

## ***Interactive comment on “Assimilating compact phase space retrievals of atmospheric composition with WRF-Chem/DART: a regional chemical transport/ensemble Kalman filter data assimilation system” by A. P. Mizzi et al.***

### **Anonymous Referee #1**

Received and published: 22 December 2015

Many thanks for revising the paper following my previous comments and questions. Please note my further comments below, which should require minor revisions.

#### Responses to the General Comments

1) I agree with the authors that here it is not important to distinguish between the use of an eigenvector decomposition and of an SVD (as I did in my first review of this paper) given that the two transformations are identical when the transformed matrix is symmetrical. In 10, however, the term  $U_0^T E_m U_0 = \Phi \Sigma \Psi$  should be

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replaced with  $U_0^T E_m U_0 = \Phi \Sigma \Phi^T$ . My comment about applying an eigenvector decomposition to  $E_m$  rather than to  $U_0^T E_m U_0$  is still unanswered. I think the authors should at least note in the paper that the latter option is also possible.

2) My understanding of the transformation as in Eq. 24 of Migliorini et al., 2008 is that it is not based, as the authors say in their replies at (iv), on an eigenvalue decomposition of the ensemble forecast error covariance but rather of  $H' P H'^T$ , where  $P$  can be the ensemble forecast error covariance. The eigenvalues of  $H' P H'^T$  are signal-to-noise ratios, so that a truncation according to eigenvalues of  $H' P H'^T$  is based on the measurements' signal to noise ratio. It is of course also possible to truncate according to the singular values of  $H'$ , which in Migliorini et al. 2008 is given by  $H' = E_m^{-1/2} A$ , where  $E_m$  is the measurement error covariance and  $A$  the averaging kernel matrix. The truncation is then applied by left-multiplying by  $U^T$ , the left singular vectors of  $H'$ , which are identical to the eigenvectors of  $H' H'^T$ , i.e. identical to the eigenvectors of the covariance of  $H' x^t$  when  $P = I$  (this is why this procedure is a particular case of the more general eigenvector decomposition of  $H' P H'^T$ ). Your approach is to decompose  $U_0^T E_m U_0$  to find its square root (rather than the square root of  $E_m$ ) and then to project according to the left singular vectors of  $A$  rather than the left singular vectors of  $H'$ . The two approaches are, if not mathematically identical, practically equivalent. This should be noted in your paper.

#### Responses to the Specific Comments

No further comments on 1) and 2)

3) As discussed above, the correct form of the eigenvector decomposition of  $U_0^T E_m U_0$  (equivalent to an SVD given the matrix is symmetrical, as noted by the authors) is  $\Phi \Sigma \Phi^T$  not  $\Phi^T \Sigma \Psi$ .

No further comments on 4) to 8)

9) I agree with the authors' derivation. Their last expression can (should?) be simplified

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as  $(I - A) C_a A^T$  as noted in my previous comment.

No further comments on 10) to 22)

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Interactive comment on Geosci. Model Dev. Discuss., 8, 7693, 2015.

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