_r}{\partial t} = g\frac{\partial}{\partial p}\left[R_{l,r} - R_{r,v} - P_r\right] \tag{4}$$

$$\frac{\partial q_i}{\partial t} = g\frac{\partial}{\partial p}\left[R_{v,i} - R_{i,s} + \frac{q_i(P_r + P_s)}{1 - q_r - q_s} - D_i\right] \tag{5}$$

$$\frac{\partial q_s}{\partial t} = g\frac{\partial}{\partial p}\left[R_{i,s} - R_{s,v} - P_s\right] \tag{6}$$

$$\frac{\partial q_d}{\partial t} = g\frac{\partial}{\partial p}\left[\frac{q_d(P_r + P_s)}{1 - q_r - q_s} - D_d\right] \tag{7}$$

In these equations, P_k denotes precipitation fluxes, and D_k denotes diffusive fluxes. Note that it is necessary that $\sum_{k=d,v,i,l} D_k = 0$ to ensure that all terms on the right hand sides cancel out. The terms R_{k_1,k_2} denote pseudofluxes and represent mass transfer between two water species. The concept of pseudofluxes is essential to the presented system and deserves some more explanation. The common and more intuitive way to express a mass transfer between two species is through a time tendency. For instance, consider the microphysical process of condensation, which is a mass transfer from water vapour to liquid cloud water droplets. The effect of this process on the specific humidities could be expressed as

$$\frac{\partial q_v}{\partial t} = -\left(\frac{\partial q}{\partial t}\right)^{cond}$$

$$\frac{\partial q_l}{\partial t} = \left(\frac{\partial q}{\partial t}\right)^{cond}$$

The pseudoflux $R_{v,l}$ expresses exactly the same effect, only as a flux instead of as a tendency. This flux is determined by taking the vertical integral of the tendency:

$$R_{v,l} = \frac{1}{g}\int_0^p \left(\frac{\partial q}{\partial t}\right)^{cond} dp$$

Although a pseudoflux is arguably more difficult to interpret than a tendency, writing conversions between species in terms of pseudofluxes offers the possibility to write the evolution equations in a flux-conservative form. The benefit of this is explained further. Also note that this does not mean that the internals of the physics parameterizations should be formulated in terms of pseudofluxes. Instead, it is only at the moment when the contributions of the physics parameterizations are added to the prognostic variables, that pseudofluxes are beneficial. They can be determined at that point from the more conventional tendencies using the expression above.

The thermodynamic equation for the system with 4 hydrometeors is as follows:

$$\begin{aligned}\frac{\partial}{\partial t}(c_pT) = -g\frac{\partial}{\partial p}\big[&(c_l - c_{pd})P_rT + (c_i - c_{pd})P_sT \\ &-(\hat{c} - c_{pd})(P_r + P_s)T + J_s + J_{rad} \\ &-L_l(T_0)(R_{v,l} - R_{r,v}) - L_i(T_0)(R_{v,i} - R_{s,v})\big]\end{aligned} \tag{8}$$

where $\hat{c} = \frac{c_{pd}q_d + c_{pv}q_v + c_iq_i + c_lq_l}{1 - q_r - q_s}$, and J_s and J_{rad} are the diffusive and radiative heat fluxes, respectively. c_p is the total heat capacity of the parcel, given by

$$c_p = c_{pd}q_d + c_{pv}q_v + c_l(q_l + q_r) + c_i(q_i + q_s)$$

It should be noted that Eq. (8) expresses only the thermodynamic effect of the physical parameterizations. The complete thermodynamic equation of the atmospheric model would also include terms that are resolved by the dynamics of the model.

A full discussion of these equations is given in CGTBT07, but we would like to stress the following characteristics:

- All equations are flux-conservative, i.e. every right hand side is a divergence of a summation of fluxes. The importance of this property cannot be underestimated, because it means that this system intrinsically conserves mass and energy. Put somewhat simplistically, in a flux-conservative system, the only way energy or mass can leave one model layer, is by transporting it to an adjacent layer. Therefore, mass and energy are conserved by design of the system.

- The precipitation fluxes P_r and P_s are relative to the (moving) center of mass of the parcel. They relate to the absolute precipitation fluxes P_r^* and P_s^* through

  $$P_r = (1 - q_r)P_r^* - q_rP_s^* \tag{9}$$

  $$P_s = -q_sP_r^* + (1 - q_s)P_s^* \tag{10}$$

  To derive these relations, one starts from the definition of a flux as a product of a density with a velocity. For instance, for rain, one writes

  $$P_r^* = \rho_{tot}q_rw_r^*$$

  The absolute velocity w^* of the center of mass of the parcel is given by the weighted average of the velocities of the components. In a system where only rain and snow are precipitating, this means that

  $$w^* = q_rw_r^* + q_sw_s^*$$

  The relative velocity of rain is then given by $w_r = w_r^* - w^*$, so the relative precipitation flux becomes

  $$P_r = \rho_{tot}q_rw_r = \rho_{tot}(q_rw_r^* - q_r^2w_r^* - q_rq_sw_s^*) = (1 - q_r)P_r^* - q_rP_s^*$$

– The latent heat values of sublimation and condensation L_i and L_l that appear on the right hand side, are evaluated at $T_0 = 0$K. This does not mean that the temperature-dependency of these latent heat values is neglected. Instead, it is accounted for by considering the time derivative of the enthalpy c_pT. Considering only the process of condensation, the traditional way to express its thermodynamic effect would be

$$c_p \frac{\partial T}{\partial t} = L_l(T) \frac{\partial q_l}{\partial t}$$

Using the before-set assumption that L_l varies linearly with temperature, and the fact that, still only considering condensation, $\partial q_v/\partial t = -\partial q_l/\partial t$, so $\partial c_p/\partial t = (c_l - c_{pv})\partial q_l/\partial t$, this expression becomes

$$c_p \frac{\partial T}{\partial t} = L_l(T_0) \frac{\partial q_l}{\partial t} - T \frac{\partial c_p}{\partial t}$$

which can be rewritten as

$$\frac{\partial}{\partial t}(c_pT) = L_l(T_0) \frac{\partial q_v}{\partial t}$$

This shows how the temperature-dependence of the latent heat values can be accounted for by considering the tendency of enthalpy.

