response to reviewer 1

1.1 – grammar/usage

We tried to correct all grammar errors. As for usage, none of us is a native English speaker, so it is a bit difficult for us to identify the points that need correction.

1.2 – evaluation against observations

We understand the importance of validation vs. real data and we are planning a future presentation accordingly. However this is only the technical presentation of the development where we only deal with verification and not validation.

1.3 pg 8240 ln 24 – Sentence modified to:
Lagrangian description is the natural framework for tracer dispersion modelling, and etc ...

1.4 pg 8241 ln 9-11 – Joined two paragraphs and modified to:

Moist convection is widespread in the Earth's atmosphere where it displays a wide range of space and time scales in response to the variability of environmental parameters, ranging from the sub-kilometer / tens-of-minutes of individual cumuli to hundred-of-km / several days of mesoscale convective complexes \citep[see, \textit{e.g.},]{emanuel-1994}.

1.5 pg 8241 ln 23-27 – The paragraphs has been modified to address the Reviewer's comment trying to make the point more clear.

1.6 pg 8242 ln 14 – Added explanation:

In the absence of a convective parameterization, explicit convection can occur, and thus some vertical transport of tracers was present in the previous version of the model. However, since the scales of convection are in the sub-km range, any explicit representation of it at coarser resolution is bound to misrepresent most of those scales, and create updrafts that are incorrect in location and strength. Therefore the inclusion of a moist convection Lagrangian redistribution mechanism is essential to the completeness of the model.

1.7 pg 8242 ln 20 – Removed "with great detail'.

1.8 pg 8242 ln 25 – corrected accordingly

1.9 pg 8243 ln 16 – Changed "The updraft is very detailed and accounts for the thermodynamics" to: "The updraft is a detailed account of the thermodynamics"

1.10 pg 8244 ln 7 – No change

In the code, vertical mass flux is defined as vertical velocity times area times density. Closure then determines the value of an adjustment factor to be applied to mass flux as a whole. The scheme makes explicit use of vertical velocity in the trigger function and in the computation of precipitation and condensate loading. We therefore interpret the adjustment factor as applied to the area occupied by the convective ensemble, since any change of vertical velocity would influence nonlinearly the geometric and physical characteristics of the plume and render inconsistent a closure expressed by a single scalar factor. As far as we understand it, not having seen the code, both Tiedke, and Gregory and Rowntree use different approaches to closure and plume models that do not require such a distinction. This level of detail seems outside the scope of this presentation so we have not included it in the text.

1.11 pg 8245 ln 18 etc – Paragraph rewritten in order to present first the general formula, and then the special case of cloud top.

1.12 pg 8246 ln 15 – Equation corrected according to the Reviewer's comment
1.13 pg 8248 ln 21 – Changed to “uniformly distributed”

1.14 pg 8250 ln 8-9 – Sentence removed. We meant exactly what the reviewer wrote, just expressed it badly.

1.15 pg 8253 ln 6-9 – Changed to:

The structure of convective updrafts (see e.g figure 1) is such that most of the upward moving mass comes from the lowest levels of the atmosphere (below cloud base) and is returned to the environment in the upper troposphere, in the strong outflow at the top of the cloud, while areas of weak subsidence surround the updrafts.

1.16 pg 8253 ln 10-18 – The first part of Conclusions have been rewritten to make the novel aspect more clear. “Fully consistency” is now explained using the theoretical equivalence of Lagrangian and Eulerian description of fluid flows.

1.18 pg 8254 ln 6 – OK

1.19 pg 8254 ln 7 – OK

1.20 pg 8254 ln 17 – OK

1.21 Figure 2 – OK

1.22 Figure 4 – OK

1.23 Figures 3-5 – OK
response to reviewer 2

general comments:

2.1 – Resolution
We agree with the reviewer and took the time to perform a simulation at higher resolution, that is now shown in the renewed Figures. We did not change figure 1, since it is only used as an illustration of the effect of the activation of the convective scheme. Results are qualitatively unaltered.

2.2 – In the old figure 5 we only showed the tracer parcels released in the tropics. We added to the figure also tracer released in the other two subdomain, as it is shown in the renewed figure 5. Different behaviour is observed between tropics and extra-tropics release. This is reported in the text.

2.3 pg 8241 ln 10-11
Our intention was to inform the reader that convection will appear in many places and times during the model's run, not to go into a physical explanation of its occurrence, but since this sentence seems to incur the disapproval of both reviewers we removed it and substituted it with:

Moist convection is widespread in the Earth's atmosphere where it displays a wide range of space and time scales in response to the variability of environmental parameters, ranging from the sub-kilometer / tens-of-minutes of individual cumuli to hundred-of-km / several days of mesoscale convective complexes \citep[see, \textit{e.g.},][]\cite{emanuel-1994}.

2.4 pg 8241 ln 16 – Changed “below” to “smaller than or close to”

2.5 pg 8242 ln 10 – OK

2.6 pg 8243 ln 24 – LET: no change

The Kain-Fritsch code uses LET, and we left it so in order to help anyone who would look at the code. We like LNB as well, but other acronyms appear in the literature beside those mentioned by the reviewer (e.g. Bechtold 2001 uses ETL) and we thought there was no harm in using a different one.

