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Adaptive wavelet simulation of global ocean dynamics

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Abstract

In order to easily enforce solid-wall boundary conditions in the presence of complex coastlines, we propose a new mass and energy conserving Brinkman penalization for the rotating shallow water equations. This penalization does not lead to higher wave
5 speeds in the solid region. The error estimates for the penalization are derived analytically and verified numerically for linearized one dimensional equations. The penalization is implemented in a conservative dynamically adaptive wavelet method for the rotating shallow water equations on the sphere with bathymetry and coastline data from NOAA's ETOPO1 database. This code could form the dynamical core for a future global
10 ocean model. The potential of the dynamically adaptive ocean model is illustrated by using it to simulate the 2004 Indonesian tsunami and wind-driven gyres.

1 Introduction

The goal of this paper is to propose a new Brinkman volume penalization of the rotating shallow water equations and implement them in our dynamically adaptive wavelet
15 model on the sphere (Dubos and Kevlahan, 2013; Aechtner et al., 2014) to simulate oceanic flows with realistic coastlines and bathymetry over scales ranging from sub-kilometre to global.

Brinkman penalization methods for the numerical solution of the Navier–Stokes equations with solid boundaries were originally introduced by Angot et al. (1999) follow-
20 ing the pioneering work of Arquis and Caltagirone (1984). Like all penalization methods, their goal was to avoid having to adapt the discretization scheme to account for complex solid boundaries by instead modifying the dynamical equations such that as a control parameter tends to zero the solution of the modified equations with simple boundary conditions (e.g. periodic) tends to the solution of the original equations with
25 the desired boundary conditions. The physical analogy is that the regular fluid is replaced by a porous medium where the porosity and permeability tend to zero in the

solid portion of the computational domain and the porosity is one (i.e. a regular fluid) in the fluid part of the domain. Angot et al. (1999) proved the method converges and gave (non-sharp) estimates of the error in terms of the control parameter. Because it is a volume penalization, Brinkman penalization methods are easy to implement be-
5   cause the geometry of the boundary need not be known: it is sufficient to know the indicator function (or mask) defining points as being either in the solid or fluid parts of the computational domain.

Since its introduction Brinkman penalization has been applied to a wide range of fluid flow problems and numerical schemes, including spectral methods (Kevlahan and
10   Ghidaglia, 2001), moving boundaries (Kevlahan and Wadsley, 2005; Kolomenskiy and Schneider, 2009), the wave equation (Paccou et al., 2005), the compressible Euler equations (Liu and Vasilyev, 2007) and the shallow water equations (Perret et al., 2003; Reckinger et al., 2012).

Penalization methods are particularly well-suited to dynamically adaptive methods
15   since these methods automatically refine the computational grid in the boundary layers and can use very coarse grids in the solid part of the computational domain where the solution is irrelevant (Kevlahan et al., 2000; Schneider and Farge, 2002; Vasilyev and Kevlahan, 2002; Kevlahan and Vasilyev, 2005). In addition, because penalization methods enforce the boundary conditions to only first-order accuracy adaptive methods
20   can provide the required level of accuracy by local grid adaptation (i.e. h refinement).

In this paper we propose a new volume penalization for the shallow water equations and then implement it in the adaptive wavelet method for the rotating shallow water equations on the sphere that we have recently developed (Dubos and Kevlahan, 2013; Aechtner et al., 2014). Our method is a modification of the one proposed by Reckinger
25   et al. (2012) to ensure that mass and energy are conserved and that the wave speed is the same in both the solid and fluid parts of the domain. We also modify the velocity penalization (i.e. permeability) term to ensure better control of the overall error using the porosity parameter alone.

Previous volume penalization methods for the shallow water equations are reviewed in Sect. 2. The new penalization is derived from the porous shallow water equations in and Sect. 3. The new penalization is verified for the linearized one-dimensional equations in Sect. 4. Finally, we illustrate the potential of the new method by applying it to two global ocean flows: tsunami propagation and wind driven gyres. These simulations have realistic bathymetry and coastlines from the 1 arc min NOAA ETOPO1 global relief data base (Amante and Eakins, 2009). The two examples show how the Brinkman penalization of the shallow water equations works with a dynamically adaptive wavelet method for both fast (tsunami) and slow (global ocean circulation) dynamics and in the inertia–gravity (tsunami) and quasi-geostrophic (global ocean circulation) regimes. We intend to extend the methods presented here to build a full dynamically adaptive global ocean circulation model.

2   Previous penalization methods for the shallow water equations

In vector-invariant form, Reckinger et al. (2012) proposed the following set of penalized shallow water equations,

$$\frac{\partial h}{\partial t} + \frac{1}{\phi(\boldsymbol{x})}\mathrm{div}h\boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\mathrm{curl}(\boldsymbol{u})}{h} \times h\boldsymbol{u} + \mathrm{grad}\left(gh + \frac{1}{2}|\boldsymbol{u}|^2\right) = -\sigma(\boldsymbol{x})\boldsymbol{u}, \tag{2}$$

where h is the height of the fluid column, $\boldsymbol{u}$ is the vertically averaged horizontal velocity and g is gravity. In this section, as well as in Sects. 3 and 4, the Coriolis force is omitted for simplicity. It will be reintroduced in the numerical experiments of Sect. 5. The corresponding momentum equation is

$$\frac{\partial \boldsymbol{m}}{\partial t} + \mathrm{div}(\boldsymbol{m} \otimes \boldsymbol{u}) + \phi\mathrm{grad}\left(\frac{1}{2}gh^2\right) = -\sigma(\boldsymbol{x})\boldsymbol{u}, \tag{3}$$

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where momentum $\boldsymbol{m} = h\boldsymbol{u}$ coincides with the mass flux. $\phi(\boldsymbol{x})$ and $\sigma(\boldsymbol{x})$ are respectively the variable porosity and linear friction terms characterizing the porous medium. In order to model a fluid with solid boundaries these terms have the following discontinuous forms

$$(\phi(\boldsymbol{x}), \sigma(\boldsymbol{x})) = \begin{cases} (\alpha, 1/\epsilon) & \text{in the penalized region,} \\ (1, 0) & \text{in the fluid,} \end{cases} \tag{4}$$

where the parameters α and ϵ control the accuracy of the boundary condition approximation. (For stable numerical implementation of the penalization the discontinuities in ϕ and σ are smoothed over a few grid points.) Physically, a large jump in porosity leads to a large jump in impedance that causes inertia–gravity waves to be almost perfectly reflected at the solid boundary, while a strong linear friction term rapidly damps velocity fluctuations approximating a no-slip velocity boundary condition.

