

# ***Interactive comment on “PLUME-MoM 1.0: a new 1-D model of volcanic plumes based on the method of moments” by M. de’ Michieli Vitturi et al.***

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# PLUME-MoM 1.0: A new integral model of volcanic plumes based on the method of moments

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## Responses to Referee 2

Dear Referee, thank you very much for the thorough review of the manuscript. You can find below the reply to the comments and a revised version of the manuscript where we have highlighted the changes and the new text added to clarify the points raised by the referees. Please note that some additional details can be found in the answers to the other referees.

**R2.** *In the Introduction, your description of the value of using a continuously variable function is too vague to present a clear image of the problem and its importance. Much of problem is in a single sentence on lines 12-14 of page 3747, which describes the limitations of models that do not use continuously variable functions. I would change*

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this line by deleting the “. . . “ in the following: “Both approaches make proper treatment of the continuous variability . . . of particles and gas bubbles difficult”.

**A.** We have modified the text as suggested by the referee and added a few more references.

**R2.** *In Section 2, it would be worthwhile adding a qualitative statement at the beginning or end of this section saying how the number-based moments, Sauter mean diameter in particular, differ from corresponding moments calculated by mass-based methods. The Sauter mean diameter appears to be an average diameter based on the number of particles rather than mass. Because there are more small particles than big ones, this mean diameter should be less than a mass-based mean diameter. Is this correct? Would the same apply for the moments of density and other properties?*

**A.** The Sauter mean diameter is not defined as a function of the number of particles but it is defined as the ratio between total volume and total surface area, where the total values are obtained integrating over all the particles. The fact that there are more small particles than big ones results in the fact that the diameter  $L_{10} = M^{(1)}/M^{(0)}$ , based on the number of particles of the different sizes (represented by  $M^{(0)}$ ), will be smaller than the diameter  $L_{43} = M^{(4)}/M^{(3)}$ , based on the volumes of the particles (proportional to  $M^{(3)}$ ). To clarify we added this physical interpretation also in the text.

**R2.** *A few key assumptions of the model should be stated more explicitly. For example I could see from the energy equation that phase changes in water were not considered, but I didn't see this point explained in the text.*

**A.** The main assumption of the model now are stated more explicitly at the beginning of Section 3: "In this section we describe the assumption and the equations of the model. As in Bursik 2001, the model assumes an homogeneous mixture of particles and gases with thermal and mechanical equilibrium between all phases. Aggregation and breakage effects are not considered and consequently density does not change with time. Finally, the model does not consider effects of humidity and water phase

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changes."

**R2.** *Page 3747: Lines 12-13: deleting “of fundamental physical and chemical properties of the dispersed phases” will make this sentence more concise and meaningful. Lines 14-19: could you cite some of the literature that you say makes the point in this sentence? Line 24: Many readers (including me) will be unfamiliar with the method of moments. What fundamental physical quantities are balanced in a population balance equation? And what are “moments” in physical terms? A general statement relevant to the Introduction. I am familiar with only one other 1-D plume model that treats the GSD as a continuous function and analyzes its change with height. Veitch and Woods [2001] looked at changes in the GSD with height in a plume due to particle aggregation. Veitch and Woods isn’t mentioned in this paper, but perhaps should be. Could one modify PlumeMom to calculate changes in GSD due to aggregation? This might be a selling point of this approach, since particle aggregation has proven very hard to model quantitatively by other methods.*

**A.** We removed the text as suggested by the referee. The following references have been added:

Costa, A.; Folch, A. and Macedonio, G. A model for wet aggregation of ash particles in volcanic plumes and clouds: 1. Theoretical formulation *Journal of Geophysical Research: Solid Earth* (1978–2012), 2010, 115.

Pal, R. Rheological behavior of bubble-bearing magmas *Earth and Planetary Science Letters*, 2003, 207, 165-179.

Llewellyn, E. W.; Mader, H. M. and Wilson, S. D. R. The constitutive equation and flow dynamics of bubbly magmas *Geophysical Research Letters*, 2002, 29.