– Although the equations only describe the evolution of water species, similar flux-conservative equations could be formulated for other atmospheric variables like momentum, turbulent kinetic energy, etc. In this manuscript, only water species and their effect on the thermodynamic equation are studied.

2.3 The generalized flux-conservative equations

Despite the clear strength of the equations proposed by CGTBT07, their application is not straightforward because of the fixed number of water species, and because of the fixed set of interactions between them (six pseudofluxes). More advanced microphysics schemes often consider more water species, for instance by including graupel and/or hail (Lascaux et al., 2006), or by separating convective and nonconvective fractions of hydrometeors (Piriou et al., 2007). Also the fact that only six transfer mechanisms between the water species are possible is limiting. For instance, snow melting cannot be represented directly, but it should be written as a combination of snow sublimation ($R_{s,v}$) and rain evaporation ($R_{r,v}$). Although thermodynamically fully correct, it would be better to have a system that digests all kinds of transfers between water species.

It is, however, possible to generalize the equations from CGTBT07, without touching the important characteristics. We introduce the following notation: n is the number of water species, the index $k = 1,\ldots,n$ denotes a single water species, and by convention, $k = 0$ denotes the dry air component. The specific heat capacity values at constant pressure of the different species are written generically as c_k, the latent heat of evaporation or sublimation at $0\,K$ is written as L_k^0. The index j denotes a conversion process between a source water species k_j^s and a target water species k_j^t. The effect of this process is expressed through the pseudoflux R_j. We consider an arbitrary number m of such conversion processes. We now define variables $\lambda_{kj} = \delta_{k,k_j^s} - \delta_{k,k_j^t}$ for $k = 1,\ldots,n$ and for $j = 1,\ldots,m$, where the usual definition of the Kronecker delta is used. This variable takes

Table 1. Variables λ_{kj} for the system of CGTBT07

		process	$r \to v$	$v \to l$	$l \to r$	$s \to v$	$v \to i$	$i \to s$
		j	1	2	3	4	5	6
species	k							
v	1		-1	1	0	-1	1	0
l	2		0	-1	1	0	0	0
r	3		1	0	-1	0	0	0
i	4		0	0	0	0	-1	1
s	5		0	0	0	1	0	-1

Table 2. Variables Λ_j^0 for the system of CGTBT07

process	$r \to v$	$v \to l$	$l \to r$	$s \to v$	$v \to i$	$i \to s$
j	1	2	3	4	5	6
Λ_j^0	$L_l(T_0)$	$-L_l(T_0)$	0	$L_i(T_0)$	$-L_i(T_0)$	0

value 0 if a species k is not involved in the conversion process j; it takes value -1 if it is the target species of this process, and it takes value 1 if it is the source species of this process. The variable λ_{kj} will allow to write the time tendency of a water species by summing over all conversion processes, regardless of the role this specific water species plays in each process. Furthermore, a variable $\Lambda_j^0 = L^0_{k_j^s} - L^0_{k_j^t}$ is defined. This variable is the latent heat released at temperature T_0 under a conversion process with source species k_j^s and target process k_j^t. To clarify these notations, consider the original system of CGTBT07 with 5 water species and 6 conversion processes between them. By convention, we assign $k = 1, \ldots, 5$ to water vapour, liquid cloud water, precipitating rain, cloud ice crystals and precipitating snow, respectively. Tables 1 and 2 give the values of λ_{kj} and Λ_j^0 for the different conversion processes.

Next, a precipitation flux P_k is defined for each component, even for the non-precipitating species (dry air, vapour, liquid cloud water droplets and cloud ice crystals). Contradictory as this may sound, it should be stressed that in our barycentric system, these fluxes express the motion of the species with respect to the center of mass of the parcel. When precipitating species are present, the suspended species will move upward with respect to the mass center. Using a similar calculation as before to describe the motion with respect to the center of mass of the parcel, the relative precipitation fluxes P_k are determined from the absolute fluxes P_k^* as

$$P_k = P_k^* - q_k \sum_{i=0}^{n} P_i^* \tag{11}$$

where the absolute precipitation fluxes of suspended species can be taken to be zero. It should be noted that the strict distinction in CGTBT07 between suspended and precipitating species is somewhat arbitrary and scale-dependent. Indeed, also the so-called suspended cloud water species can undergo a slow sedimentation. This arbitrary distinction is no longer necessary in the generalized set of equations that is presented here. Similarly to defining (relative) precipitation fluxes for all species, also diffusive fluxes D_k are defined for all species, where the diffusive fluxes of precipitating species can be taken equal to zero.

These notations make it possible to formulate the specific mass equations and the thermodynamic equation as follows:

$$\frac{\partial q_k}{\partial t} = -g\frac{\partial}{\partial p}\left[\sum_{j=1}^{m}\lambda_{kj}R_j + P_k + D_k\right], \qquad \text{for } k = 0,\dots,n \tag{12}$$

$$\frac{\partial}{\partial t}(c_pT) = -g\frac{\partial}{\partial p}\left[\sum_{k=0}^{n}c_kP_kT + J_s + J_{rad} - \sum_{j=1}^{m}\Lambda_j^0 R_j\right] \tag{13}$$

These equations generalize the ones from CGTBT07 in three ways: (i) an arbitrary number n of water species is considered; (ii) an arbitrary number m of inter-species conversion processes is considered; and (iii) the strict distinction between suspended and precipitating species can be abandoned. The fact that quite compact equations are obtained, which are valid for all components of the atmosphere, is an additional indication of the strength of the barycentric approach.

2.4 Remarks

Some comments should be given on the application area of the physics-dynamics interface presented in Eqs. (12–13).