Equations (1)–(3) are derived from Liu and Vasilyev (2007)'s similar penalized equations for the compressible Euler equations. Both penalizations have the property that mass and momentum do not move at the same speed and so it is impossible to conserve mass or to define an energy equation.

The lack of mass conservation is easy to see from the mass equation (Eq. 1), which can be rewritten as

$$\frac{\partial \phi(\boldsymbol{x}) h}{\partial t} + \text{div}\,\boldsymbol{m} = 0, \tag{5}$$

where $\boldsymbol{m} = h\boldsymbol{u}$ is the height (i.e. mass) flux. In order to conserve mass, the mass flux should actually be $\boldsymbol{m} = \phi(\boldsymbol{x})\boldsymbol{u}$ to take into account the changing volume fraction of the fluid in the porous medium. The penalized momentum equation (Eq. 3) also uses a non-porous mass flux (i.e. $h\boldsymbol{u}$ instead of $\phi h\boldsymbol{u}$). Therefore, it is impossible to derive an energy budget from Eqs. (1) and (3).

Reckinger et al. (2012)'s penalization also has the property that inertia–gravity wave speeds are $1/\sqrt{\alpha}$ times faster in the porous medium. This introduces a stiffness in time associated with the small porosity α that enforces an artificially small time step.

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The earlier shallow water equation penalization used by Perret et al. (2003) is even simpler in that only the velocity field is penalized using the friction term $-\sigma(\boldsymbol{x})\boldsymbol{u}$. Therefore, only the no-slip velocity boundary condition is approximated and not the perfect reflection of inertia–gravity waves at the boundary. This penalization can therefore be approximately valid in the quasi-geostrophic regime where wave motion is insignificant compared to vortical motion.

In the following section we derive the shallow water equations for a porous medium using Euler–Poincaré theory and then use these physical equations to propose a new Brinkman penalization for the shallow water equations in complex geometries. The final equations differ only slightly from those proposed by Reckinger et al. (2012), but they conserve both mass and energy and the wave speed is the same in both the fluid and penalized parts of the domain. Although our penalization is better justified on physical grounds, it is not yet clear whether it has any computational advantages apart from eliminating the stiffness constraint associated with the small porosity α.

3 New volume penalization for the shallow water equations

3.1 Derivation of porous shallow water equations

Euler–Poincaré theory (Holm et al., 2002) states that Hamilton's least action principle applied to the action

$$\mathcal{L} = \int L(h, \boldsymbol{u}, \boldsymbol{x})\,\mathrm{d}x\mathrm{d}y\mathrm{d}t$$

generates momentum equations for a particular choice of Lagrangian density $L(h,(\boldsymbol{u}),(\boldsymbol{x})) = T - V$. The Lagrangian density is the difference in kinetic and potential energy density and is assumed to depend on a scalar h, velocity vector field $\boldsymbol{u}(\boldsymbol{x})$ and position vector $\boldsymbol{x} \in \mathbb{R}^2$. If the conservation equation for the scalar h is

$$\frac{\partial h}{\partial t} + \mathrm{div}(h\boldsymbol{u}) = 0,$$

then locally conservative vector-invariant equation for momentum $\boldsymbol{m}$ is

$$\frac{\partial \boldsymbol{m}}{\partial t} + \mathrm{div}(\boldsymbol{m} \otimes \boldsymbol{u}) + \mathrm{grad}(p) = \frac{\partial L}{\partial \boldsymbol{x}}, \tag{6}$$

and the vector-invariant equations of motion are

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{\nabla \times \boldsymbol{v}}{h} \times h\boldsymbol{u} + \nabla B = 0, \tag{7}$$

where

$$\boldsymbol{m} = \frac{\partial L}{\partial \boldsymbol{u}} = h\boldsymbol{v}, B = \boldsymbol{u} \cdot \boldsymbol{v} - \frac{\partial L}{\partial h}, p = L - h\frac{\partial L}{\partial h}, \boldsymbol{v} = \boldsymbol{u}.$$

The total energy

$$E = \iint (\boldsymbol{m} \cdot \boldsymbol{u} - L)\mathrm{d}x\mathrm{d}y$$

is conserved.

We now use Euler–Poincaré theory to derive standard and modified shallow water equations. The fluid has free surface perturbations $\eta(\boldsymbol{x})$ from the mean free surface $\eta = 0$ and the depth of the fluid is given by $b(\boldsymbol{x}) > 0$ so the total depth is $h(\boldsymbol{x}) = \eta(\boldsymbol{x}) + b(\boldsymbol{x})$ as shown in Fig. 1. (In ocean modelling b is called the bathymetry, and $b = 0$ corresponds to coastlines.) The shallow water approximation assumes that η is small compared to depth b and that the wavelength of surface waves is much longer than the depth b. Note that h is proportional to the total mass density of the fluid column.

