Geoscientific
Model Development
Discussions

# *Interactive comment on* "A High-order Staggered Finite-Element Vertical Discretization for Non-Hydrostatic Atmospheric Models" *by* J. E. Guerra and P. A. Ullrich

**Anonymous Referee #3**

Received and published: 22 February 2016

The manuscript "A high-order staggered finite-element vertical discretization for non-hydrostatic atmospheric models" by J.E. Guerra and P. Ullrich describes the numerical formulation of a staggered nodal finite-element method for the vertical coordinate in nonhydrostatic models of atmospheric dynamics. The method is tested on benchmark cases of small and mesoscale flows. The paper is very well written and the results presented are comprehensive, the content is well structured and the analysis and discussion of the results are thorough and systematic.

However, given the validating character of the test cases presented, more could be done in order to assess the accuracy and efficiency properties of the method as well as its motivation and limitations. Once the comments listed below are addressed I am

happy to recommend publication of the manuscript in GMD.

## General comments

1. The context where the authors place their study could be explored more in depth. Although only small to mesoscale test cases in Cartesian geometry are considered here, some scope could be provided to the reader on the eventual application of the present method. Do the authors target global or limited-area models, with adaptive grids or otherwise?

2. No reference is made in the manuscript to parallel developments of mixed finite element formulations, see e.g. Cotter and Shipton 2012, Cotter and Thuburn 2014, Thuburn and Cotter 2015. It would be useful to the reader if the authors clarified the points of contact of their manuscript with the mixed finite element literature, in particular regarding the different requirements in terms of artificial viscosity.

3. Some more formal accuracy validation could be useful in evaluating the performance of the method. Besides the data in Table 2 and the differences in Figure 9, convergence or self-convergence tests could be performed for example in the Straka or bubble case that will reproduce the theoretical accuracy of the method, see for example the graphs in Restelli and Giraldo 2009. It would be enough to perform a test at an earlier final time than the one used in the run (e.g., 300 s for Straka), ie before grid-scale effects kick in.

4. In terms of computational performance, it would be useful to see for all the test cases the results in Table 3. Is there a reason why the authors report results for a different number of cores? Aside of scalability not being optimal, the stress there appears to be on the effect of higher vertical order on performance. If the

emphasis is on testing scalability then the method should be tested on a much larger number of cores.

5. At least for the bubble runs, it would be helpful to test mass, momentum, and energy conservation properties of the method as seen for example in time series of these quantities throughout the simulation. What do the authors expect?

6. A previous study by the authors (Ullrich and Guerra 2015) is not referenced in the manuscript. Is the present manuscript an extension of that one? Some discussion on this point should be added in the introduction.


**Specific comments**

1. Page 3, lines 25 ff.: please list the test cases in the order they are presented in the paper.

2. Page 4, the objectives are probably best listed before the paragraph on the test cases.

3. Page 5, since all the test cases are in Cartesian coordinates, the authors should ask themselves whether the paper could gain conciseness and clarity by reducing or eliminating the discussion on arbitrary coordinate frames. At least the formulation in Cartesian coordinates could be reported at the end of Section 2 for the reader's convenience.

4. Notation, please use the 'g' symbol only once. It is currently used for the local coordinate vectors, the metric tensor, as well as for gravity in expression (13), the flux correction functions in expression (33), and for equation (48).

5. Section 3.2.2. is very detailed, especially in comparison with section 3.2.5., and the information contained in the latter appears somehow inconspicuous despite

its importance. Some material could be relegated to an appendix and Section 3.2. slightly restructured to better guide the reader from the aims of the paper to the tests.

6. Section 3.3, some more detail should be provided to the reader regarding CFL limitations of the employed time integration strategy. The information about the time splitting at page 31 should be placed in this section instead. Is the method second order in time? Again, a convergence test could be useful in backing up this claim.

7. Page 18, the list of choices is very clear, however point (5) is unnecessarily iterated in the tests, e.g. at page 21 line 25.

8. Page 21, lines 18 ff. Does the sponge layer cover 10 out of 25 km in the vertical, i.e. is 40% of the computational power just used for the sponge? Is this sponge layer the same as the Rayleigh layer discussed at page 23? If so please use only one of the two terms.

9. Section 4.3 could include standard 1d cuts at 5000 m height as seen in the literature as well as the convergence tests suggested above.

10. Point 1 page 31, was the analysis of wave dispersion not claimed to be left for future work on page 4?

11. Figure 3, please define 'VO' in the caption as well for clarity.

12. Figure 7 contains discussion of the results that should be put in the text instead.

13. Legibility of the lines in Figure 10 could be increased by either plotting the differences with respect to the reference or splitting the four lines into two panels.

14. Figures' captions should contain information on the contour interval values.

**Technical corrections**

- Expression (35) page 12: please replace $\partial g/\partial x(x \to 1)$ with $\lim_{x \to 1} \partial g/\partial x$.

- Page 18, line 11, 2 x finer $\to$ twice as fine as.

- Page 20, line 1, "in are" $\to$ 'are'.

- Page 25, what is a "truncation order"?

- Figure 1, should $v_{np}$ be $n_{vp}$ instead?

**References**

Cotter, C.J. and Shipton, J. Mixed finite elements for numerical weather prediction. Journal of Computational Physics 231, 7076–7091, 2012.

Cotter, C.J. and Thuburn, J. A finite element exterior calculus framework for the rotating shallow-water equations. Journal of Computational Physics 257, 1506-1526, 2014.

Restelli, M. and Giraldo, F.X. A conservative discontinuous Galerkin semi-implicit formulation for the Navier-Stokes equations in nonhydrostatic mesoscale modelling. SIAM Journal of Scientific Computing 31, 2231–2257, 2009.

Thuburn, J. and Cotter, C.J. A primal–dual mimetic finite element scheme for the rotating shallow water equations on polygonal spherical meshes. Journal of Computational Physics 290, 274–297, 2015.

Ullrich, P.A. and Guerra, J.E. Exploring the effects of a high-order vertical coordinate in a non-hydrostatic global model. Procedia Computer Science 51, 2076–2085, 2015.

C5