

# ***Interactive comment on* “Open-source modular solutions for flexural isostasy: gFlex v1.0” by A. D. Wickert**

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*I would like to thank Referee 1 for the time they spent making these thorough comments, which have been very helpful to me in improving the understandability, readability, and presentation of this work.*

## **MAJOR COMMENTS**

1. The implementation is of course continuous. However, some users would wonder to what extent they can model very steep gradients in flexural rigidity, and narrow weak zones of very low elastic thickness (major fault zones or crustal discontinuities). The

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application example contains such weak zone (ridge) but as the author clearly states, it sorts of low pass filter this sharp boundary. It is important to explain to the reader to what extent gflex can approximate major faults or zone of weakness inside the plate and what their response would be. This is particularly important for the non-modellers.

*The low-pass filtering effect that was mentioned early in the paper, in the section on the development of the theory, is a fundamental characteristic of bending of plates of finite thickness. The equations always maintain continuity, even for a plate of 0 thickness – in this case, the flexural solution goes to full local isostatic equilibrium where plate thickness is 0, and then responds accordingly where plate thickness is finite. This is an important distinction between the treatment of a 0-thickness but continuous plate and a true discontinuity, e.g., as given by the 0Moment0Shear (“broken plate”) boundary condition. If the  $T_e = 0$  condition is in the middle of the load, then the response to the load on either side of it will be continuous with it, creating a cantilever-style condition at the edge of the weak zone. If the  $T_e = 0$  condition is over a place in which there is no load, then the solution in the rest of the region will be forced to approach this. See the figures at the end of the response for a better description. One final note is that the finite difference solution requires that the zone of  $T_e = 0$  be at least 3 cells wide in order to be seen as an isolated block. This is because of numerical diffusion inherent in the solution method and the high order (4) of the PDE. At first glance, this does seem like a disadvantage, or at least that there should be some more flexible way to include a non-straight broken plate condition. However, there are very few (no?) places on Earth’s surface where we truly reach a broken-plate solution, as this implies absolutely no physical interaction between the crust on opposite sides. The closest to this is a rift zone, but even if these have no mantle lithosphere, they have some (thin) crust.*

*I have included a paragraph on this in the paper; see the last paragraph in the a new section titled, “Limit as  $T_e \rightarrow 0$ ”.*

2. Delta rho is included in equations 1, 2 and 4. However it is not clear what this term is. It is only until pp. 4258/line 15 that rho\_f is mentioned. It would be important to

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explain the meaning of rho\_delta (rho\_mantle - rho\_filling) and the way it can be used to model different scenarios (marine vs. sedimentary basin, etc.).

*This was part of a big oversight on my part! I rearranged some of the paper before submitting, but did not add the descriptions of the variables into the proper sections. I have added the following:*

“ Here,  $w$  is vertical deflection of the plate,  $q$  is the applied surface load, and  $\Delta\rho = \rho_m - \rho_f$  is the density of the mantle minus the density of the infilling material. This last term represents the feedback by which flexural subsidence can lead a depression to be filled by material, which leads to additional flexural subsidence. This can occur, for example, in a system that is fully underwater (e.g., an underwater volcano load) or one in which the depression is completely filled with sediments. If this infilling material is not uniform in density and/or spatial extent—for example, due to onlap or offlap of water along a shoreline—then one may solve this feedback instead via iteration, by solving for  $\rho_f = \rho_{air} \approx 0$  and adding water (or another load) to regions that match certain conditions after every cycle of the iteration. ”

3. Can variable rho\_f (density of infilling) be modelled in gflex? Page 4260/line 15 says yes, but it is not sure how this can be accomplished. Can variable rho\_f be defined as input?

*It can, but not explicitly so: the quantity  $\rho_f$  is a scalar. I have changed the “As Part of a Python Script” section to explain this better than I had before:*

“ gFlex may also be imported as a Python module to be run either as a standalone simulation or as a component in a multi-model integration effort. This allows it, for example, to be a part of a flexural backstripping toolchain or a model of glacial–isostatic adjustment. A programmatic approach is also useful for scenarios in which material infills a depression, but not over the whole domain and/or not with uniform density.

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While the flexure equations require that  $\rho_f$  be constant, a more flexible way to solve for the effect of infilling material is to compute flexural response with  $\rho_f = \rho_{air} \approx 0$ , add loads based on some set of rules, and then re-calculate flexure iteratively until convergence is achieved. This can occur in regions with a complex set of sedimentary deposits and/or to be used for seawater loading across a shoreline. ”

4. Figure 5 is rather confusing. It does not completely explain how the different boundary conditions in Figure 4 can be related to different geological processes (line 5 of p. 4255). It is not clear also what the role of figure 5b is ("provides a contrived field of variable elastic thickness"?). Is that the elastic thickness distribution used in 5c and 5d? It does not seem so: for example in 5d the upper right mountain belt load produces more deflection than the lower left one. One would expect the contrary for the elastic thickness distribution in 5b.