The standard shallow water equations are obtained using the Lagrangian density for the shallow water system

$$L(h, \boldsymbol{u}) = \frac{1}{2}h(|\boldsymbol{u}|^2 - g(\eta - b)),$$

from which one derives

$$m = hu,$$

$$B = g\eta + \frac{1}{2}|u|^2, \quad p = \frac{1}{2}gh^2,$$

$$E = \frac{1}{2}\iint h(u^2 + g(\eta - b))\mathrm{d}x\mathrm{d}y.$$

Thus, the shallow water equations of motion are the equations of motion

$$\frac{\partial u}{\partial t} + \frac{\mathrm{curl}(u)}{h} \times hu + \mathrm{grad}\left(g\eta + \frac{1}{2}|u|^2\right) = 0. \tag{8}$$

We now assume a porous medium with volume fluid fraction given by the variable porosity $\phi(x)$. We define a new variable $\widetilde{h} = \phi h$ satisfying the conservation law

$$\frac{\partial \widetilde{h}}{\partial t} + \mathrm{div}(\widetilde{h}u) = 0, \tag{9}$$

and the action

$$\mathcal{L} = \iiint \frac{1}{2}h(|u|^2 - g(h - 2b))\,\phi\mathrm{d}x\mathrm{d}y\mathrm{d}t. \tag{10}$$

The Lagrangian density for the new variable $\widetilde{h}$ is then

$$L(\widetilde{h}, u, x) = \frac{\widetilde{h}}{2}\left(|u|^2 - g\frac{\widetilde{h} - 2\widetilde{b}}{\phi}\right), \tag{11}$$

where $\widetilde{b} = \phi b$, from which

$$m = \widetilde{h}u, \quad v = u, \quad B = g\eta + \frac{1}{2}u^2,$$

$$p = \frac{1}{2}\phi gh^2, \quad \frac{\partial L}{\partial x} = \frac{1}{2}gh(h - 2b)\,\mathrm{grad}(\phi).$$

The momentum equation for the porous shallow water system is

$$\frac{\partial \boldsymbol{m}}{\partial t} + \operatorname{div}(\boldsymbol{m} \otimes \boldsymbol{u}) + \phi \operatorname{grad}\left(\frac{1}{2}gh^2\right) = 0.$$

However, surprisingly, the vector-invariant form of the equations of motion for the shallow water system are identical to the usual shallow water equations (Eq. 8); only the mass budget has changed to Eq. (9). States of rest correspond to constant h and inertia-gravity waves travel at speed $\sqrt{gh}$ if the porosity ϕ is constant, independent of the actual value of ϕ.

The non-dissipative equations of motion derived above do not fully model flow in porous media since they do not include the friction force per unit volume that resists flow through the medium. Including the friction force, the full vector-invariant equations of motion for the porous shallow water system are

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\operatorname{curl}(\boldsymbol{u})}{h} \times h\boldsymbol{u} + \operatorname{grad}\left(g\eta + \frac{1}{2}|\boldsymbol{u}|^2\right) = -\frac{\mu \phi(\boldsymbol{x})}{K(\boldsymbol{u}, h, \boldsymbol{x})}\boldsymbol{u}, \tag{12}$$

where μ is the fluid viscosity and $K(\boldsymbol{u}, h)$ is the effective permeability of the medium due to various friction terms. However, for the purposes of this paper we will assume the simple linear friction term of the form

$$-\frac{\phi(\boldsymbol{x})}{K}\boldsymbol{u}, \tag{13}$$

with constant permeability K which, like ϵ, has the dimensions of a time.

If the porosity is not small, it is better to use an empirical nonlinear friction law that includes both bottom and wall shear stresses (Guinot and Soares-Frazao, 2006). For example, the Strickler law approximates the friction term as

$$-\frac{g\widetilde{h}|\boldsymbol{u}|}{k^2 h^{4/3}}\boldsymbol{u}, \tag{14}$$

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where k is the so-called Strickler coefficient that depends empirically on the bottom roughness k_s, e.g. Ramette's formula gives $k = 8.2\sqrt{g}/k_s^{1/6}$ (Hervouet, 2007). Strickler's law is used by Guinot and Soares-Frazao (2006) in their porous shallow water model for large-scale flooding of urban areas.

3.2 Volume penalization of the shallow water equations

Our goal in this paper is to derive a volume penalization for solid boundaries in the shallow water model (e.g. coastlines or islands in an ocean model). As in all penalization methods, the idea is to implement boundary conditions implicitly by modifying the equations in a suitable way. In the limit as certain control parameters tend to zero the solution of the modified equations tends to the solution of the original equations with the desired boundary conditions. Such penalization techniques are particularly well-suited to adaptive numerical methods since, although the solid region is technically part of the computational domain, it can be resolved very coarsely except near the boundary.

We propose modelling the solid parts (e.g. continents and islands) of the computational domain as a porous medium with vanishingly small porosity ϕ and permeability K. The fluid part of the computational domain remains a regular fluid. The jump in porosity causes inertia-gravity waves to be reflected physically at the coastline and the small permeability approximates a no-slip boundary condition for velocity, i.e. $\boldsymbol{u} = \boldsymbol{0}$.

The vector-invariant penalized shallow equations based on Eq. (12) are

$$\frac{\partial \widetilde{h}}{\partial t} + \mathrm{div}\,\widetilde{h}\boldsymbol{u} = 0,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\mathrm{curl}(\boldsymbol{u})}{\widetilde{h}} \times \widetilde{h}\boldsymbol{u} + \mathrm{grad}\left(\frac{g\widetilde{\eta}}{\phi(\boldsymbol{x})} + \frac{1}{2}|\boldsymbol{u}|^2\right) = -\sigma(\boldsymbol{x})\boldsymbol{u},$$

(15)

where $\widetilde{\eta} = \phi(\boldsymbol{x})\eta$. The porosity $\phi(\boldsymbol{x})$ and porous friction coefficient $\sigma(\boldsymbol{x})$ are discontinuous such that the fluid portion of the domain is unaffected and the solid portion is

penalized as a very impermeable medium,

$$(\phi(\boldsymbol{x}), \sigma(\boldsymbol{x})) = \begin{cases} (\alpha, \alpha/K) & \text{in the penalized region,} \\ (1, 0) & \text{in the fluid,} \end{cases} \tag{16}$$

with $K \ll \alpha \ll 1$. The solid regions are defined by the indicator function $\chi(\boldsymbol{x})$,

$$\chi(\boldsymbol{x}) = \begin{cases} 1 & \text{in the solid,} \\ 0 & \text{in the fluid.} \end{cases} \tag{17}$$

5 When implemented numerically the indicator function $\chi(\boldsymbol{x})$ is smoothed over a few grid points, as discussed in Reckinger et al. (2012). The porosity $\phi(\boldsymbol{x})$ and friction coefficient $\sigma(\boldsymbol{x})$ are then defined based on $\chi(\boldsymbol{x})$ and the control parameters $\alpha \ll 1$ and $K \ll \alpha \ll 1$ as

$$\phi(\boldsymbol{x}) = 1 + \chi(\boldsymbol{x})(\alpha - 1), \tag{18}$$

10 $$\sigma(\boldsymbol{x}) = \frac{\alpha}{K}\chi(\boldsymbol{x}). \tag{19}$$

Note that the prognostic variables for the penalized shallow water equations (Eq. 15) are $(\widetilde{h}, \boldsymbol{u})$ and that $\widetilde{h} = h$ in the non-penalized (i.e. non-porous) region.