More details on the moments and their physical interpretation have been added both in the introduction and further in the text.

Also in Veitch and Woods the GSD is not treated as a continuous function. This is

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explained in their appendix A.3: "To determine the evolution of the particle size distribution in the plume, we discretize the range of particle sizes and evaluate  $g(j)$  for integer values of  $j$  only."

Regarding the modification to the code to calculate changes in GSD due to aggregation, this is something that we are currently implementing in the model, and we agree with the referee that it is a selling point of the approach. This has been added in the abstract and conclusions.

**R2.** *page 3748, line 7: what do you mean by "the implementation of the quadrature"? Line 10: Uncertainty in what? What physical properties are you studying, whose uncertainty you want to incorporate?*

**A.** Part of text was missing. Now the text is "the implementation of the quadrature method of moments" and a reference has been added. The text has been extended writing "epistemic uncertainty in input parameters (characterizing lack-of-knowledge)".

**R2.** *Page 3749: Equation 1: What terms on the right-hand side of this equation are different for different moments? It seems that this  $f_j(D)$  would have to be different, but it's not clear to me how. Also, perhaps mention that the second and third moment assume a spherical shape. Or if this is not the case, what do they assume? And what does the superscript  $i$  on  $D_i$  mean? Line 20: it would be helpful to explain briefly what the Sauter mean diameter means physically. I thought this was a diameter based on number concentration of particles but in Fig. 2 it shows the Sauter mean diameter as being larger than the mass-based mean, which one would not expect from a numbers-based diameter.*

**A.** The term on the right-hand side changing for different moments is the exponent  $i$  in  $D^i$ . The text has been changed clarifying the assumption of spherical particles for the expressions of the moments. The Sauter diameter is not based on number concentration but it is defined as the ratio between total volume and total surface area. To clarify we added this physical interpretation also in the text.

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**R2.** Page 3750: line 14: add “such” before “as settling velocity”. line 15: change “function” to “functions”.

**A.** We changed the text as suggested by the referee.

**R2.** Page 3751: Equation 3: actually, using the method of Bonadonna and Phillips, density at diameters intermediate between  $D1$  and  $D2$  are interpolated on a log scale of  $\phi$ , not on a linear scale of  $D$  as this equation indicates. Equation 4: what is the physical meaning of the different moments of density? Line 8: What are  $D^*$  and  $\rho^*$ ? Lines 11- 12: you define volumetric averaged density as the mass of particles per unit volume. Does “per unit volume” mean per unit volume in the jet or plume? Per unit volume of each particle? If per particle (as implied later), perhaps say “average mass per unit volume of particles”

**A.** Eq. 3 has been removed to avoid confusion and the text has been changed in the following way: “density of pumices  $\rho_{s,pum}(D)$  with diameter  $< 2$  mm is assumed to decrease and to reach the lithic density value when the fragment diameter decreases below  $8 \mu\text{m}$ .”

The physical meaning of the different moments of density is explained in the text: “Otherwise, there is no reason, e.g., for  $\rho_{s,j}^{(1)}$  and  $\rho_{s,j}^{(3)}$  to be the same, as they represent length and volume weighted density averages, respectively.” This means that the moment of order 1 is an average with a weight given by the particle length, the third moment is an average with the weight given by the volumes. In the same way, for example, the moment of order zero will just integrate the densities and divide by the total number of particles.

$D^*$  and  $\rho^*$  are the constant values assumed for a monodisperse distribution. This has been now clarified in the text.

The referee is right and the definition has been corrected writing “average mass per unit volume of particles”.

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**R2.** Page 3752: line 10: Put “Pfeiffer et al. (2005); Textor et al. (2006a, b)” in parentheses. Equation 8: what is the physical meaning of the moments of settling velocity? Maybe this could be addressed by just moving the statement (line 3 of the next page) that they represent surface and volume-weighted averages to the line immediately above the equation.