*I have updated the figure to include all of the available boundary conditions. One of those in the figure had an old name from a development version of the code, and that has been fixed. Panel D does include more subsidence below the lower left mountain belt, as the reviewer noted would be expected, but this was obscured by the shading for the mountain belt. I have therefore replaced it with a pattern of lines that allows the underlying colors to be seen.*

5. p. 4255/lines 25 to 29: This sentence is not clear.

*The reason for the lack of clarity is because I somehow wrote a sentence that spanned 15 lines! This has now been broken up and re-worded, see the revised third paragraph in Section 2.3. "Boundary Conditions"*

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6. p. 4256/paragraph lines 13-24: Periodic boundary. Not clear. I read many times this paragraph, but still I don't understand how to model a continuous mountain belt with the Periodic boundary.

*I was concerned that many geologists might not understand what was meant by a periodic boundary condition (I certainly didn't until I started using numerical methods), so I tried to explain it as a loop. I have added additional words noting that you essentially connect the west and east edges (or north and south edges), and hopefully this provides some additional help. See the revised 6th paragraph in Section 2.3., "Boundary Conditions", and please let me know if this still is unclear.*

## MINOR COMMENTS

p. 4246/line 24: change "of Earth" by "of the Earth"

*Not changed: generally still correct, and it is better to have it without the article in the context of other planets (e.g., we would say "of Mercury").*

p. 4247/line 9: change "Analytical" by "The analytical"

*There is more than one plate-bending theory, so can't use the article, but the word "analytical" isn't needed (should just be implied), so I have removed it instead.*

p. 4248/line 15: change "and may" by "which could"

*This change would imply that boundary conditions relate to the linear response of the analytical solution, but I really want to say that the boundary conditions are applied to the loading scenarios, meaning that I must keep the current wording.*

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p. 4249/line 8: remove "greatly"

*Done.*

Eq. 5: What is kei?

*It is a Kelvin function. I didn't make this clear above, and in fact, I found my description of it unsatisfactory. So I have added some more background about what it actually is, and made clear that the Kelvin function mentioned in the text is kei.*

p. 4252-4253: Is the long sentence at the end of p. 4252 and ending at line 4 of p. 4253 necessary? It is rather confusing and in my opinion does not contribute much to the discussion.

*It explains the method for efficiently calculating the space-domain analytical solution, so is necessary. I have re-ordered the concepts in the sentence and broken it into multiple sentences for clarity. (It was ordered in a way that worked in my mind at the time, but probably wasn't the best explanation.) As a result of these changes, I have rewritten the entire paragraph as follows:)*

“ For a given elastic thickness, each flexural response to a line or point load is similar in shape, but different in amplitude. Therefore, I optimize solution speed by pre-calculating the flexural response to a unit load in the center of a template array. This array has twice the linear dimensions of the solution array, and is subsampled and re-scaled to compute the distributed response to each cell in the grid that contains a load. This technique works for all for rectilinear grids with uniform x- and y-grid-spacing, though the x- and y-grid-spacing do not have to be equal to one another. A similar optimization is possible for one-dimensional solutions, but these are so rapid that this has not been found to be necessary. Within gFlex, this solution type is termed “SAS”, which stands for “Superposition of Analytical Solutions”. ”

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p. 4260/line 13: "use repeat forward modeling" is awkward. Please consider changing. Also not clear how gflex can be used as a flexural backstripping tool.

*In answer to the awkward phrase, I have removed this and rewritten the paragraph to give the more general idea. In answer to the flexural backstripping tool portion, I have included (1) a citation to a paper that provides a general flexural backstripping method, for which gFlex and any other flexural isostatic calculation tool or method would work, and (2) a reference to a newly-accepted paper in which we use gFlex to backstrip a foreland basin and reconstruct the loading history of a mountain belt.*

p. 4262/line 15: "presents cause" is awkward.

*It is, and so is much of the sentence. I have rewritten it as follows:*

“ Earth’s crust at Iceland has been built by the unique intersection of the Iceland hotspot and the Mid-Atlantic Ridge, which together produce a weak lithosphere with spatially-variable elastic thickness, resulting in short-wavelength variability in solid Earth response to loading. ”

Figure 5c: What is "Dirichlet0" to the left of the scale bar?

*This was an old and poorly-described name for the 0Displacement0Slope boundary; this figure has been updated to fix that.*

Figure 6: Caption: Clearly indicate a, b and c: a. before "The ungridded". b. before "The gridded". and c. before "Finite difference"

*Thanks for catching this! Done.*

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Figure 6: scales of x and y axes in a, b and c are quite different. This can be misleading.

*They are not only different scales, but they are fundamentally different axes because of the different drivers of compute time for each of the different model runs. While it is nice to put fully-comparable plots above and below each other, I think in this case, the reader will be left to read the axis labels. Figure 7 has a comparison among the different solution types. I have fixed an issue with transparency in both of these figures (6 and 7) and updated the axes labels to be more intuitive.*

Figure 7: Put labels to the scale bars, including units (as in Figure 5).