- The fact that these equations are very general, opens the road for a 'plug-compatible' view of physics parameterizations. Indeed, the only output that is needed from a parameterization are diffusive and precipitative transport fluxes, pseudofluxes for phase changes and the radiative and diffusive energy fluxes. The physics-dynamics interface then receives these quantities and determines the effect on the prognostic variables of the model, thereby ensuring satisfaction of the conservation of mass and energy, as well as consistency in the thermodynamic assumptions.

  However, it should be kept in mind that other conditions should be met before parameterizations can really be considered plug-compatible. A first aspect is that interactions exist between parameterizations. For instance, the parameterization of cloud processes will affect the radiation scheme. This kind of interactions should properly be accounted for when plugging a new parameterization into a model. In this context, it is interesting to see that the technical recommendations that were made in Kalnay et al. (1989) regarding the design of parameterizations and their interactions, are still relevant at present. A second aspect is that parameterizations should also obey the second law of thermodynamics (Gassmann and Herzog, 2014). This condition cannot be enforced at the higher level of the physics-dynamics interface, and should be taken care of at the level of the parameterization itself.

- A common assumption in atmospheric modeling (although it is often made implicitly) is that all vertical mass transport due to the physics parameterizations is compensated for by a fictitious flux of dry air (Courtier et al., 1991). This assumption ensures the conservation of total mass in the atmosphere, but makes it impossible to express a net mass exchange with the surface due to for instance precipitation. From a barycentric point of view, this approximation means that the center of mass of an air parcel does not move vertically. The equations (12–13) remain valid under this assumption, if the relative flux of dry air is defined as $P_d = -\sum_{k=1}^{n} P_k$.

- The Eqs. (12–13) are theoretically only valid for a model using the hydrostatic primitive equations. In a fully compressible system, the diabatic heating from the physics parameterizations does not only affect the temperature equation, but also the continuity equation (Laprise, 1998). CGTBT07 present the extension of their flux-conservative system to the fully compressible case. An entirely equivalent development can be made for the generalized equations presented in this paper.

  However, as shown by Malardel (2010), the impact of including the heat from parameterizations as a forcing in the continuity equation is quite limited. In other words, one can apply the thermodynamic equation (13) also in a non-hydrostatic model.

- The fact that Eq. (13) describes the evolution of enthalpy $h = c_p T$, does not mean that this variable should become the prognostic thermodynamic variable of the model. A model that uses temperature T as the prognostic thermodynamic variable, can also use the presented interface. After all, one can easily calculate the total heat capacity tendency as follows:

$$\frac{\partial c_p}{\partial t} = \sum_{k=0}^{n} c_k \frac{\partial q_k}{\partial t}, \tag{14}$$

  which in turn can be used to determine the temperature tendency from the enthalpy tendency:

$$\frac{\partial T}{\partial t} = \frac{1}{c_p}\left(\frac{\partial}{\partial t}(c_p T) - T\frac{\partial c_p}{\partial t}\right) \tag{15}$$

  The importance of writing Eq. (13) as a time evolution of enthalpy only becomes clear in the time-discretized case.

$$c_p^{t+\Delta t} = c_p^t + \Delta t \sum_{k=0}^{n} c_k \frac{\partial q_k}{\partial t} \tag{16}$$

$$T^{t+\Delta t} = \frac{1}{c_p^{t+\Delta t}}\left(c_p^t T^t + \Delta t \frac{\partial}{\partial t}(c_p T)\right) \tag{17}$$

  where a superscript t denotes variables at the current timestep, while a superscript $t+\Delta t$ denotes variables at the next timestep. Using an enthalpy-based formulation of the interface is reflected in the use of $c_p^{t+\Delta t}$ in the right-hand side of Eq. 17. Although this appears to be a small detail, it is crucial in ensuring the conservation of energy. The importance of appropriately discretizing a conserved nonlinear variable such as enthalpy is also indicated by Gassmann and Herzog (2008).

  As a side remark, it can be noted that simply adding temperature tendencies from several parameterizations cannot lead to an energy-conserving atmospheric model, at least not for a process-split coupling strategy (Williamson, 2002). For example, consider a model containing two parameterizations (indicated with a and b), yielding a respective change in temperature of ΔT^a and ΔT^b, and a respective change in heat capacity of Δc_p^a and Δc_p^b. Suppose that each of these parameterizations is energy-conservative in itself, meaning that the enthalpy changes are respectively $\Delta h^a = (c_p^t + \Delta c_p^a)(T^t + \Delta T^a) - c_p^t T^t$ and $\Delta h^b = (c_p^t + \Delta c_p^b)(T^t + \Delta T^b) - c_p^t T^t$. Then the joint effect of the parameterizations cannot be expressed as $\Delta T = \Delta T^a + \Delta T^b$, but it should be determined as

$$\Delta h = \Delta h^a + \Delta h^b, \tag{18}$$

from which the total change in temperature is determined as

$$\Delta T = \frac{(c_p^t + \Delta c_p^a)\Delta T^a + (c_p^t + \Delta c_p^b)\Delta T^b}{c_p^t + \Delta c_p^a + \Delta c_p^b}$$

This expression is only valid for a process-split coupling. For a time-split coupling, the total enthalpy change is still equal to the sum of the enthalpy changes of the separate processes, as indicated in Eq. (18). However, the fact should should be taken into account that process b does not start from c_p^t and T^t, but rather from the atmospheric state after accounting for process a, i.e. $\tilde{c_p} = c_p^t + \Delta c_p^a$, and $\tilde{T} = T^t + \Delta T^a$. So for a time-spit coupling, the enthalpy change of process b becomes

$$\Delta h^b = (\tilde{c_p} + \Delta c_p^b)(\tilde{T} + \Delta T^b) - \tilde{c_p}\tilde{T}$$

Working out the heat capacity and the temperature at the end of the timestep now gives

$$c_p^{t+\Delta t} = c_p^t + \Delta c_p^a + \Delta c_p^b$$
$$T^{t+\Delta t} = \frac{c_p^t T^t + \Delta h}{c_p^{t+\Delta t}} = T^t + \Delta T^a + \Delta T^b$$

So with time-split coupling, the total temperature change can be obtained as the summation of the temperature changes from the separate parameterizations. However, it is better to use an enthalpy-based system, as this works both for the process-split and the time-split cases.