**A.** The parentheses have been added. The physical meaning of the different moments is simply a different way to evaluate the weighted average, i.e. weighting the settling velocities of particles of different sizes with a quantity proportional to the particle surface ( $D^2$ ) or to the particle volume ( $D^3$ ).

**R2.** Page 3753: Line 5: change “particles” to “particles”, to make it possessive.

**A.** Done!

**R2.** Page 3756: Line 13-14: bulk density means mass of particles per unit volume of particles? Mass of particles per unit volume of plume mixture? If the latter, maybe say “mass of particles per unit volume of plume mixture”, or something similar.

**A.** The definition has been added to the text: “bulk density of the particles of the  $j$ -th family (i.e. the mass of particles of the  $j$ -th family per unit volume of gas-particles mixture)”.

**R2.** Page 3757: Equations 22, 23: If this is a 3-D coordinate system, shouldn't there be three momentum conservation equations? Also, in the equation for horizontal momentum (22), I've generally thought of the change in momentum of the gaseous phase (first term on the right-hand side) as being equal to the horizontal momentum contained in the entrained air ( $2 * r * r_{ho_a} * U_e * w$ ). Your formulation is a little different. Perhaps you could add a sentence explaining your formulation.

**A.** As stated in the introduction, in this work we present an extension of the Eulerian steady-state volcanic plume model presented in Barsotti et al. (2008) (derived from Bursik, 2001). Eqs. 22 and 23 use the same formulation of Eqs. 4 and 5 in Barsotti et

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al. (2008), with the only difference that the loss of particles is computed in terms of the moments. The reference to the original work has now been added to the text.

**R2.** *Page 3758: Equation 24 (energy equation): Do you mention anywhere that you are not considering phase changes of water? It looks from this equation like you are not considering it, but I don't see that you mention this point in the anyplace in the text. Line 18: What do the superscript B's represent? Also, defining the rho terms on this line as bulk densities seems misleading (at least to me). In order for the denominators on the top and bottom of eq. 26 to be consistent, the rho's must be the mass (or air, water vapor etc.) per unit volume plume mixture, not per unit volume of air or water vapor. Perhaps refer to them as the mass of each component per unit volume of plume fluid. (maybe I missed it).*

**A.** A paragraph has been added at the beginning of the section where the main assumptions, as the fact that we are not considering phase change of water, are listed. The superscript  $B$  is used for the bulk properties and now it is defined in the text when the bulk density is introduced. As stated before, the bulk density refers to the mass of a phase per unit volume of the mixture, and thus the definition given by Eq.26 is consistent with the interpretation given by the referee.

**R2.** *Page 3760: Line 15: change "particles number" to "particle numbers"*

**A.** The text has been changed.

**R2.** *Page 3762: Equation 40: If these are ODE's, the LHS of eq. (40) should be  $dy/ds$  rather than  $\text{partial}(y)/\text{partial}(s)$ . Equation 41: perhaps add a multiplication symbol on the RHS between  $ds$  and  $f()$ .*

**A.** The partial derivatives have been changed to  $dy/ds$ .

**R2.** *Page 3764: Line 14 and elsewhere: change "particles family" to "particle family" Line 23: change "particles size distribution" to "particle size distribution"*

**A.** The text has been changed.

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**R2.** Page: 3765: Line 9: change “then writes as” to “is then written as” Equations 44: does “log2” mean the log base 10 of 2? Maybe write as  $\log_{10}(2)$ . Or, if you mean the natural log, then write  $\ln(2)$ . Pages 3765-3768: this is quite a slog through this section. It’s not clear exactly where you’re going. Adding a sentence at the beginning of Section 4.2.1 stating your objective in deriving these formulas might help readers keep their interest.

**A.** “then writes as” has been changed to “is then written as”. All the occurrences of “log” have been changed to “ln”. Furthermore, a sentence has been added before the subsection 4.2.1 to better introduce what is the objective in deriving the formulas.