2.7 pg 8245 ln 5 – OK

2.8 pg 8245 ln 15 – OK

2.9 pg 8245 ln 17 – OK

2.10 pg 8246 ln 1 – Paragraph rewritten in order to present first the general formula, and then the special case of cloud top.

2.11 pg 8246 ln 7 – The sentence starting at 8246 ln 5 is changed to: “In terms of probability this is expressed by the complementary probability ...”

2.12 pg 8247 ln 4 – OK, added: “where the redistribution of mass is applied”

2.13 pg 8250 ln 8-9 – OK – see general comments
IL-GLOBO (1.0) – Development and verification of the moist convection module

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Abstract. The development and verification of the convective module of IL-GLOBO, a Lagrangian transport model coupled online with the Eulerian general circulation model GLOBO, is described. The online-coupling promotes the full consistency between the Eulerian and the Lagrangian components of the model. The Lagrangian convective scheme is derived based on the Kain-Fritsch convective parameterisation used in GLOBO. A transition probability matrix is computed using the fluxes provided by the Eulerian KF parameterisation. Then, the convection redistribution of Lagrangian particles is implemented via a Monte Carlo scheme. The formal derivation is described in details and, consistently with the Eulerian module, includes the environmental flux in the transition probability matrix to avoid splitting of the convection and subsidence processes. Consistency of the Lagrangian implementation with its Eulerian counterpart is verified by computing environment fluxes from the transition probability matrix and comparing them to those computed by the Eulerian module. Assessment of the impact of the module is made for different latitudinal belts, showing that the major impact is found in the tropics, as expected. Concerning vertical distribution, the major impact is observed in the boundary layer at every latitude, while in the tropical area, the influence extends to very high levels.

1 Introduction

Long range transport of atmospheric tracers plays an important role in several fields ranging from atmospheric composition and chemistry to climate change, with applications spanning from air pollution to natural or anthropogenic disaster management and assessment.

Dispersion processes are Lagrangian “by nature.” Lagrangian description is the natural framework for tracer dispersion modelling, and the use of Lagrangian particle dispersion models (LPDM) for both theory and application is very effective and widespread. In particular, Lagrangian models are used often to retrieve information about the sources contributing to the concentration at a specific location (known as “backtrajectories”). The consistent implementation of LPDM requires the careful consideration of all the processes involved in the atmospheric dispersion.

Depending on the geographical area and season, the redistribution of tracers released in the atmosphere can be largely affected by the vertical transport due to moist convection events. In particular, convection is very efficient in mixing the boundary layer with the free troposphere air (Cotton et al., 1995) contributing to the long range spread of local emissions.

Moist convection is widespread in the Earth’s atmosphere, taking place wherever water vapor is present in conjunction with appropriate, fairly common, environmental conditions. In response to the variability of environmental parameters, moist convection where it displays a wide range of space and time scales in response to the variability of environmental parameters, ranging from the sub-kilometer/ tens-of-minutes of individual cumuli to hundred-of-km / several days of mesoscale convective complexes (see, e.g., Emanuel, 1994).

For all the scales that lie below or near smaller than or close to the grid size of the numerical models, explicit resolution is inadvisable, and numerical models resort to parameterisation schemes. A discussion of the theoretical issues and field of application of the different convective parameterisations is beyond the scope of this introduction - the interested reader
Rossi et al.: IL-GLOBO: moist convection module

is referred to Arakawa (2004) and references therein. In this work, a slightly modified Kain and Fritsch (1990, hereafter KF) scheme is adopted, which will be described briefly in Sect. 2.

In contrast to For the turbulent diffusion processes (that are predominant in the boundary layer), for which the theoretical framework provides a scheme whose typical space- and time-scales are small compared to the resolution of a general circulation model, a well-founded theoretical framework exists and allows for the formulation in terms of stochastic models of turbulence (Thomson, 1987). In contrast, mass-flux theories of moist convection in LPDM can be implemented as a simple, do not provide sufficient details to implement stochastic models. Therefore, moist convection effects are simulated using particle redistribution mechanisms which reproduce the expected mass fluxes obtained from an Eulerian convective scheme parameterization, usually via a Monte-Carlo scheme that redistributes masses according to the fluxes computed by the mass-flux Eulerian scheme (see, e.g., Forster et al., 2007).

While Coupled Chemistry and Meteorology Models (CCMM) are now at the front edge of research in atmospheric composition studies (Baklanov et al., 2014), popular LPDMs are designed for offline usage and need to reconstruct necessary quantities that are not included in the normal output of meteorological models. In particular, offline recalculation of convective mass fluxes is needed, from quantities made available from the meteorological (Eulerian) model such as temperature and moisture, and is often performed with a mass-flux scheme different from the one that produces the meteorological output (see, e.g., Forster et al., 2007), possibly leading to dynamical inconsistencies in the results.