- The Eqs. (12)–(13) only describe the evolution of the atmospheric prognostic variables. The prognostic variables of the surface scheme are not part of this system. In this context, the work of Best et al. (2004) should be mentioned. They present a method to separate the surface scheme from the atmospheric model. The core of this method is to describe the interaction between atmosphere and surface with fluxes. In this sense, their work matches perfectly with the flux-based equations (12)–(13).

3 Application of the flux-conservative equations in the AROME model

AROME is a limited area model that was developed at Météo-France and is now a configuration inside the ALADIN system. It became operational in France in 2008, and it is currently used in many European countries of the ALADIN and HIRLAM consortia. AROME uses a nonhydrostatic, fully compressible dynamical core (Bubnová et al., 1995; Bénard et al., 2010), with the same spectral semi-implicit semi-Lagrangian space-time discretization as the ECMWF's IFS model. The height coordinate is terrain-following mass-based (Laprise, 1992). The physics parameterizations in AROME originate from the Méso-NH research model (Lafore et al., 1998). The Méso-NH model has a dynamical core which is explicit in time, with a staggered spatial grid and a height-based vertical coordinate, so it is substantially different from the AROME dynamical core. The plugging of the physics from this model to a different dynamical core was quite challenging, and several approximations were made during this process.

A first approximation that is made in the existing AROME physics-dynamics interface concerns the heat transport by precipitation. From Eq. (13), it is clear that precipitation has two thermodynamic effects. Falling species modify the composition of the atmosphere, so they also change the specific heat capacity $c_p = \sum_{k=0}^{n} c_k q_k$. Secondly, if a vertical temperature gradient exists, falling species are heated, thus cooling down the surrounding air. The effect on the enthalpy due to a change in c_p is given by

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec,c_p} = T \sum_{k=0}^{n} c_k \frac{\partial q_k}{\partial t} = -gT \frac{\partial}{\partial p} \sum_{k=0}^{n} c_k P_k \tag{19}$$

while the second effect due to a vertical temperature gradient is given by

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec,heat} = -g \sum_{k=0}^{n} c_k P_k \frac{\partial T}{\partial p} \tag{20}$$

The combination of these two effects indeed corresponds to the effect of precipitation in the right-hand side of Eq. (13):

$$\left(\frac{\partial}{\partial t} c_p T\right)^{prec,c_p} + \left(\frac{\partial}{\partial t} c_p T\right)^{prec,heat} = -g \frac{\partial}{\partial p} \sum_{k=0}^{n} c_k P_k T \tag{21}$$

The approximation made by the existing physics-dynamics interface in AROME is that it neglects the heat transport effect of precipitation, i.e. the term given in Eq. (20).

A second approximation concerns the effect of diffusive moisture transport (shallow convection and turbulence) in the energy budget. Similar to the effect of precipitation, diffusive moisture transport modifies the total specific heat capacity c_p, and this effect should be accounted for in the energy budget. However, this effect is neglected in the existing AROME physics-dynamics interface.

A third approximation is that the values of specific heat capacity c_p and latent heat $L_{i|l}(T)$ are not consistent between the different parameterizations. For instance, the heat capacity in the radiation scheme only accounts for water vapour and neglects the other hydrometeors ($c_p^{rad} = (1-q_v)c_{pd} + q_v c_{pv}$). This situation stems from the fact that the different physics parameterizations are developed by different teams, each using their own conventions.

A final approximation by the existing physics-dynamics interface in AROME is that the total temperature tendency is obtained by summing the temperature tendencies from the individual parameterizations. As indicated in the previous section, such an approach cannot lead to an energy-conserving system.

Although it can be expected that the overall effect of these approximations and inconsistencies is quite limited, the generalized physics-dynamics interface as presented in the previous section offers the possibility to get rid of them in order to take a (admittedly small) step towards a more accurate model. A second motivation to equip the AROME model with the generalized flux-conservative physics dynamics interface is that this opens the route towards importing physics parameterizations from other NWP models, thus allowing a fair comparison of different parameterizations and stimulating scientific progress.

4 Impact on weather forecast

The impact of the presented flux-conservative formulation of the physics-dynamics interface is investigated with the AROME operational high-resolution LAM model running at Météo-France. Before April 2015, this model ran on a 739×709 grid with a resolution of 2.5 km. Figure 1 shows the model domain. The timestep is 60 s. The model is provided with lateral boundary conditions by the operational global model 'ARPEGE' from Météo-France. The initial conditions are generated with a 3D-Var data-assimilation (Fischer et al., 2005; Brousseau et al., 2011).

At the surface level, precipitation and evapotranspiration imply a net mass-flux across the surface. Since the vertical coordinate of the AROME model is mass-based, correctly accounting for such net mass exchange between atmosphere and surface has far-reaching implications, especially in the surface boundary condition of the nonhydrostatic dynamical core. Currently, this has not been implemented in the dynamical core of the AROME model. Instead, the above-mentioned approximation is made that all vertical transport due to the parameterizations is compensated by a fictitious flux of dry air. Taking full advantage of the barycentric framework of Eqs. (12–13) would require an adaptation of the dynamical core of AROME, which falls outside the scope of this work.

All these settings are identical for the operational run (denoted REF) with the temperature-tendency based interface and for the run with the flux-conservative interface (denoted FCI).