Equation (19) shows that the velocity penalization friction term $\sigma(\boldsymbol{x})$ depends explicitly on both the porosity α and the permeability K. In contrast, in Reckinger et al. (2012)
15 the velocity friction parameter ϵ is formally independent of porosity. Since $\epsilon = K/\alpha$ for a porous medium this implies that they effectively modify the permeability as the porosity changes in order to keep ϵ constant.

The flux form of the equations is

$$\frac{\partial \widetilde{h}}{\partial t} + \mathrm{div}(\boldsymbol{m}) = 0,$$

20 $$\frac{\partial \boldsymbol{m}}{\partial t} + \mathrm{div}\left(\frac{\boldsymbol{m} \otimes \boldsymbol{m}}{\widetilde{h}}\right) + \phi\,\mathrm{grad}\left(\frac{g\widetilde{h}^2}{2\phi^2}\right) = -\sigma\boldsymbol{m},$$

where the mass flux $\boldsymbol{m} = \phi h \boldsymbol{u}$. This shows clearly that both mass and momentum move at the same speed $\boldsymbol{u}$.

Although this penalization scheme is similar to that proposed by Reckinger et al. (2012), it does have some important physical and numerical differences that could prove advantageous. In addition, we fully characterize the error and convergence properties of penalization by deriving analytical estimates for the exact solution of the linearized one-dimensional wave propagation problem.

3.3 Properties of the penalization

We now summarize the main numerical properties of the volume penalization of the rotating shallow water equations introduced in the previous section.

The impedance mis-match at the solid boundary means that inertia-gravity waves are reflected with reflection coefficient

$$\mathcal{R} = \frac{\alpha^{-1} - 1}{\alpha^{-1} + 1} = 1 - 2\alpha + O(\alpha^2),$$

whereas the exact behaviour at the boundary is perfect reflection, $\mathcal{R} = 1$. Therefore, some height amplitude will be lost since part of the wave is transmitted and the size of the error is $O(\alpha)$.

There are two main differences compared with the method proposed in Reckinger et al. (2012). First, mass and energy both move at the same speed $\boldsymbol{u}$ and so energy is conserved. In particular, total energy decreases as

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{1}{2} \iint h(g\eta + |\boldsymbol{u}|^2)\,\phi(\boldsymbol{x})\,\mathrm{d}x\mathrm{d}y = -\iint \sigma(\boldsymbol{x})h|\boldsymbol{u}|^2\phi(\boldsymbol{x})\,\mathrm{d}x\mathrm{d}y,$$

which implies that the penalization is stable. Secondly, ignoring friction, the linear wave speed is the same in both the fluid and porous regions,

$$c = u \pm \sqrt{\frac{g\widetilde{h}}{\phi(x)}} = u \pm \sqrt{gH},$$

where $\widetilde{h} = h\phi(x) = (H + O(\eta))\phi(x)$, with $\eta \ll 1$, independent of α. This means that, un-like Reckinger et al. (2012)'s method, the height penalization does not affect the time step or stability properties of the numerical method.

The velocity penalization term is stiff in time, and limits the time step to $\Delta t = O(\epsilon)$ for explicit methods. Note that the height penalization parameter α does not place any additional constraints on the spatial resolution Δx or the time step Δt unless we choose to set $\epsilon = K/\alpha$.

Because height and velocity are governed by diffusion (and not wave) equations in the penalized solid region a wave will not be emitted from the boundary if there is no incoming wave. Therefore, the penalization is stable according to GKS stability theory for numerical stability of hyperbolic problems (Gustafsson et al., 1972).

The error and convergence properties of this method are derived analytically and verified numerically for a simple linear one-dimensional example in following section.

4 Analysis of linearized 1-D equations and guidelines for use

4.1 Exact solution and error analysis

We consider the one-dimensional penalized shallow water equations linearized about the state of rest with depth H and speed $\boldsymbol{u} = 0$,

$$
\begin{aligned}
\frac{\partial \widetilde{h}}{\partial t} &= -H\frac{\partial}{\partial x}(\phi(x)u), \\
\frac{\partial u}{\partial t} &= -g\frac{\partial}{\partial x}\left(\frac{\widetilde{h}}{\phi(x)}\right) - \sigma(x)u,
\end{aligned}
\tag{20}
$$

where the penalization functions $\phi(x)$ and $\sigma(x)$ are as given in Eq. (4). The geometry of the domain is defined by the indicator function $\chi(x) = H(x)$, where $H(x)$ is the Heaviside function. This means that $x < 0$ is fluid and $x \geq 0$ is solid. (Note that in a numerical

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implementation the indicator function is smoothed over a few grid points to avoid numerical oscillations.) The initial conditions are $u(x,0) = 0$ and

$$h(x,0) = \begin{cases} H_w, & x < -L - 1, \\ -\frac{H_w}{L}(x+1), & -L-1 \le x \le -1, \\ 0, & x > -1, \end{cases} \tag{21}$$

i.e. a linear ramp wave front with (non-dimensional) width L and amplitude H_w.

Following Kevlahan and Ghidaglia (2001) we solve the problem by taking separate Laplace transforms in time for the regions $x < 0$ and $x \ge 0$ and solving the resulting ordinary differential equations in x. The resulting four constants are determined by the requirement of finite solutions as $x \to \pm\infty$ and from the jump conditions at $x = 0$,

$$\widetilde{h}(x_-) = \widetilde{h}(x_+)/\alpha, u(x_-) = u(x_+)\alpha. \tag{22}$$

These jump conditions are found by integrating Eq. (20) across the fluid–solid boundary $x = 0$.

