Reply to referee's comments

Reviewers' comments are in plain and the author's reply is in italic text.

General response:

We would like to thank the two Reviewers for their in depth perspicacious comments that contributed to improving the presentation of our paper.

In summary, to address the comments of the reviewers, the following work has been carried out:

1. *The first two cases have been redone to show more convincing convergence as the mesh is refined. For the 1st and 2nd cases, the time step is reduced by a factor to ensure a small Courant number with the smaller elements sizes. We have thus re-plotted the results in Figs 2~9.*
2. *The 3rd case has been redone with a negative concentration background of -0.2 in the subdomain [0.24,0.76]×[0.12,0.88] as suggested by reviewer. The maximum number of nodes for adaptive schemes is set to be 15000. Table 1 and Figs 10~15 have been updated to reflect these new results. A new Fig 16 has been added to show the distribution of CFL number over the domain.*
3. *A new case, case 4 based on a real – large scale atmospheric geometry – and flow, has been added to demonstrate the capability of this new adaptive multiscale model. Figs 17~20 show the results obtained from this new case.*
4. *Case 4 is the simulation of the dispersion of power plant plumes. Diffusion and source terms have therefore been introduced into the equations in section 2.*
5. *Section 3 has been revised and more details of the adaptive mesh techniques have been added.*
6. *Section 2.2 has been rewritten and details of numerical schemes have been provided.*

<u>Anonymous Referee #1</u>

Summary: the authors present details of an adaptive grid-resolution approach to solving the classic tracer advection problem whereby locally movable higher resolution grids are employed in areas of tracer distribution where steep gradients and small features are better simulated with finer resolution. Overall the paper is sound and presents promising results. I suggest a couple minor comments related to the tests presented, and all the tests should be repeated using a nonzero (preferably a negative) background rather than zero background, since there is nothing special about zero, but many algorithms assign inappropriate significance to zero. Positive-definite schemes are not necessarily useful with tracers with large backgrounds are present, and the value of this scheme increases if it can be shown to advect negative tracers. Please redo the 3rd test with a negative background.

RESPONSE:
Thanks for your comments. As suggested, the 3rd test has been redone with a negative concentration background of -0.2 in the subdomain [0.24,0.76]$\times$[0.12,0.88]. The maximum number of nodes for the adaptive schemes is set to be 15000. Figs 10~15 in the paper have been replotted. A new Fig. 16 has been added to show the distribution of CFL number over the domain.

Other minor points would improve the manuscript

1. In Eq (6) the "sup" operator is used. I am not familiar with this nomenclature. The authors should briefly qualitatively explain this operator and maybe provide a reference.

   RESPONSE:
   Here "sup" is the abbreviation of supremum. This has been explained in the paper.

2. At line 220 the authors note that computation efficiency is obtained by reducing the number of grid cells. However, this reduction is very dependent on the nature of the scalar fields being advected. For many air pollution scenarios, tracer fields can be very noisy with multiple point sources and advection feature, and it is possible that this adaptive grid will use considerably greater grids than a fixed grid approach. For example, in Fig. 1 if another tracer blob were added to the tracer field, or of one of the three shapes were removed, the number of grid cells would change dramatically.

   RESPONSE:
   The computation efficiency has been estimated through a comparison between the fixed and adaptive schemes where the minimum mesh size of the adaptive grid is set to be the same with the mesh size of the fixed uniform grid. Detailed discussion has been provided in cases 1-3.

   Here as suggested, we have added a new case (case 4) to show the computational efficiency, where there are 100 emission sources in the domain. Again, it is shown that the results using adaptive meshes are in agreement with those using fixed meshes with a high mesh resolution of 2.5 km while the number of nodes decreases by a factor of 16 with use of adaptive meshes. The corresponding discussion is provided in section 5.4.

3. At line 270 the authors describe the Staniforth swirling test. This is an interesting test where the advected tracer distribution becomes sheared into smaller and smaller swirls that become infinitesimally small as time progresses, and this raises an interesting question about comparing the "exact" solution with numerical approximations. As time goes to infinity, Walcek & Aleksic (their fig 13) show that the tracer distribution turns into an essentially unchanging button/pillow-like appearance. This "pillow" appearance might in fact be an EXACT solution, AT

THE RESOLUTION OF THE NUMERICAL SIMULATION. When comparing their algorithm with fixed-grid modeling domain, the authors should average the "exact" solution over the identical averaging volumes used in the fixed or adaptive grid models. Even here, the adaptive grid should be averaged onto the same fixed grid and then compared. I think it unfair to compare simulations at different resolutions.

RESPONSE:
The "exact" solutions were plotted at times after initialization by calculating back trajectories along streamlines of these swirling flow. As displayed in Figs 11-12, the distribution becomes sheared into smaller and smaller swirls as time evolves. To adequately represent the "exact" solution, sampling interval Δx or Δy near the edges of the vortex should be set to 1/25600 so that these small-scale features can be qualitatively represented. However, for a given spatial resolution, there is a limiting time beyond which it is no longer possible to adequately represent all the space scales of the exact solution in a qualitative manner.

Therefore, Staniforth et al.(1987) defined two flow regimes (short time periods and long time periods) that have different evaluation criteria for the numerical advection schemes. For long time periods, it is necessary to average the "exact" solution over the identical averaging volumes used in the fixed or adaptive grid models. However, in this paper, we focus on the evaluation of the first regime (short time periods) so that the numerical solutions should be compared with the "exact" one in a qualitative manner. As shown in Figs 11-12, the anisotropic adaptive schemes can effectively represent infinitesimally small-scale features using almost the same number of fixed uniform grid nodes. But if the adaptive grid has been averaged onto the same fixed grid, these small features would disappear due to insufficient resolution.

Further clarification has been made in section 5.3

4. At line 300 the authors state that the regular grid contains 40000 grid cells. This simulation domain consists of a grid of 4x4 (16) swirling vortex circulations that are materially isolated from one another. The initial tracer is spread over only six of those vortices, and all tracer mass remains within those six swirling cells. Therefore the regular grid really only needs 6/16 (or 3/8ths) of the 40000 cells to simulate this tracer evolution with time or 15000 cells. This is the true number of cells required for any non-adaptive grid. The authors should reduce the domain size for this test to be restricted to the six cells containing tracer mass. All of the remaining domain is only advecting a constant. Either reproduce this test using the reduced domain, or change the reference from 40000 cells to 15000 cells.

RESPONSE:
The 3rd test has been reproduced using the reduced domain $[0.24,0.76]\times[0.12,0.88]$ that only cover those 6 swirling vorticies. The maximum number of nodes for the adaptive schemes is set to be 15000. The results have been presented in Figs 10-15.

5. Again for the Staniforth test: Fig. 15 shows that the adaptive grid method is using considerably greater number of grid cells than the 15000 cells required (not 40000,

see note above) by the fixed grid beyond a critical time, and this might even be a problem for this method. I assume the authors utilize some method for stopping grids from becoming infinitesimally small? Please explain how to stop this grid-adaptive method from going too small in size.

RESPONSE:
For robustness of the mesh adaptivity procedure, and to limit refinement/coarsening of the mesh it is possible to set the maximum and minimum allowed edge length sizes. These constraints are achieved through manipulations to the metric, which in turn controls an optimization procedure. Section 3 has been re-written and more details of adaptive mesh techniques have been added.