Interactive comment on “PLUME-MoM 1.0: a new 1-D model of volcanic plumes based on the method of moments” by M. de’ Michieli Vitturi et al.

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Received and published: 24 July 2015
PLUME-MoM 1.0: A new integral model of volcanic plumes based on the method of moments

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24 July 2015

Responses to Referee 3

Dear Referee, thank you very much for the thorough review of the manuscript. You can find below the reply to the comments and a revised version of the manuscript where we have highlighted the changes and the new text added to clarify the points raised by the referees. Please note that some additional details can be found in the answers to the other referees.

R3. In Section 5.3 – 5.4, the logic is complicated and messy, and there are many technical jargons that are not defined nor fully explained. “Response surface” is not clearly defined, and therefore, it is difficult to understand Figs. 7 and 8. For these uncertainty and sensitivity analysis, more systematic parameter study is required using C1512
a lot of simulation settings such as vent and atmospheric conditions to obtain the general conclusion about a small change of the size distribution along the plume. In my opinion, in order to clarify the focus of this paper, it is better to describe the discussion of Section 5.3 – 5.4 in another paper after thorough analyses and discussion.

A. Accordingly to the suggestions of the referees, we have tried to clarify some of the terms used, as "response surface" and more references have been added. For the uncertainty and sensitivity analysis, a more systematic parameter study was out of the scope of this paper. This is actually the main topic of a parallel paper we are working on. Our aim here was to apply the sensitivity analysis and uncertainty quantification techniques to the parameters described by the moments of the particle size distribution, since the paper focuses on this specific issue. Sections 5.3 and 5.4 also provide informations on the changes in the parameters describing the grain size distribution for the three different test cases, otherwise presented only in Figure 5. For these reasons, we do not think that the content of the sections should be moved in another paper but that it helps in generalizing the results presented in Sections 5.1 and 5.2.

R3. p.3755, Eq.(16): It needs a reference that clearly expresses the formulation. For example, Bursik et al. (1992).

A. The reference has been added.

R3. p.3759, Eq.(28): In order to evaluate this equation, it will be helpful to briefly describe the derivation from Eq.(26) to Eq.(28) in appendix.

A. I've reported here the steps for the derivation of the equation. There is nothing particularly interesting because it mostly consists in expanding the derivatives and re-arranging and canceling some terms. For this reason we do not think that there should be an appendix, but if the referees and the editor thinks that this would be helpful, we can add these steps in an appendix.
\[ C_{\text{mix}} = \frac{\rho_{\text{atm}} C_{\text{atm}} + \rho_{\text{wv}} C_{\text{wv}} + \sum_{j} \rho_{s,j} \bar{C}_{s,j}}{\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j}}. \] (1)

From this expression, if we multiply all the terms at the numerator and the denominator of the right-hand side by \( U_{sc} r^2 \), and then we derive, we obtain:

\[ \frac{\partial C_{\text{mix}}}{\partial s} = \]

\[ \left[ \frac{C_{\text{atm}}}{(\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j}) U_{sc} r^2} - \frac{\rho_{\text{atm}} C_{\text{atm}} + \rho_{\text{wv}} C_{\text{wv}} + \sum_{j} \rho_{s,j} \bar{C}_{s,j}}{(\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j})^2 U_{sc} r^2} \right] \frac{\partial}{\partial s} \left[ \rho_{\text{atm}} U_{sc} r^2 \right] \]

\[ + \sum_{j} \left[ \frac{\bar{C}_{s,j}}{(\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j}) U_{sc} r^2} - \frac{\rho_{\text{atm}} C_{\text{atm}} + \rho_{\text{wv}} C_{\text{wv}} + \sum_{j} \rho_{s,j} \bar{C}_{s,j}}{(\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j})^2 U_{sc} r^2} \right] \frac{\partial}{\partial s} \left[ \rho_{s,j} U_{sc} r^2 \right] \]

\[ + \sum_{j} \frac{\rho_{s,j} U_{sc} r^2}{(\rho_{\text{atm}} + \rho_{\text{wv}} + \sum_{j} \rho_{s,j}) U_{sc} r^2} \frac{\partial \bar{C}_{s,j}}{\partial s} \]