4.1 Monthly scores

The daily forecasts during two periods are considered in this section: 1–30 November 2014 and 6 January – 6 February 2015. The first month is characterized by exceptionally mild weather, with numerous episodes of heavy precipitation in the South-West of France. The second month was characterized by strong winds and episodes of heavy snowfall. Figures 2 and 3 show bias and rmse for several meteorological variables for the two periods, respectively. These scores are calculated by comparing the AROME forecasts with observations throughout the French territory. Figures 4 and 5 compare the forecasted precipitation over the two periods. To avoid the problem of the double penalty, the precipitation is verified with the neighbourhood observation Brier skill score (Amodei and Stein, 2009). This score is determined by calculating the probability that a precipitation threshold is exceeded in the vicinity of an observation. By choosing the threshold, one focuses the verification more on light or on heavy precipitation.

The scores indicate that the impact of using the flux-conservative set of equations appears quite limited when considering time- and space-averaged scores as the ones considered here. It should be stressed that no retuning has been done for the experiments with the flux-conservative equations. As a result, compensating errors can be responsible for masking an improvement of the scores. The fact that the scores do not change substantially, merely indicates that the approximations that are made in the existing temperature-tendency based interface are indeed small on a domain-wide scale. In this context, the limitations of

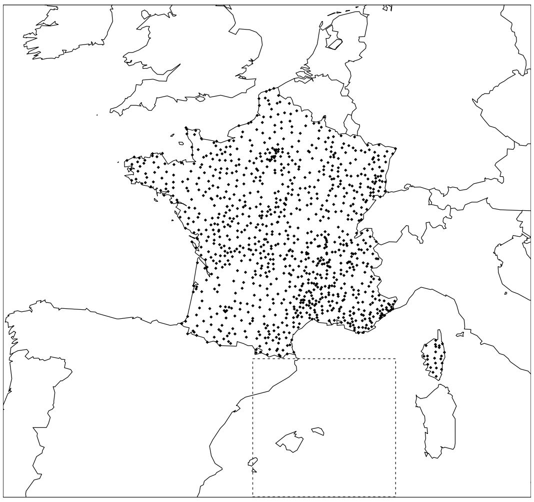


Figure 1. Operational AROME domain with a resolution of 2.5 km. The markers indicate the temperature stations used for the monthly scores. The dashed line indicates the area of the case-study of section 4.2.

this standard verification against station data should also be mentioned. By taking the average score over a large number of stations, important local differences may be hidden in the scores. In a similar way, the fact that monthly averaged scores are considered, only allows to detect differences that are systematic in time. Therefore, notwithstanding the neutral impact on the standard scores, some significant differences are observed under specific circumstances. A case study is presented in the next section to illustrate this.

4.2 Case study of a cold pool originating from heavy precipitation

When precipitation evaporates while falling through unsaturated air, it cools its environment. As such, a region of relatively cool air, the so-called cold pool, originates when heavy, localized precipitation occurs, for instance in precipitating convective systems (Fujita, 1959). It has been shown that the cold pool is in fact a key element in the lifecycle of a such systems. On the one hand, new convective cells originate at the border of the cold pool and its warmer surroundings, but on the other hand, if the cold pool becomes too strong, it may cut off the supply of warm air to the updraft (Engerer et al., 2008). The cold pool is also accompanied by a meso-scale high pressure area (Fujita, 1959) which plays a crucial role in the wind gusts that go with heavy precipitation. For these reasons, it is no surprise that the representation of the cold pool is also crucial in a NWP model (Engerer et al., 2008; De Meutter et al., 2014).

Although evaporative cooling is the main cause for a cold pool, a second mechanism may enhance it. As precipitation falls from colder layers aloft to hotter layers below, it will be heated by the surrounding air, which in response will cool down