The exact Laplace transforms of penalized height and velocity in the fluid solid regions are

$$\widetilde{h}_{\text{fluid}}(x,s) = \widetilde{h}_1(x,s) + \frac{cH_w}{2Ls^2} e^{sx/c}(e^{-s/c} - e^{-s(1+L)/c}) \frac{(1+\alpha^2)\epsilon s + 1 - 2\alpha\sqrt{\epsilon s(\epsilon s + 1)}}{(1-\alpha^2)\epsilon s + 1},$$

$$u_{\text{fluid}}(x,s) = u_1(x,s) - \frac{c^2 H_w}{2HLs^2} e^{sx/c}(e^{-s/c} - e^{-s(1+L)/c}) \frac{(1+\alpha^2)\epsilon s + 1 - 2\alpha\sqrt{\epsilon s(\epsilon s + 1)}}{(1-\alpha^2)\epsilon s + 1},$$

$$\widetilde{h}_{\text{solid}}(x,s) = -\frac{\alpha c H_w}{Ls^2} \frac{\epsilon s + 1 - \alpha\sqrt{\epsilon s(\epsilon s + 1)}}{(1-\alpha^2)\epsilon s + 1} e^{-\frac{x}{\sqrt{\epsilon}c}\sqrt{s}\sqrt{\epsilon s + 1}}(e^{-s/c} - e^{-s/c(1+L)}),$$

$$u_{\text{solid}}(x,s) = \frac{g H_w}{Ls^{3/2}} \frac{\sqrt{\epsilon s + 1} - \alpha\sqrt{\epsilon s}}{(1-\alpha^2)\epsilon s + 1} e^{-\frac{x}{\sqrt{\epsilon}c}\sqrt{s}\sqrt{\epsilon s + 1}}(e^{-s/c} - e^{-s(1+L)c}), \tag{23}$$

where the wave speed $c = \sqrt{gH}$, and $\widetilde{h}_1(x,s)$ and $u_1(x,s)$ do not depend on the penalization. Now, taking the leading order series expansions in $\alpha \ll 1$ we have the following approximate expressions for the Laplace transforms of the penalized solutions,

$$\widetilde{h}_{\text{fluid}}(x,s) = \widetilde{h}_{\text{exact}}(x,s) - \frac{\alpha \epsilon^{1/2} c H_w}{L} \frac{e^{sx/c}(e^{-s/c} - e^{-s(1+L)/c})}{s^{3/2}\sqrt{\epsilon s + 1}} + O(\alpha^2),$$

$$u_{\text{fluid}}(x,s) = u_{\text{exact}}(x,s) + \frac{\alpha \epsilon^{1/2} c^2 H_w}{HL} \frac{e^{sx/c}(e^{-s/c} - e^{-s(1+L)/c})}{s^{3/2}\sqrt{\epsilon s + 1}} + O(\alpha^2),$$

$$\widetilde{h}_{\text{solid}}(x,s) = \frac{\alpha c H_w}{L s^2} e^{-\frac{x}{\sqrt{\epsilon}c}\sqrt{s}\sqrt{\epsilon s + 1}}(e^{-s/c} - e^{-s(1+L)/c}) + O(\alpha^2),$$

$$u_{\text{solid}}(x,s) = \frac{g H_w}{L s^{3/2}} \frac{e^{-\frac{x}{\sqrt{\epsilon}c}\sqrt{s}\sqrt{\epsilon s + 1}}}{\sqrt{\epsilon s + 1}}(e^{-s/c} - e^{-s(1+L)/c}) + O(\alpha),$$

(24)

where we recall that the exact solution in the solid region is zero.

Taking the inverse Laplace transform of Eq. (24) gives the following results for the penalizations errors in the fluid part of the domain,

$$\widetilde{h}_{\text{fluid}}(x,t) - \widetilde{h}_{\text{exact}}(x,t) = \frac{\alpha H_w}{L}[f_1(x + ct - (1 + L)) - f_1(x + ct - 1)],$$

$$u_{\text{fluid}}(x,t) - u_{\text{exact}}(x,t) = -\frac{c}{H}(\widetilde{h}_{\text{fluid}}(x,t) - \widetilde{h}_{\text{exact}}(x,t)),$$

(25)

where

$$f_1(x) = H(x)xM\left(\frac{1}{2}, 2, -\frac{x}{c\epsilon}\right),$$

and $M(1/2, 2, -z)$ is a hypergeometric function with leading order asymptotic expansion for large argument z

$$M(1/2, 2, -z) \sim \frac{2}{\sqrt{\pi}}z^{-1/2}.$$

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Note that the error is exactly zero until the wave reflects from the boundary. After reflection the error is zero at the leading edge of the wave $x = 1 - ct$ and maximal at the trailing edge $x = 1 + L - ct$. The maximum relative penalization errors are therefore

$$\frac{\|\widetilde{h}_{\text{fluid}} - \widetilde{h}_{\text{exact}}\|_\infty}{H_w} = \alpha M\left(\frac{1}{2}, 2, -\frac{L}{c\epsilon}\right) \sim 2\sqrt{\frac{c}{L}}\alpha\epsilon^{1/2},$$

$$\frac{\|u_{\text{fluid}} - u_{\text{exact}}\|_\infty}{c} = \alpha\frac{H_w}{H}M\left(\frac{1}{2}, 2, -\frac{L}{c\epsilon}\right) \sim 2\frac{H_w}{H}\sqrt{\frac{c}{L}}\alpha\epsilon^{1/2}, \tag{26}$$

where we have assumed that $\epsilon \ll L/c$ and recall that $\epsilon = K/\alpha$.

The asymptotic estimates Eq. (26) show that the penalization converges as $\epsilon \to 0$ and $\alpha \to 0$ and that the relative errors the penalized equations are $O(\alpha\epsilon^{1/2}\sqrt{c/L})$ for height and $O(\alpha\epsilon^{1/2}\sqrt{c/L}\,H_w/H)$ for velocity. As expected, the error is exactly zero until the wave reaches the solid boundary at $t = 1$.