In contrast to avoid this inconsistency, IL-GLOBO (Rossi and Maurizi, 2014) was designed as an online-coupled model that makes use of the full availability of Eulerian fields. In its first step of development, the vertical transport and dispersion of tracers were the result of the vertical advection and diffusion only. Although vertical transport by explicit convection, even in absence of convective parameterisation, can partly account for In the absence of a convective parameterisation, explicit convection can occur, and thus some vertical transport of tracers to high levels were present in the previous version of the model. However, since the scales of convection are in the sub-km range, any explicit representation of it at coarser resolution is bound to misrepresent most of those scales, and create updrafts that are incorrect in location and strength. Therefore the inclusion of a moist convection-moist convection Lagrangian redistribution mechanism is essential to the completeness of the model and is expected to be far more effective. The online coupling requires full consistency with the meteorological (Eulerian) sub-model and benefits from the full availability of all meteorological variables at every time step. Therefore IL-GLOBO moist convection module is developed consistently with the modified KF scheme adopted in GLOBO (Malguzzi et al., 2011) (see Sect. 2). With the online coupling this module benefits from the full availability of all meteorological variables at every time step.

In this article the development of the online-coupled convective module of IL-GLOBO is presented with great detail and its features will be assessed through some application examples. In Section 2 the Eulerian convective parameterisation is presented while the implementation of the Lagrangian scheme is described in Section 3 with emphasis on Eulerian consistency and providing full details of the constructive procedure. Verification of the scheme and some evaluation of the inclusion of convective effects in IL-GLOBO are presented in Section 4.

2 The Kain-Fritsch scheme

A convective parameterisation that makes explicit use of the vertical fluxes of mass, the Kain-Fritsch (hereafter KF) scheme, was adopted by the GLOBO developers, and in the present work, for its ready availability, ease of implementation and widespread use by the meteorological research community.

The original formulation, and its successive evolution, were presented in a series of papers (Fritsch and Chappel, 1980; Kain and Fritsch, 1990; Kain, 2004) to which the reader is referred for more details. Recent presentations of its performance in the simulation of meteorological events can be found, e.g., in Liu and Wang (2011) and Bullock et al. (2015).

The scheme is based on a steady-state entraining-detraining plume model and a closure based on release of convective available potential energy (CAPE). Three streams of mass are present: updraft and downdraft of the convecting ensemble, and an weak environmental flow (subidence) which maintains the balance of mass at each model level.

The updraft is very detailed and accounts for a detailed account of the thermodynamics of moist air and entrainment-detrainment of moisture and condensate at every level between cloud base and the cloud top. Briefly, mixtures of low-level air are tested for instability. Once a lifted condensation level (LCL) is identified, the parcel buoyancy at each upward level is computed, and an estimate of the kinetic energy gained by latent heat release obtained. The, as yet unspecified, upward mass flux is then fractionally increased/reduced by entrainment/detrainment of environmental air based on a buoyancy-sorting principle. The dilution of the originally unstable air with drier and cooler air from the environment reduces its buoyancy, up to an equilibrium level (LET) where the rising air has no more acceleration from thermodynamic processes. Upward of the LET, the rising air
is decelerated until the remaining kinetic energy of the vertical motion is reduced to zero, which defines the top of the cloud.

Downdrafts are generated by re-evaporation of water condensate expelled by the rising motion. Environmental air is assumed to be entrained uniformly into the downdraft in a layer around cloud base, and detrained, again linearly in pressure, at lower levels. The empirical evidence for this structure is discussed at length in Kain (2004).

Only at this point the dimensional mass fluxes are determined by applying the closure assumption that requires at least ninety per cent of the CAPE to be consumed by the ensemble of convective clouds. This finally determines the fraction of a grid box covered by the ensemble of clouds and the environmental subsidence needed to maintain the balance of mass at each level.

The tendencies of thermodynamic quantities to be returned to the model are spread over a “convective time scale” ΔT_C, ranging from half an hour to an hour, covered by several advective time step Δt, so that each model time step only receives a fraction of the convective tendencies.

An example of the effect of the scheme on an unstable atmospheric profile is shown in Figure 1.

Figure 1. SkewT-logP thermodynamic diagrams for a deep tropical convective episode, before (left panel) and after (right panel) the action of convection as determined by the parameterisation scheme. Pressure on left axis in hPa, temperature on right and top axes in °C. Middle panel: profiles of vertical mass fluxes as computed by the convection scheme: updraft ($F_u^i$, thick solid line), downdraft ($F_d^i$, thin solid line), environmental subsidence ($F_e^i$, dashed) needed to maintain the balance of mass at each level (see Figure 2 for definitions). Vertical coordinate for the middle panel is pressure, with the same scale indicated for left and right panels. Horizontal coordinate is kg/s/m^2 with arbitrary scale: the flux per unit area depends on the area attributed to the convective ensemble by the convection scheme. In this instance it is about 7 per cent of the grid box. Also shown on the central axis are the locations of model levels. Significant levels for the convection computation are labelled on the graph. The air in the updraft source layer (USL) becomes saturated when raised to the lifted condensation level (LCL), and is unstable when further pushed to its level of free convection (LFC - at the same model level in this instance). Vertical acceleration of the rising air parcel ceases at the level of equilibrium temperature (LET) and vertical motion stops at cloud top. The convective downdraft begins at the level of free sink (LFS) and extends down to the ground.