*I'm guessing that you mean Figure 8? In which case, done, and thanks for noticing this!*

### Additional changes:

- Updated overview sketch (Figure 1) for better 3D perspective
- Defined  $T_e$  and  $D$  and their relationship towards the beginning of the paper
- Changed the left-handed coordinate system in the appendix schematic to a right-handed one (oops!)

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**Fig. 1.** 0 elastic thickness in N–S orientation in the in the middle (E–W) of the array; 0Displacement0Slope boundary conditions elsewhere.

**Fig. 2.** Broken plate on the eastern edge; 0Displacement0Slope boundary conditions elsewhere.

**Fig. 3.** 0 elastic thickness in N–S orientation on the eastern edge of the array with a broken-plate b.c. on the East. The broken plate b.c. now only applies to the 0-Te material, which responds to local isostatic equilibrium (here with no load), and the rest of the block must respond to be continuous with this 0-displacement boundary. Essentially, the 0-Te section absorbs the effects of the boundary condition locally, and then imposes a 0-displacement boundary condition on the interior.

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Interactive comment on Geosci. Model Dev. Discuss., 8, 4245, 2015.

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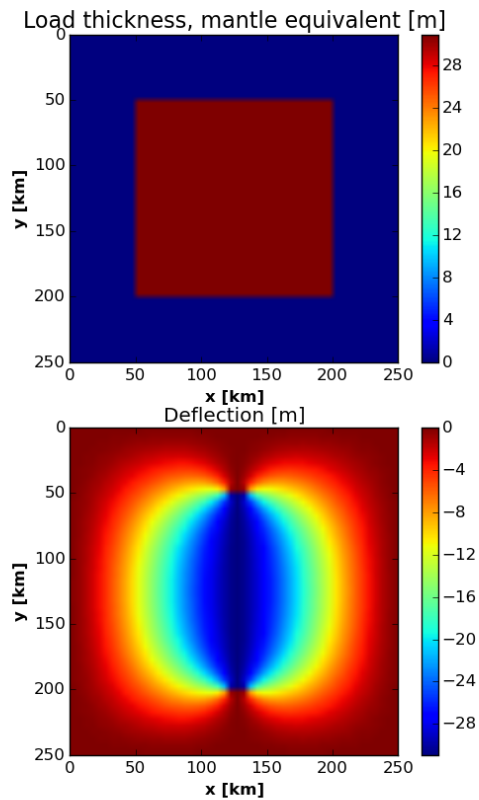
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Fig. 1.

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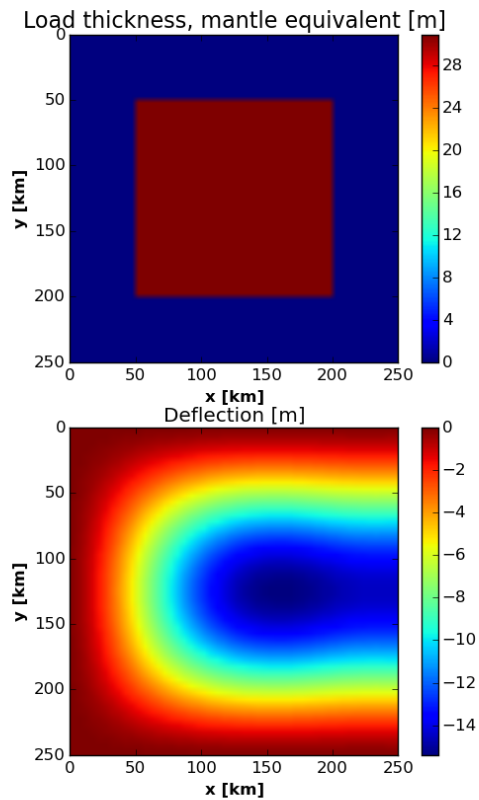


Fig. 2.

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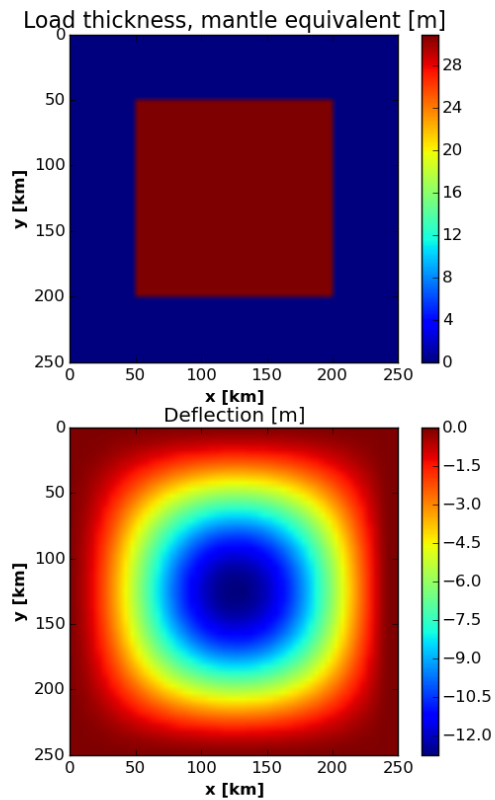
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Fig. 3.

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