Now, taking the inverse Laplace transform in the solid region we find that

$$\widetilde{h}_{\text{solid}}(x,t) = \frac{\alpha c H_w}{L}\left[\int_{x/c}^{t-1/c} e^{-\tau/2\epsilon}I_0\left(\frac{1}{2\epsilon}\sqrt{\tau^2 - \left(\frac{x}{c}\right)^2}\right)e^{-\frac{t-1/c-\tau}{\epsilon}}M\left(\frac{3}{2}, 1, \frac{t-1/c-\tau}{\epsilon}\right)d\tau\right.$$

$$\left. - \int_{x/c}^{t-(1+L)/c} e^{-\tau/2\epsilon}I_0\left(\frac{1}{2\epsilon}\sqrt{\tau^2 - \left(\frac{x}{c}\right)^2}\right)e^{-\frac{t-(1+L)/c-\tau}{\epsilon}}M\left(\frac{3}{2}, 1, \frac{t-(1+L)/c-\tau}{\epsilon}\right)d\tau\right]$$

$$u_{\text{solid}}(x,t) = \frac{gH_w}{L}\int_{t-(1+L)/c}^{t-1/c} e^{-\tau/2\epsilon}I_0\left(\frac{1}{2\epsilon}\sqrt{\tau^2 - \left(\frac{x}{c}\right)^2}\right)d\tau. \tag{27}$$

If we now assume that $\epsilon \ll t - (L+1)/c$ to approximate $I_0(z) \sim e^z/\sqrt{2\pi z}$ for $z \gg 1$, $x \ll ct - (L+1)$ to approximate $\sqrt{\tau^2 - (x/c)^2} = \tau(1 - 1/2(x/c\tau)^2) + O(x/c\tau)^4)$ and $\epsilon \ll$

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x/c to approximate $M(3/2, 1, -z) \sim 2z^{1/2}/\sqrt{\pi}$, the above Laplace transform integrals become

$$\widetilde{h}_{\text{solid}}(x,t) = \frac{2\alpha c H_w}{\pi L} \left[\int\limits_{x/c}^{t-1/c} \left(\frac{t-1/c}{\tau} - 1 \right)^{1/2} \exp\left(-\frac{x^2}{4c^2\epsilon\tau} \right) d\tau \right.$$

$$\left. - \int\limits_{x/c}^{t-(1+L)/c} \left(\frac{t-(1+L)/c}{\tau} - 1 \right)^{1/2} \exp\left(-\frac{x^2}{4c^2\epsilon\tau} \right) d\tau \right], \qquad (28)$$

$$\widetilde{u}_{\text{solid}}(x,t) = \frac{g H_w}{L} \sqrt{\frac{\epsilon}{\pi}} \int\limits_{t-(1+L)/c}^{t-1/c} \tau^{-1/2} \exp\left(-\frac{x^2}{4c^2\epsilon\tau} \right) d\tau.$$

Again, assuming $\epsilon \ll x/c$ the integrand in the first equation decays exponentially as $\tau \to x/c$ and we can approximate the lower integration limit x/c by zero. Evaluating the integrals in Eq. (28) gives the final results,

$$\frac{\widetilde{h}_{\text{solid}}(x,t)}{H_w} \sim \frac{\alpha c}{L} [f_2(x, t-1/c) - f_2(x, t-(L+1)/c)],$$

$$\frac{u_{\text{solid}}(x,t)}{c} \sim \frac{g H_w}{Lc} [f_3(x, t-1/c) - f_3(x, t-(L+1)/c)], \qquad (29)$$

where

$$f_2(x,t) = H(t)t \left[\left(1 + \frac{x^2}{2c^2\epsilon t} \right) \text{erfc}\left(\frac{x}{c\sqrt{\epsilon t}} \right) - \frac{x}{2c\sqrt{\pi\epsilon t}} \exp\left(-\frac{x^2}{4c^2\epsilon t} \right) \right],$$

$$f_3(x,t) = H(t) \left[\frac{x}{c} \text{erf}\left(\frac{x}{2c\sqrt{\epsilon t}} \right) + 2\sqrt{\frac{t\epsilon}{\pi}} \exp\left(-\frac{x^2}{4c^2\epsilon t} \right) \right]. \qquad (30)$$

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Assuming an interaction time $t \approx L/c$, the results (Eqs. 29 and 30) show that the penalized solution penetrates a distance $O(\sqrt{cL\epsilon})$ into the solid region. This numerical boundary layer must be resolved, so we require a local grid size near the boundary $\Delta x \leq \sqrt{cL\epsilon}/2$. If the wavefront is well-resolved, i.e. L is much larger than the grid size Δx, then the penalization is first-order accurate in space with a relative height error $O(\alpha\Delta x/L)$. However, if the wavefront is only marginally resolved, i.e. $L \approx \Delta x$, then the relative error is $O(\alpha)$, independent of the grid resolution. In this case a sufficiently small error can be achieved for any grid by choosing α appropriately.

4.2 Numerical verification on linearized 1-D wave propagation

The error estimate $O(\alpha\epsilon^{1/2}) = O(\sqrt{\alpha K})$ for height and velocity derived in the previous section is verified here for one-dimensional linear wave propagation with reflection. The computational domain is $x \in [0, L_x]$ with periodic numerical boundary conditions. The penalized (i.e. solid) region is $x \leq x_1$ and $x \geq x_2$ defined by indicator functions,

$$\chi(x) = \frac{1}{2}\left(\tanh\left(\frac{x - x_2}{\Delta}\right) - \tanh\left(\frac{x - x_1}{\Delta}\right)\right),$$

$$\phi(x) = 1 + \chi(x)(\alpha - 1),$$

$$\sigma(x) = \frac{1}{\epsilon}(H(-(x - x_1)) + H(x - x_2)).$$

A smoothed porosity is used since $\phi(x)$ must be differentiated. However, the permeability $\sigma(x)$ is not smoothed since otherwise the penalization error begins to grow for sufficiently small ϵ (depending inversely on α). (If $\epsilon = K/\alpha$, a smoothed $\sigma(x)$ may be used.) When $\epsilon = K/\alpha$ we choose $K = (4\Delta x)^2$. A good choice for the smoothing parameter is the smallest value that ensures stable solutions and linear error convergence with α, i.e. $\Delta = \Delta x$ which smooths the indicator function over about four grid points as shown in Fig. 2. We use these choices for the K and Δ in the remainder of this section.