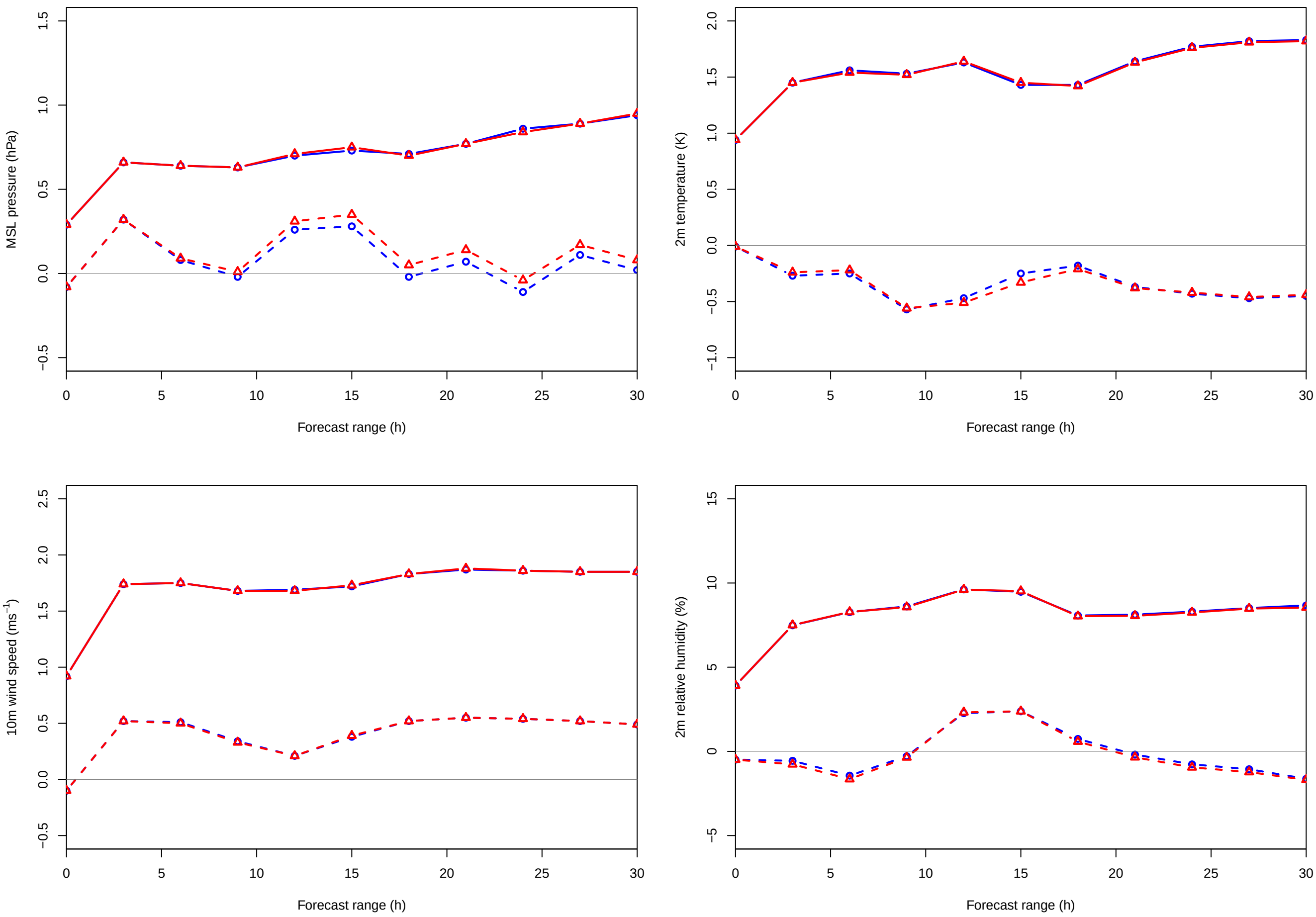


Figure 2. RMSE (solid line) and bias (dashed line) over the period 1 November 2014 – 30 November 2014, for REF (blue circles) and FCI (red triangles).

(Johnson and Hamilton, 1988). As explained in Sect. 3, this secondary thermodynamic effect (the transport of sensible heat) of precipitation is neglected in the existing AROME physics-dynamics interface, while it is correctly accounted for with the presented set of flux-conservative equations. One can thus expect that the intensity of a forecasted cold pool depends on which set of equations is used.

This is confirmed when looking at the AROME forecasts over the Balearic islands on 19 January 2015. This case is characterized by convection developing ahead of an active cold front coming from the south. Figures 6a and 6b show the forecasted 1200UTC–1800UTC accumulated precipitation with the existing AROME interface (REF) and with the flux-conservative interface (FCI). It is observed that the overall structure of the precipitation is quite similar. However, when comparing the coldpool characteristics of both experiments, important differences appear. Figures 6c and 6d show the differences between both experi-

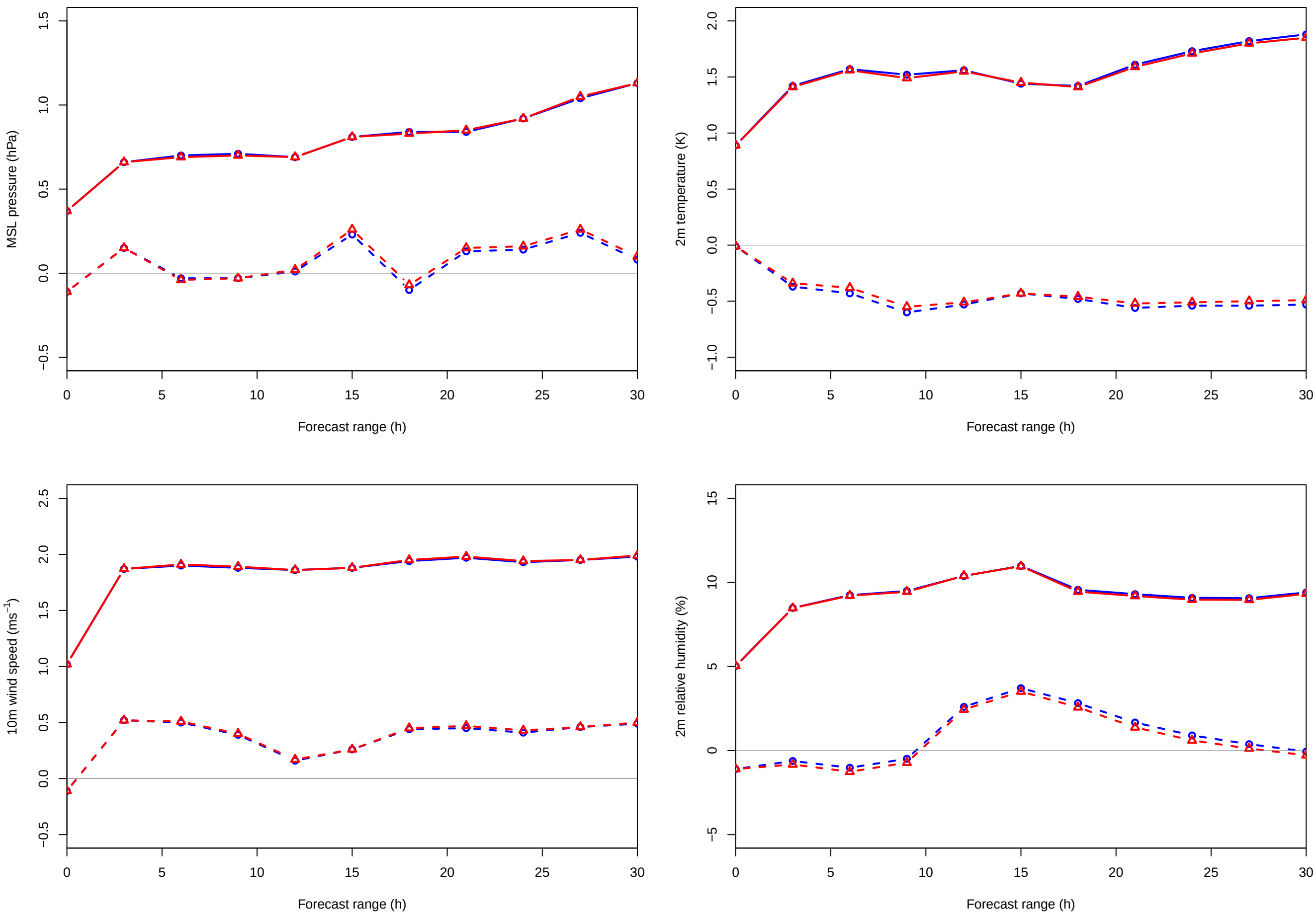


Figure 3. RMSE (solid line) and bias (dashed line) over the period 6 January 2015 – 6 February 2015, for REF (blue circles) and FCI (red triangles).

ments for the 2m temperature and the surface pressure. The temperature is significantly lower with FCI (up to 5 K cooler), and the surface pressure is higher (up to 1.4 hPa).