**R2.** Page 3769: Line 5: the term “weak plume without wind” makes no sense to me, since a weak plume is generally defined as one that is bent over by wind [Sparks et al., 1997, p. 279]. “Low-flux plume without wind” may be more accurate. Line 16: you use a normal distribution? Not lognormal? Line 13: can you provide a reference for the standard atmosphere cited here? Line 17: change “expresses” to “expressed”. Line 18: Are you describing three different model runs, or a single model run with the output portrayed in three different ways?

**A.** The text has been changed using the term suggested by the referee “Low-flux plume without wind”. The normal distribution is referred to the phi scale, it is a lognormal when the diameter is expressed in meters. A reference for the standard atmosphere has been added:

“Champion, K.; Cole, A. & Kantor, A. Standard and reference atmospheres Handbook of Geophysics and the Space Environment, Air Force Geophysics Laboratory, 1985, 14”

In the figure we present three different model runs with the same initial condition, but with different modeling approaches used to describe the GSD: discretization in bins, moments of the number of particles as a function of diameter expressed in meters, moments of the mass fraction of particles expressed as a function of the diameter ex-

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pressed in the phi scale.

**R2.** *Page 3772: a nicely written and illuminating summary of the Latin Hypercube and gPCE alternatives to MCMC modeling.*

**A.** Thank you!

**R2.** *Page 3773: Still illuminating. I'm not a specialist in the mathematics of these techniques and can't critique them. It's a little unclear to me what the form of equation 55 would be if fully expanded. For example, if zeta were a vector of 3 variables, would P1 represent three families of polynomial coefficients; one for each variable?*

**A.** Thank you again for the positive comment. Some references have been added where the technique is explained in more details. If zeta were a vector of 3 variables, P1 would be a polynomial of the 3 variables. The polynomials P1,P2,...Pn have to be orthogonal, i.e. the integral of the product of two polynomials with index  $i$  and  $j$  (with  $i \neq j$ ) has to be equal to zero.

**R2.** *Page 3774: the values contoured in the lower panels of Fig. 7 were not initially very clear to me. You call them response functions in the caption, and on page 3773 (line 17). Are these the values of gamma(zeta) in eq. 55? Perhaps referring to gamma(zeta) as a response function would clarify. Also, it would help to mention that the values contoured in the lower panels are the same as those plotted in the upper panels (e.g. top mean phi for panel 1). Lines 22-30: I would say that the points you make in these few lines are the most significant of the paper, for readers interested in geologically relevant findings.*

**A.** As suggested by the referee, we refer now to eq.55 when presenting the response functions. We also added that the values contoured in the lower panels are the same as those plotted in the upper panels: "The variables contoured in the lower panels are the same as those on the horizontal axes in the upper panels".

**R2.** *Page 3775: Line 4: change "reduce" to "reduces". Line 5: change "relvance"*

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to “relevance”. Line 6: change “entrained aid” to “entrained air”. lines 13-14: “The mean and the SD of the TGDS at the top of the eruptive column clearly reflects the corresponding values at the bottom, with a small effect on the mean size at the top of larger values of the bottom SD”. What does this mean?

**A.** All the typos have been corrected. The sentence was really confusing and it has been rephrased in the following way: “a small effect of the bottom standard deviation on the mean size at the top, resulting in an increase in the average grain size with increasing values of the initial standard deviation”.

**R2.** Page 3776: Line 10: I’m a little confused about which of the sensitivity indices ( $S_i$  or  $T_i$ ) is displayed in Fig. 9.

**A.** The indices plotted are the main indices  $S_i$ . This is now written in the caption of figure 9.

**R2.** Figure 2 caption, line 4: change “forth” to “fourth”. Figure 5: the light gray curves on this figure are hard to see on my computer screen. Darkening them should make them more visible. Figure 6 caption: change “Two parameters Latin Hypercube” to “Two-parameter Latin Hypercube”

**A.** The caption of figure 2 has been corrected. In Figure 5 now the lines are more visible. The caption of figure 6 has been corrected.

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