The initial condition is a Gaussian wave for height and zero velocity,

$$h_0(x) = \exp\left[-\left(\frac{x - L_x/2}{L}\right)^2\right], \tag{31}$$

$$u_0(x) = 0.$$

The initial conditions and porosity are shown in Fig. 2. The computational domain is [0, 0.6] (i.e. $L_x = 0.6$), with the fluid part of the domain [0.05, 0.55] (i.e. $x_1 = 0.05$ and $x_2 = 0.55$) of length 0.5 and the left and right solid boundaries are penalized regions of width 0.05 each.

The exact solution with initial conditions (Eq. 31) and solid boundary conditions $u = 0$ and $\partial h / \partial x = 0$ is

$$h(x, t) = \frac{1}{2}(h_0^p(x - t) + h_0^p(x + t)) + \frac{1}{2}(u_0^p(x - t) - u_0^p(x + t)),$$

$$u(x, t) = \frac{1}{2}(u_0^p(x - t) + u_0^p(x + t)) + \frac{1}{2}(h_0^p(x - t) - h_0^p(x + t)),$$

where $h_0^p(x)$ and $u_0^p(x)$ are odd periodic extension of the initial conditions outside the fluid interval $[x_1, x_2]$.

The linearized one-dimensional equations (Eq. 20) are solved using a standard second-order finite volume/finite difference scheme with third-order Runge–Kutta integration in time on a uniform grid with $N = 2400$ grid points. A typical penalized solution is shown at time $t = 0.22$ in Fig. 3, when the wave is strongly interacting with the walls. This figure confirms the expected behaviour of the penalized solution near the walls: the velocity boundary condition has an error and internal boundary layer of size $O(\epsilon^{1/2})$, while the height perturbation does not penetrate into the solid.

In order to measure the effect of the penalization on the error of the global solution after reflection we measure the L^∞ error at $t = 0.5$ when the exact solution should

precisely reproduce the initial conditions. The prediction that the error should scale proportional to the porosity α if α and ϵ are independent and like $\alpha^{1/2}$ if $\epsilon = K/\alpha$ (as in a porous medium) is verified in Fig. 4. Note that the error at small $\alpha < 10^{-4}$ is effectively limited by the error of the underlying finite-volume/finite-difference numerical scheme, which is about 6×10^{-5} for the exact boundary conditions at this resolution $N = 2400$.

Figure 5 (left) confirms that the error scales like $K^{1/2}$ when $\epsilon = K/\alpha$. Finally, Fig. 5 (right) confirms that the error for this penalization scheme, with permeability $K = \Delta x^2$, is first-order accurate. Since we implement this penalization in a dynamically adaptive simulation, sufficient accuracy is achieved by refining the grid at the boundary (i.e. by h refinement) and choosing α appropriately as explained in Sect. 4.3.

As mentioned in Sect. 3.2, Reckinger et al. (2012) assume that α and ϵ are formally independent. However, in practice they advise that ϵ should be smaller than α, and choose $\epsilon/\alpha = 10^{-2}$ for their simulations. This restriction is not necessary in our case since the error is $O(\alpha\epsilon^{1/2})$. This means that α can be chosen smaller than ϵ, as shown in Fig. 4. In fact, to ensure scaling of the error like $O(\epsilon^{1/2})$ when α is fixed it is necessary to choose $\epsilon\alpha = K$ (constant) when the indicator function defining the solid region is smoothed. Although Reckinger et al. (2012) interpret Fig. 8 for $\alpha = \epsilon$ as showing a weaker error convergence $O(\alpha^{1/2})$, it actually appears to show the expected scaling $O(\alpha)$, but over a small range of α of about one decade.

4.3 Guidelines for choosing penalization parameters

The parameters ϵ, α and Δ determining the penalization are chosen as follows.

The permeability parameter ϵ is set first, based on the spatial resolution of the simulation Δx near the coastlines. As explained in Sect. 4.1, the smallest permissible value for ϵ is $4\Delta x^2/cL$. However, the velocity penalization term is stiff, restricting the time step to $\Delta t \leq C_1\epsilon$ (with C_1 an order one constant) for an explicit method. It is therefore often preferable to choose a larger ϵ so the penalization does not enforce an artificially small time step. For example, set $\epsilon = \Delta t = C_2\Delta x/c$ according to the Courant–

Friedrichs–Lewy (CFL) stability condition for hyperbolic equations. Note that this is also the smallest permissible ϵ when the smallest wavefronts are only marginally resolved so $L \sim \Delta x$, where $\epsilon \geq 4\Delta x/c$. Using this choice of ϵ, and in the least favourable case where the smallest wavefronts are only marginally resolved, the relative error in height

5 is $O(\alpha)$ and the relative error in velocity is $O(\alpha H_w/H)$ independent of ϵ and Δx.

Now, since ϵ has been determined by the resolution of the simulation, the desired accuracy is controlled by setting the porosity α. Recall that the choice of α does not affect the numerical stability of the simulation. Typically, $\alpha = O(10^{-3})$ is appropriate for a second-order accurate simulation. In a dynamically adaptive method like the one

10 used here, α should be set about ten times smaller than the tolerance ε. Recall that the parameter ϵ also enforces the no-slip (i.e. tangential) velocity condition to a relative accuracy $O(\epsilon^{1/2}\sqrt{u/l})$, where u and l are the velocity and length scales of the flow tangential to the boundary (Kevlahan and Ghidaglia, 2001).

The smoothing scale Δ of the indicator function $\chi(x)$ is set to smooth over a few grid

15 points (e.g. two to four). The smoothing scaling should be much smaller than the scale L of the smallest waves and also smaller than $\sqrt{Lc\epsilon}$.

These choices ensure the penalization is well-resolved, produces sufficiently accurate results and is consistent. When implemented in the adaptive wavelet method we must also ensure α is not too small, i.e. $\alpha > 7.5 \times 10^{-4}$, in order to avoid negative heights

20 near the boundary due to the linear interpolation used in the wavelet transform.