Now, we obtain after the collection of some terms and some algebra manipulations the
following equation for the variation of the mixture specific heat with $s$:

$$\frac{dC_{mix}}{ds} = $$

$$\frac{1}{\rho_{mix} U_{sc r^2}} \left[ (C_{atm} - C_{mix}) \frac{d}{ds} \left( \rho_{atm} U_{sc r^2} \right) \right. $$

$$+ \sum_j \left( \bar{C}_{s,j} - C_{mix} \right) \frac{d}{ds} \left( \rho_{s,j} U_{sc r^2} \right) $$

$$+ \sum_j \frac{\rho_{B}}{\rho_{mix}} \left[ \frac{\frac{d}{ds} \left( \bar{C}_{s,j} \rho_{B} U_{sc r^2} \right)}{\rho_{s,j} U_{sc r^2}} - \frac{\bar{C}_{s,j} \frac{d}{ds} \left( \rho_{s,j} U_{sc r^2} \right)}{\rho_{s,j} U_{sc r^2}} \right]. $$

Finally, with some cancellation:

$$\frac{\partial C_{mix}}{\partial s} = $$

$$\frac{1}{\rho_{mix} U_{sc r^2}} \left[ C_{atm} \frac{\partial}{\partial s} \left( \rho_{atm} U_{sc r^2} \right) $$

$$- C_{mix} \left( \frac{\partial}{\partial s} \left( \rho_{atm} U_{sc r^2} \right) + \frac{\partial}{\partial s} \left( \rho_{s} U_{sc r^2} \right) \right) + \frac{\partial}{\partial s} \left( \bar{C}_{s} \rho_{s} U_{sc r^2} \right) \right] $$

and, substituting the expressions for the partial derivatives appearing in each term on
the right-hand side:

\[
\frac{dC_{\text{mix}}}{ds} = \frac{1}{\rho_{\text{mix}}U_{sc}r^2} \left[ C_{\text{atm}}2r\rho_{\text{atm}}U_\varepsilon - C_{\text{mix}} \left( 2r\rho_{\text{atm}}U_\varepsilon - 2p_r \sum_j \frac{\pi}{6} M_j^{(3)} \left[ w_{s,j}\rho_{s,j}\right]^{(3)} \right) \right]
\]

(5)

R3.  p.3760, Eq.(30): It is also required to describe the derivation from Eq.(29) to Eq.(30) in appendix.

A.  The steps are the same used for the previous equation.

R3.  Section 4.1: For researchers in volcanology community, this section is quite difficult to understand. In order to clarify the algorithm, the wording and style of this section need careful editing. What are the meanings of “realizable” and “unrealizable”?

A.  We have tried to simplify this section removing some part and adding some reference. For the meaning of the term realizable", this was explained in the text at pag.3763, lines 20-21:

"the moment set is realizable, meaning that there exists a particle size distribution resulting in that specific set of moments."

Conversely, unrealizable set of moments means that there are no distributions resulting in that specific set of moments. In any case, we moved the definition before in the text, to make it clearer.

R3.  p.3770, paragraph from line 20: Usually, the variation of distribution is also evaluated by kurtosis. The skewness has a peak value at about 2 km and decreases
with height. What do these features of the skewness at the lower region of plume mean?

A. We agree that also the kurtosis is used to characterize distributions, and it can be obtained from the moments (as now written in the text) but the changes in the values are more difficult to interpret in the context of a particle size distribution. The same difficulty in the interpretation apply to the value of the skewness.

R3. Section 5.2: We would like to see the variations of size distributions for Cases 1 and 3 like Figure 5. It is better to carefully describe the similarity and difference between the cases on the basis of these figures. In addition, it will be helpful to show the variation of 2-D distributions in the horizontal spaces.

A. We report below the plots obtained for Cases 1 and 3. We remark that the outcomes of these plots, i.e. the changes in the mean and SD of the grain size distribution and the solid mass flux fraction lost, are consistent with the results already presented in figure 7 and 8, and discussed in the manuscript. Only the variations of skewness is not presented in figs. 7 and 8 but, as stated in the answer to the previous comment, the changes in the skewness are difficult to interpret and also quite small, in particular for the strong plumes. Regarding the 2-D distributions in the horizontal spaces, the model assumes a top-hat profile so there are not variations in the cross-sectional plane orthogonal to the plume centerline.

R3. p. 3786, caption of Figure 5: The caption should clearly indicate that these figures are the results of Case 2.

A. Done.

R3. Technical corrections

A. All the corrections suggested by the reviewer have been done.
Fig. 1. Particles distribution parameters (mean, variance and skewness) and cumulative loss of solid mass flux for test case 1.
Fig. 2. Particles distribution parameters (mean, variance and skewness) and cumulative loss of solid mass flux for test case 3.