Figure 2. Schematic representation of fluxes involved in the KF scheme. Uppercase $F$’s represent the fluxes between vertical levels (across level boundaries) while lowercase $f$’s are the fluxes within a level that represent the exchange of mass between environment ($e$) and updraft ($u$) or downdraft ($d$), respectively.

3 Lagrangian implementation of the moist convection effects

IL-GLOBO uses some of the quantities computed by the KF convective parameterisation (see Section 2) to implement a Monte Carlo scheme (KF-MC) for the particle displacement, in a way similar to other LPDMs (Collins et al., 2002; Forster et al., 2007). All these schemes compute the displace-
ment probability matrix (DPM) between levels making use of the entrainment and detrainment fluxes in updraft and downdraft. Additionally, in IL-GLOBO the environment effects (subsidence), that results from a mass balance, are directly included into the DPM and, therefore, implemented using the MC scheme, without the need of a-posteriori adjustment.

In the following, the same notation found in Figure 1 of Rossi and Maurizi (2014) is used where NLEV σ-hybrid grid levels are indexed decreasing with height.

In the Eulerian model component, every $\Delta T_C$ (or, in terms of time steps, every $n_C$ advective time steps $\Delta t$), the KF scheme checks for the conditions for the onset of convection and, if conditions are met, determines the evolution of the grid column for the whole $\Delta T_C$. Entrainment and detrainment fluxes in both updraft ($f_{u\delta}$, $f_{u\varepsilon}$) and downdraft ($f_{d\varepsilon}$, $f_{d\delta}$) terms for XIV each level (see Figure 2), from the ground to the cloud top, as computed by the KF scheme are made available to the Lagrangian model. With reference to Figure 2, the following relationships hold for fluxes at $(f)$ and between $(F)$ levels:

$$ F_i^u = F_{i+1}^u + f_{i\varepsilon} - f_{i\delta}, \quad \text{(1)} $$

for the updraft $(u)$ and

$$ F_i^d = F_{i-1}^d + f_{i\varepsilon} - f_{i\delta} \quad \text{(2)} $$

for downdraft $(d)$. The probability for a particle to be entrained in an updraft at level $i$ is expressed by the product of the probability to be entrained from the environment: $f_{u\varepsilon} \Delta t/m_i$, and the probability that the particle resides in the environment: $m_i/m_i$, where $m_i$ is the mass already present in the environment as opposed to the mass flowing through, in the convective ensemble, and $m_i$ is the total mass of the level $i$, giving XV XVI

$$ p_{i\varepsilon} = \frac{f_{u\varepsilon} \Delta t}{m_i}. \quad \text{(3)} $$

The probability that a particle captured by the updraft is detrained can be easily derived at the cloud top level $(i_{top})$ where, by definition of updraft, $F_{top}^u = 0$ so that from by rearranging Equation (1) $F_{top}^u + f_{top\varepsilon} = F_{top\varepsilon}$, that expresses the obvious fact that all the mass flowing in the updraft at level $i_{top}$ is detrained. In terms of probability, this is therefore expressed by

$$ p_{top\delta} = \frac{f_{top\delta}}{F_{top}^u + f_{top\varepsilon}} = 1 $$

which into

$$ \frac{F_i^u}{F_{i+1}^u + f_{i\varepsilon}} + \frac{f_{i\delta}}{F_{i+1}^u + f_{i\varepsilon}} = 1. \quad \text{(4)} $$

for the downdraft transition probability $p_{i\delta}$, a similar relationship holds. The probabilities $p_{i\varepsilon}$ and $p_{i\delta}$ represent the upper and lower triangular components of the total convective transition probability matrix $p_{ij}$ whose diagonal is defined by:

$$ p_{ij} = 1 - \frac{1}{1 - p_{i\varepsilon} p_{j\delta} - p_{i\delta} (1 - p_{i\varepsilon})}. \quad \text{(7)} $$

For the downdraft transition probability $p_{ij}$, which is identically equal to 1 at the cloud top where $F_{top\delta} = 0$. The denominator $F_{i+1}^u + f_{i\varepsilon}$ of Equation (4) is the flow entering the updraft volume (see Figure 2) at level $i$ and that is available to detrainment process. The mass entering the level $i$ which is not detrained must flow to the upper level satisfying the continuity for the updraft (Equation (4)). In terms of probability, this is expressed by the complementary probability $p_{i\delta} = 1 - p_{i\varepsilon}$ XVII

$$ p_{i\delta} = \frac{F_i^u}{F_{i+1}^u + f_{i\varepsilon}} \quad \text{(5)}$$

which, combined with Equation (5) gives back Equation (1) confirming the consistency of the above definitions of the probability components. XVIII XIX Using the above definitions it is possible to build the full transition probability matrix for the updraft fraction. The probability that a particle moves due to the updraft motion from a level $i$ to a level $j < i$ is equal to the probability that the particle is entrained at level $i$ (Equation 3) times the probability that it is detrained at level $j$ (Equation 5) times the probability that it is not detrained between level $i$ and $j + 1$ included. In formula:

$$ p_{i,j} = p_{i\varepsilon} p_{j\delta} \prod_{k=i+1}^{j+1} \left(1 - p_{k\delta}^n\right). \quad \text{(7)} $$