To further illustrate the impact of the heat transport by precipitation on the cold pool, the vertical profiles in the point as marked in figure 6b are studied for the experiment with the flux-conservative interface. The vertical profile of the precipitation fluxes (figure 7a) shows how snow and graupel originate aloft, they melt to form rain at around 850 hPa, and the rain starts to evaporate below 930 hPa. Figure 7b shows the vertical profile of the two phenomena that are responsible for the development of the cold pool, averaged between 1200UTC and 1800UTC: the latent heat effects from phase changes (solid line), and the falling of cold hydrometeors into warmer air layers (dashed line). It is clear that the second effect is orders of magnitude smaller than the first effect, at least when considering the full vertical extent of the model. However, as shown in figure 7c, the heat

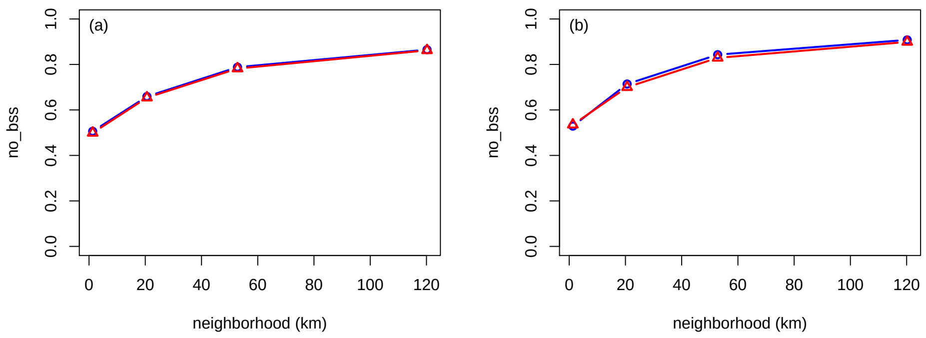


Figure 4. Neighbourhood observation Brier skill score for precipitation between 1200UTC and 1800UTC over the period 1 November 2014 – 30 November 2014, for REF (blue circles) and FCI (red triangles): (a) threshold 2 mm; (b) threshold 10 mm

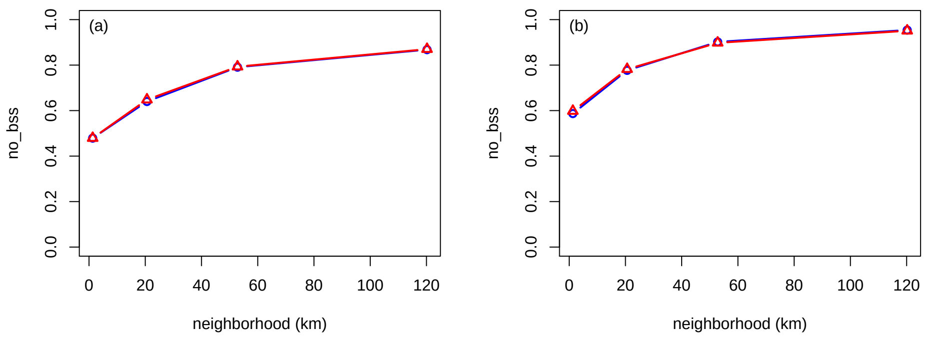


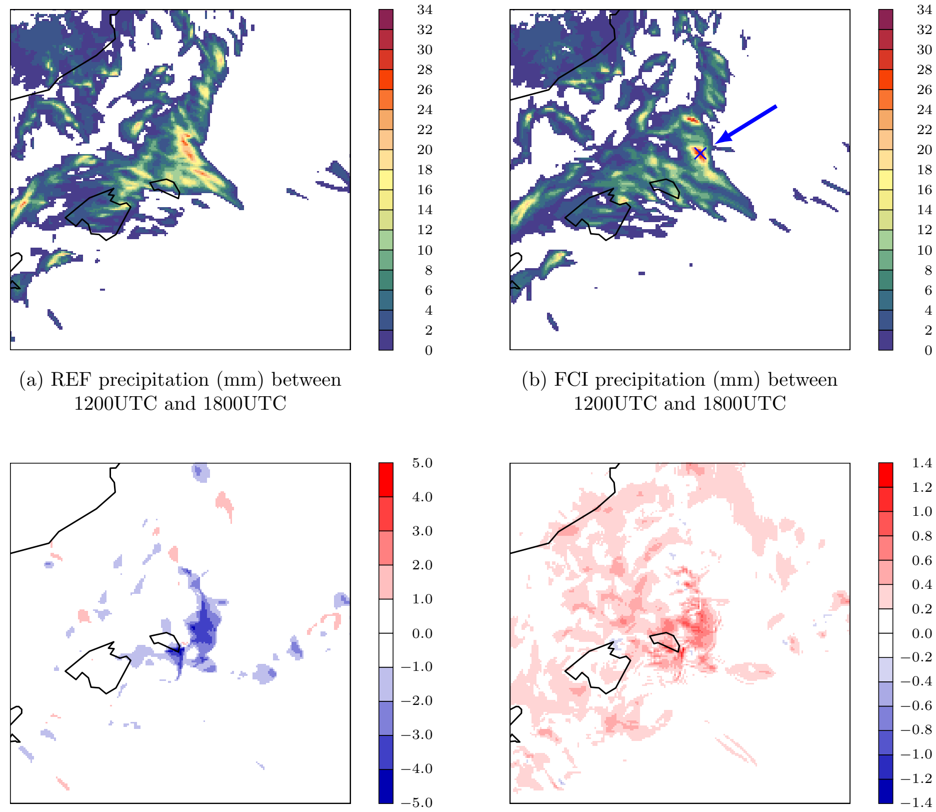
Figure 5. Neighbourhood observation Brier skill score for precipitation between 1200UTC and 1800UTC over the period 6 January 2015 – 6 February 2015, for REF (blue circles) and FCI (red triangles): (a) threshold 2 mm; (b) threshold 10 mm

transport by hydrometeors is not entirely negligible in the range between the surface and 900 hPa, and thus contributes to the intensity of the cold pool.