The mixing produced by the convective motion (updraft and downdraft) needs to be balanced by the environment flux (subsidence) to conserve the mass. For the Eulerian part this is granted by the environmental flux computed in the KF scheme. In Lagrangian terms this is equivalent to maintaining a well-mixed state where the redistribution of mass is applied, XXI and can be expressed in terms of DPM. This consistency is obtained by modifying the transition probability matrix $p_{ij}$ by imposing zero net flux at the interface between two model levels. At level $i$, the mass fluxes (assumed

Noticing that the two terms are both positive by definition, the above relationship can be used to define $p_{i\delta}^n$ at any level $n$:

$$ \tilde{p}_{i\delta}^n = \frac{f_{i\delta}}{F_{i+1}^u + f_{i\varepsilon}} \quad \text{(6)} $$
positive upward) across the two level interfaces $i$ (upper) and $i+1$ (lower) due to the sum of updraft and downdraft motion, are expressed in terms of probability as:

$$F^e_i = \sum_{k<i} [p^e(k|i) m_i - p^e(i|k) m_k]$$  \hspace{1cm} (9)

and

$$F^e_{i+1} = \sum_{k>i} [p^e(i|k) m_k - p^e(k|i) m_i]$$  \hspace{1cm} (10)

respectively. Thus, the mass conservation reads:

$$F^e_i + F^e_{i+1} = F^e_{i+1} + F^e_{i+1}$$  \hspace{1cm} (11)

where $F^e$ is the environment mass flux which is directed downward except in very peculiar cases (Kain et al., 2003). With the additional boundary condition:

$$F^e_{NLEV+1} = 0,$$  \hspace{1cm} (12)

the environment flux can be computed iteratively through Equations (9) to (11). The effect of the environment flux at surface $i$ is to increase the transition probability from $i$ to $i+1$ while reducing the probability of the “null transition” (particle remains in the same model level). This results in the modification of the elements of the diagonal:

$$p(i|i) = p^e(i|i) - \frac{F^e_{i+1} \Delta t}{m_i}$$  \hspace{1cm} (13)

and sub-diagonal:

$$p(i+1|i) = p^e(i+1|i) + \frac{F^e_{i+1} \Delta t}{m_i}.$$  \hspace{1cm} (14)

The final DPM is then defined by

$$p(j|i) = p^e(j|i)$$  \hspace{1cm} (15)

for $j < i$ or $j > i+1$ and by Equations (13) and (14) for $j = i$ and $j = i+1$, respectively.

It is worth noting that $p$ is an Eulerian quantity that can be viewed as the linear operator acting on an initial concentration vector to give the concentration distribution after the convection mixing. However, since $p$ defined in terms of a finite time step $\Delta t$, it may become unstable (flux in one time step comparable or larger than the mass of the level). In fact, KF uses a reduced time step $\Delta t_{KF} = \Delta T_C/\chi_{KF}$, with integer $\chi_{KF}$, internally computed to maintain linear stability of the numerical scheme. Consistently, the same $\Delta t_{KF}$ is used to compute the transition probability that will be iterated $n_{KF}$ times using the MC scheme.

In order to implement the MC scheme, it is convenient to compute the cumulative transition probability matrix $P$ as:

$$P_{j,i} = \sum_{k=NLEV}^{j} p(k|i).$$  \hspace{1cm} (16)

The MC scheme is applied in grid columns affected by convection to the particles that are below the cloud top by extracting a random number $\chi$, homogeneously uniformly distributed between 0 and 1, and comparing its value to $P_{j,i}$, $j = NLEV, i_{top}$ until a $\chi_f$ is found so that $\chi < P_{j,i}$. A position is then attributed to the particle within the arrival grid cell using the same $\chi$ number to interpolate linearly between the grid cell boundaries (Forster et al., 2007):

$$\sigma_p = \sigma(j_f) + (\chi - P_{j-1,i})/(P_{j,i} - P_{j-1,i}) \Delta \sigma.$$  \hspace{1cm} (17)

The MC scheme is iterated $n_{KF}$ times to obtain the final position after $\Delta T_C$. Then, as for the tendencies of thermodynamic quantities in the Eulerian part, the total particle displacement is spread over the $n_{advC}$ advective time steps that cover the convective period.

4 Model verification

In order to identify the main features of the KF-MC scheme, to verify its implementation and to assess its impact on dispersion, some numerical experiments were performed.

A number of convective episodes were extracted from a model simulation performed using an horizontal regular grid of 362 × 242 points, 1200 × 832 cells of 0.3° × 0.22°, that corresponds to a resolution of about 400–23 km at mid-latitudes in longitude, and a regular vertical grid of 50 points in the $\sigma$-hybrid coordinate. The advective time step used was $\Delta t = 432 \Delta t = 150$ s. The convective time scale is $T_C \simeq 30$ min, i.e., the KF scheme is executed every $n_{advC} = 11$ advective time steps.

4.1 Displacement Probability Matrix

An example selected in the tropical area around noon, is shown in Figure 3. The central part of the figure show the DPM for that specific event along with the vertical profiles of entrainment (bottom) and detrainment (left) fluxes in both updraft and downdraft. In this example the significant levels, as defined in Figure 1 are: $\sigma_{cloud.top} = 0.41$, $\sigma_{LET} = 0.47$, $\sigma_{LS} = 0.79$, $\sigma_{CL} = 0.93$ and the USL is a mixture 60 hPa thick based on the ground. The two most likely transitions are:

1. the particle to stay within the starting grid cell (highest values are found in the diagonal because most of the model grid cell is not influenced by convection);
2. the particle is transferred in the cell just below, due to the environment flux (high values in the matrix sub-diagonal).
The other transitions are directly caused by convective motion and are consistently less likely, with probabilities in the range $10^{-2} - 10^{-5}$. This can be partly ascribed to the coarse horizontal resolution used in these simulations, where convection is expected to cover only a small portion of a cell column at a given time. It is expected that finer model resolution would increase the ratio between the volume involved in convection to the total volume of the model column. Updrafts generate transitions with highest likelihood for displacements from levels next to the ground to levels just below the cloud top, while the downdraft transitions are permitted only from levels between 0.8 and 0.9 $\sigma$ to levels between 0.9 and 1.0 $\sigma$. This reflects the hypotheses underlying the formulation of the KF parameterisation.

4.2 Algorithm verification

In order to verify the consistency of the implementation, the distribution of initially well mixed particles were verified after convection to be still well mixed. This is performed in one-dimension-like configuration by selecting 12 convectively active grid columns, releasing $4 \times 10^4$ well mixed particles in each and integrating the model for a full $\Delta T_C$. It is found that the distribution after such integration remains well mixed within the same error interval used in Rossi and Maurizi (2014). However, this only provides a test of the numerical implementation and not of the theoretical formulation and correct calculation of $p^e$. In fact, in contrast to the formulation of a Lagrangian turbulent diffusion model for which well mixing provides a necessary and sufficient condition (Thomson, 1987), well mixed state in the present scheme is maintained by construction of the environmental flux, whether $p^e$ is correct or not. Therefore an independent
verification is necessary. This is done by comparing the environment fluxes computed using the DPM from Equations 9, 10 and 11 to those provided by the KF scheme. Such a verification, performed for a number of convective episodes, confirms that within the roundoff error (10^{-7}) Eulerian and Lagrangian KF fluxes are the same.

4.3 Impact of KF-MC on dispersion

The impact of convection on the particle dispersion in a fully three-dimensional experiment is considered. The aim is to assess the importance of the moist convection mechanism with respect to diffusion and advection. Simulations start on 11 March 2011. After a 12h spin-up of the Eulerian model, particles (particles) are released and then dispersed for 5.5 days, and their position is sampled every 40 time steps (72 minutes) hour. The source consists of $N_p \approx 4 \times 10^5$ $N_p \approx 7.4 \times 10^3$ pair of particles, each pair sharing the same initial position. Particles are released between $\sigma = 1.0$ and $\sigma = 0.9$ proportionally to the average vertical density profile, and homogeneously distributed in the horizontal within 3 zonal areas: around the Equator, within the Tropical area ($-15^\circ, +15^\circ$), at mid-latitudes in the northern hemisphere and ($+30^\circ, +60^\circ$) and southern hemisphere ($-30^\circ, -60^\circ$). For each emission area, two different simulations were performed with the KF-MC switched on and off. Values of relative and absolute dispersion are shown in Fig. 4. Absolute dispersion is computed as:

$$\Delta^2 = \frac{1}{2N_p} \sum_{p=1}^{2N_p} (x_p - x_{p0})^2$$

where $x_p$ is a generic particle coordinate that can indicate both, the particle vertical position (in Figure 4 represented as the height above the model surface), and the horizontal distance along the earth surface, and $x_{p0}$ is the starting position of the same particle. Relative dispersion is computed considering the ensemble of pair of particles sharing the same initial position and is defined as:

$$\Sigma^2 = \frac{1}{N_p} \sum_{p=1}^{N_p} (x_{p1} - x_{p2})^2$$

where $x_{p1}$ and $x_{p2}$ are the position of each particle of the pair.

Results of the experiments, reported in Figure 4 show that absolute dispersion is influenced by convection mainly at the tropics, where the convective activity is more intense and the tropopause higher. Moreover, the effect is far more relevant on the vertical which is the direction directly influenced by the scheme. Concerning the relative dispersion, the moist convection scheme has a relevant impact on both the vertical and horizontal directions. The effect is important in all of the zonal areas but is still more pronounced at the tropics. The larger impact on horizontal relative dispersion compared to the absolute dispersion can be explained by considering that as particles separate due to convection, they are captured by different horizontal structures that, in turn, rapidly decorrelate the motion of the two particles of the pair.