No comparison with observations is done for this case, because the purpose of this case study is merely to illustrate that even small terms can have a significant impact under certain conditions. The conclusions from this case-study are in line with the results from Bryan and Fritsch (2002), where neglecting a supposedly small term in the energy budget unexpectedly leads to the worst results.

5 Conclusions

This paper starts from the equations presented in Catry et al. (2007) that describe how the effect of physical parameterizations on the dynamical core of an NWP model can be expressed in a flux-conservative way. The main advantage of these equations



(a) REF precipitation (mm) between 1200UTC and 1800UTC

(b) FCI precipitation (mm) between 1200UTC and 1800UTC

(c) 2m temperature difference (K) between FCI and REF at 1800UTC

(d) Surface pressure difference (hPa) between FCI and REF at 1800UTC

Figure 6. Case of heavy precipitation on 19 January 2015. The arrow and the marker in subfigure (b) indicate the location of the profiles of Figure 7.

is that they impose the constraints of energy- and mass-conservation at a higher level in the model than at the level of the individual physical parameterizations. The presented equations only guarantee conservation of mass and energy regarding the effect of the physics contributions, not for the dynamical core of the model. A second advantage of the presented equations is that by gathering the thermodynamic calculations of all physics parameterizations in a single equation, it is also guaranteed that a predefined framework of hypotheses is consistently respected.

Notwithstanding these clear advantages, the equation set in the mentioned paper also faces limitations that hinder its application in existing NWP models. This paper presents a generalized set of thermodynamic equations that overcomes these restrictions without touching the sound theoretical foundations. More specifically, the presented equations are valid for an arbitrary number of hydrometeors, and can be applied in a model with an arbitrary number of conversion processes between these water species. This has allowed to use this set of equations in the AROME NWP model, which currently uses a physics-

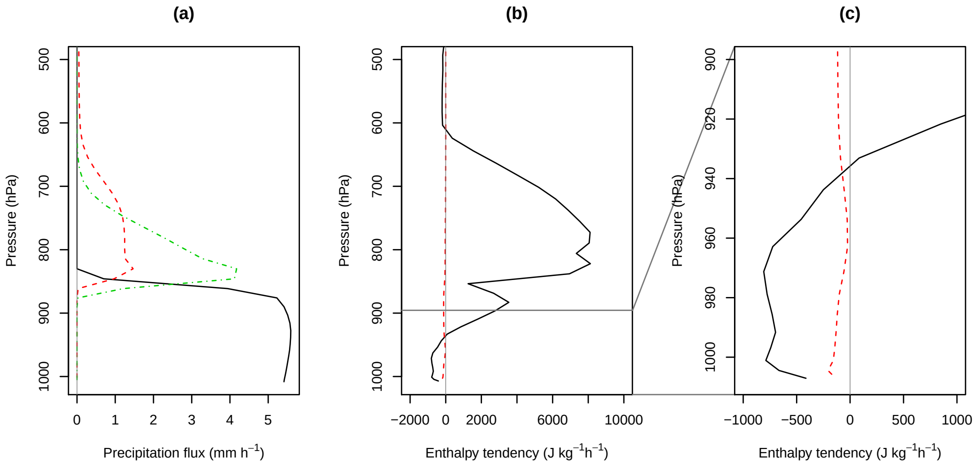


Figure 7. Vertical profiles at 1800UTC in the point indicated in figure 6b for the run with the flux-conservative interface. (a) precipitation fluxes: rain (black solid line), snow (red dashed line) and graupel (green dash-dotted line); (b) cold pool-generating phenomena: latent heat effects due to phase changes (black solid line) and sensible heat advection (red dashed line); (c) same as (b) but focused on near-surface.

dynamics interface that makes some ad-hoc approximations. By moving to the generalized flux-conservative equations, the effect of these approximations can be studied.

Monthly verification scores show that the overall effect of introducing the flux-conservative equations in AROME is quite limited. There is no significant improvement or degradation of these scores. Given the mentioned theoretical benefits of the presented equations, this means that the presented work is a valuable advancement of the AROME model. Moreover, it appears that substantial differences may exist in specific cases. A detailed study of a heavy-precipitation case gives the example of the formation of a cold-pool, which is an essential mechanism in the life-cycle of a convective event. As it appears, one mechanism that contributes to the formation of this cold pool is the heat transport by precipitation. This effect is neglected in the existing AROME physics-dynamics interface, while it is correctly accounted for in the presented flux-conservative set of equations. In this specific case, this leads to a different surface temperature and surface pressure within the cold pool. A more systematic study of the effect of heat transport on the life-cycle of a cold-pool is left for future research. In this manuscript, this case serves as an illustration of the importance of correctly accounting for supposedly small terms in the energy budget, something that is achieved with the presented set of thermodynamic equations.

Besides offering a direct improvement of the thermodynamic budget of the physics parameterizations of the AROME model, the presented set of equations also paves the way for interesting future research. Especially the impact of the heat from physics parameterizations on the continuity equation, and the effect of accounting for the net mass exchange between the atmosphere and the surface, are topics that deserve to be studied in detail.

Code availability

The used ALADIN Codes, along with all related intellectual property rights, are owned by the Members of the ALADIN consortium. Access to the ALADIN System, or elements thereof, can be granted upon request and for research purposes only.

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