The effect on concentration is shown in Figure 5, where the final concentration is displayed for vertical (Figure 5a) and with and without moist convection scheme. For the horizontal Figure 5b a difference map is shown. Particles were counted for intervals of equal size and normalised so that the starting concentration between 0.9 and 1.0 $\sigma$ is around 1. Figure 5a shows that moist convection has an important effect close to the surface in all the areas, with an enhanced effect at the tropics. In the free troposphere, the effect is almost negligible except for the tropical area above $\sigma = 0.4$ where the largest effect is observed. It is worth observing that, in the tropical area, particles reach high levels even in the simulation without convection although with a concentration smaller by a factor of 2. Since the diffusivity only acts between 0.8 an $\sigma$ in the vast majority of cases, the vertical transport of particles producing the high concentration above $\sigma = 0.4$ can be attributed mainly to vertical advection that result from large scale convergence with minor contribution from horizontal advection and orographic effects.

Figure 5b displays the map of differences of the vertically integrated number of particles between simulations with and without Lagrangian convection scheme for the case of the tropical emissions. Particles are sampled for each $1^\circ 	imes 1^\circ$ $0.6^\circ \times 0.43^\circ$ column and the difference is normalised with respect to the initial number of particles per bin. For the case of release in the tropical area, it can be noted that areas of strong depletion are surrounded by relatively larger areas where the difference is weakly positive. This is clearly related to combination of the source and the convection features. Source is located close to the surface where strong updraft entrainment is active. Highest transition probability for particles at the surface is to be released just below the cloud top where horizontal wind tends to spread them around. The structure of convective updrafts (see, e.g. Figure 1) is such that most of the upward moving mass comes from the lowest levels of the atmosphere (below cloud base) and is returned to the environment in the upper troposphere, in the strong outflow at the top of the cloud, while areas of weak subsidence surround the updrafts. Particles released in the extra-tropical regions (north and south) display different qualitative behaviour showing smaller scale features with respect to those released at the tropics, in agreement with the expected horizontal scales of convective cells.

5 Conclusions

The formulation of a Monte Carlo algorithm that implements the effect of moist convection on Lagrangian tracers have been discussed in details in A Lagrangian transport scheme for moist convection is implemented online in IL-GLOBO.
in parallel with the integration of the Eulerian model. This gives the Lagrangian scheme direct access to all the prognostic variables without any need for additional diagnostics and ensures full consistency of the DPM with the frame of the IL-GLOBO model. The Lagrangian convective scheme is constrained to be fully consistent with its Eulerian counterpart, the Kain-Fritsch scheme. In deriving the Displacement Probability Matrix, the effect of environment is included to closely follow the KF Eulerian scheme parameterization scheme. As a consequence, the Lagrangian and Eulerian description of tracer dispersion in the coupled model are equivalent as it is expected on theoretical ground.

Verification of the implementation and assessment of the impact of the scheme have been presented showing that this aspect differs from the approach of other models found in literature. The quantities used in those cases to advect and diffuse Lagrangian particles are diagnosed from the meteorological thermodynamics profiles with parameterizations that may differ from that of the meteorological model, making the Eulerian-Lagrangian consistency hard to realize.

The consistency of the present scheme with the Eulerian quantities has been verified in a number of offline 1D tests, where the model is fully shown to conserve the mass and reproduce the expected fluxes.

Global experiments with tracers released close to the surface at different latitudes show that the effects of the MC-KM is strong and gives rise to large departures from the non-convective version even at mid-latitudes. Vertical distribution displays again a larger difference at the tropics. However, even with the KF-MC deactivated (but still activated in the...
Figure 5. On the left: vertical concentration profiles (in arbitrary units), as function of $\sigma$. Colors and line-types have the same meaning as in Fig. 4. In addition, the top of the release region ($\sigma = 0.9$), is displayed as a bold solid line. On the right: difference map (with convection minus without convection) of vertically integrated particle number at the end of the simulation for all the case of tropical three emission subdomain (initial emission area delimited by horizontal dashed lines, as labelled in the figure: north, equator, south). Details on the concentration normalizations for both panels can be found in Section 4.3.

Eulerian part, so generating the “correct” dynamics), at the tropics some tracer is observed to reach very high altitudes. This is found to be a combined effect of advection, both vertical and horizontal.

The next step of the IL-GLOBO development will be the validation of the models against available data for which appropriate datasets are hardly available as noted by, e.g., Forster et al. (2007).

Code availability

The numerical code of the IL-GLOBO vertical moist convection module (Fortran 90) is released under the GNU Public Licence GPL and is available at the BOLCHEM website.

The software is packed as a library using autoconf, automake and libtool, which allows for configuration and installation in a variety of systems. The code is developed in a modular way, permitting the easy improvement of physical and numerical schemes.

The GLOBO model is available upon the signature of an agreement with the CNR-ISAC Dynamic Meteorology Group (contact: p.malguzzi@isac.cnr.it).

Acknowledgements. The authors would like to thank Piero Malguzzi for making the GLOBO model available and for making himself available for the explanation of the finer points of the numerics. The software used for the production of this article (model development, model run, data analysis, graphics, typesetting) is free software. The authors would like to thank the whole free software community and, in particular, the Free Software Foundation (http://www.fsf.org) and the Debian Project (http://www.debian.org).
References


Rossi et al.: IL-GLOBO: moist convection module


Notes

1R1: please explain why dispersion processes are Lagrangian “by nature”, rather than Eulerian and Lagrangian simply being two different perspectives. - Sentence modified

2R2: Three conditions requires for convection to occur: unstable stratification, moisture and triggering, “appropriate, fairly common, environmental conditions” is an obscure expression. - Our intention was to inform the reader that convection will appear in many places and times during the model’s run, not to go into a physical explanation of its occurrence, but since this sentence seems to incur the disapproval of both reviewers we removed it and substituted

11R1: this is a very vague description of when convection occurs. It should be either more specific about the necessary conditions, or removed. - Joined two paragraphs and modified, see II

12R2: “below or near”: change it to “smaller than or close to”, - done

1R1: this paragraph appears to be addressing the similarities and differences between the representation of turbulent diffusion and moist convection in an LPDM, but the exact point is unclear – a little more explanation would be helpful. - Rewritten paragraph

VI R2: The subject is plural (something and something). Change “was” to “were”.

VIII R1: I agree that, when coupled to a coarse-resolution GCM with parameterised convection, this mechanism is likely to be far more effective at redistribution than explicit vertical motion. However, the reasons why should be explained in the manuscript. Added explanation

VIII R1: the detail is sufficient, but I’m not sure “with great detail” is warranted. - removed

10R1: IL-GLOBO should be IL-GLOBO

2R1: “very detailed” compared to what? The description sounds like a fairly standard bulk mass-flux parameterisation.- changed

12R2: Why the equilibrium level is abbreviated to LET? It is commonly expressed as LE or LNB (level of neutral buoyancy) or LOC (limit of convection).- no change: The Kain-Fritsch code uses LET, and we left it so in order to help anyone who would look at the code. We like LNB as well, but other acronyms appear in the literature beside those mentioned by the reviewer (e.g. Bechtold 2001 uses ETL) and we though there was no harm in using a different one.

XI R1: mass-flux parameterisations typically do not decompose the mass flux into updraught area and velocity. Please explain briefly how Kain–Fritsch differs from e.g. Tiedtke or Gregory & Rowntree, enabling it to do so. - no change:

In the code, vertical mass flux is defined as vertical velocity times density area times density. Closure then determines the value of an adjustment factor to be applied to mass flux as a whole. The scheme makes explicit use of vertical velocity in the trigger function and in the computation of precipitation and condensate loading. We therefore
interpret the adjustment factor as applied to the area occupied by the convective ensemble, since any change of vertical velocity would influence nonlinearly the geometric and physical characteristics of the plume and render inconsistent a closure expressed by a single scalar factor. As far as we understand it, not having seen the code, both Tiedke, and Gregory and Rowntree use different approaches to closure and plume models that do not require such a distinction. This level of detail seems outside the scope of this presentation so we have not included it in the text.

XIII R1: clear “updraft”, “downdraft” and “environment” labels on the schematic would be helpful - done

XIV R2: Typos. Change “fore” to “for”

XV R2: Do you think you need a subscript i in $p^{u\varepsilon}$? - Yes we do

XVII R2: What does the overbar mean? - definition added to the text

XIII R1: the presentation here is confusing, with the special case of cloud-top coming first but having the exact same formula as the general case. It might be clearer to present the formula first for the general case, and then note that this is equal to 1 at cloud top - Paragraph rewritten in order to present first the general formula, and then the special case of cloud top.

XVII R2: Why Equation 4 can be used to define $p^{u\varepsilon}_i$ at any level? You do not use it to define Equation 5, you just borrow a similar idea to define $p^{u\delta}_i$ - Paragraph rewritten, see XVIII

XVIII R1: why is the formula for Eq. (7) not

$$p^u(j|i) = p^{ue}_i \prod_{k=i}^{i+2} (1 - p^{u\delta}_k)$$

Please explain the derivation more clearly. - Equation corrected according to the Reviewer’s comment.

XVIII R2: Where a well-mixed state is maintained? Anywhere? Everywhere? - specified in the text

XVIII R1: “uniformly distributed” is the more usual term.

XVIII R1: show the line colours in a legend, rather than describing them in the text - done

XVIII R1: finer resolution would only be expected to increase the ratio if the calculation is restricted to grid cells where some the convective parameterisation is triggered. Averaged over all cells, including those where no convection occurs, surely the ratio should remain similar? - Sentence modified

XVIII R2: It would be interesting to see the impact of a finer resolution on the result as I raised in the main points - done. See the reply to the general comments

XVIII R1: column (absolute/relative) and row (vertical/horizontal) headings would make the subplots easier to understand at a glance. - done

XVIII R1: it’s not clear what these three sentences are trying to say, or how they relate to the results shown in the figures. Please explain further. - changed

XVIII R1: it could be made clearer what the main novel component is here (the use of the DPM approach in an online-coupled GCM-dispersion model, where it has previously been used for offline models). - The first part of Conclusions have been rewritten to make the novel aspect more clear. “Fully consistency” is now explained using the theoretical equivalence of Lagrangian and Eulerian description of fluid flows.

XVIII R1: this is very brief – in what way is the model “fully consistent”? - see XXVIII