In the following section we verify the results of the penalization analysis numerically using a dynamically adaptive second-order finite difference – finite volume scheme (Dubos and Kevlahan, 2013; Aechtner et al., 2014) on the sphere based on the TRiSK scheme (Ringler et al., 2010).

5   Applications to ocean simulation

The Coriolis force, which is omitted in the previous sections, is now included by adding the Coriolis parameter f to the relative vorticity curl($\boldsymbol{u}$) in the curl-form equations of motion (Eq. 15).

5.1   Implementation of penalization in adaptive wavelet solver on the sphere

Penalization techniques are especially well-suited to dynamically adaptive numerical simulations, where the local resolution changes in time to resolve the solution. In particular, in ocean flows we expect the resolution to be finer near coastlines in order to resolve boundary currents (e.g. wind-driven gyres in the quasi-geotrosphic regime) or wave interaction with the coast (e.g. tsunami propagation in the inertia gravity wave regime). Ocean flow is well-suited to variable resolution adaptive numerical methods since about 25 % of the surface of the Earth is land (which thus requires no resolution) and the ocean flows are highly inhomogeneous and variable in both time and space.

An explicit definition of the coastline is difficult to implement in adaptive simulations because the precise location of the coastline changes as the grid refines and coarsens. On the other hand, it is computationally inefficient to resolve the coastline to the finest resolution at all locations and at all times. Defining the coastline as a mask means the coastline is defined implicitly and automatically becomes more detailed as the grid refines to follow the local flow dynamics. In addition, smoothing the profile of the coastline over a few grid points arguably produces a better physical model than a sharp boundary (since coastlines are in fact porous). The multiscale and staggered structure of the adaptive wavelet scheme also causes problems for an explicit definition of the coast line since the hexagonal cells containing the height are shifted between adjacent scale of resolution (see Dubos and Kevlahan, 2013; Aechtner et al., 2014).

Finally, as mentioned in the previous section, grid refinement near the coastlines increases the local accuracy of the penalization through h refinement compensating for its relatively low order of accuracy.

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Interactive Discussion

The penalization defined by the variable porosity (Eq. 18) and friction (Eq. 19) is easily integrated into the dynamically adaptive second-order finite difference – finite volume scheme on the sphere presented in Dubos and Kevlahan (2013) and Aechtner et al. (2014) since it requires only straightforward modifications of the shallow water equations. The bathymetry and topographic data are from the 1 arc min NOAA ETOPO1 global relief data base (Amante and Eakins, 2009).

The raw bathymetry data from from the ETOPO1 database naturally tends to zero depth near the coast. Because we have not implemented wetting and drying in our shallow water model, we impose a minimum depth H_{min} near the coastlines,

$$
b = \begin{cases} b_r & b_r \le -H_{min}, \\ -H_{min} & -H_{min} < b_r < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{32}
$$

In practice, $H_{min} > 2\,\text{m}$ is usually sufficient.

The mask $\chi(x)$ defining the solid and fluid regions is found by setting locations with negative bathymetry to zero and regions with positive (or zero) bathymetry to one,

$$
\chi = \begin{cases} 0, & b_r < 0, \\ 1 & \text{otherwise.} \end{cases} \tag{33}
$$

This generates a mask on the regular 1 arc min latitude–longitude ETOPO 1 grid, which does not correspond to the non-uniform dual hexagonal–triangular grids used in the adaptive scheme. The value of the mask at required points on the hexagonal–triangular grid are found by using a simple exponential radial basis function (RBF), with weights $f(x; a) = \exp(-(ar)^2)$ where r is the arc distance between the ETOPO 1 mask and the location of the required grid point. The parameter a is chosen to smooth over an area equivalent to two to four hexagonal cells. This RBF procedure both interpolates from the latitude–longitude grid to the adaptive grid nodes and smooths the resulting mask. The RBF procedure can also be used to smooth the bathymetry data in the fluid part of

the domain, although this is not usually necessary. Currently, all points are smoothed although the method could be optimized by smoothing only those points in a small neighbourhood of a coastline.

During grid refinement the bathymetry is computed at the new grid-points using the
5 RBF interpolation described above. This procedure means that total mass of water is no longer conserved exactly. Instead, the total mass relative to the sea-level is conserved by the numerical scheme. This means that the mass defect introduced by the discrete model, which may accumulate over time, is still controlled to the order of round-off errors. The mass defect caused by changes in the bathymetry cannot accumulate
10 and is bounded at all times.

In the following sections the adaptive wavelet method for the shallow water equations with penalization is used to solve two characteristic ocean flows: tsunami propagation (i.e. the inertia gravity wave regime with fast dynamics) and wind-driven gyre flow (i.e. the quasi-geostrophic regime with slow dynamics). The goal of these simulations is
15 to demonstrate the potential of this method for efficient simulation of global flows with localized small scale features. It should be stressed that different degrees of physical accuracy are to be expected in each case due to the approximations inherent in the shallow water model. On the one hand, the shallow water equations model tsunami propagation quite accurately, so that a realistic tsunami simulation is expected. On the
20 other hand, the shallow water equations are quite insufficient to model the general circulation of the oceans. Only the mean gyre circulation, driven by the wind stress and Sverdrup balance, which is acceptably represented in a one-layer model, can be captured realistically. Smaller-scale features, such as vortices and jet meandering, are predominantly generated in the real ocean by baroclinic mechanisms which cannot
25 be captured by a single-layer model. Their main characteristics are not expected to be realistic. Rather, the capacity of the adaptive model to produce, say, boundary currents, should be analyzed as a qualitative demonstration of the potential of the method, rather than evaluated quantitatively for its accuracy.

5.2 Tsunami propagation

Our first example illustrates how the penalization, combined with the dynamically adaptive wavelet method (Aechtner et al., 2014), performs for global calculation of tsunami wave propagation. The flow is clearly in the inertia gravity wave regime and the dynamics are fast. Since the solution is very localized, the dynamical adaptation is particularly effective, allowing local resolutions up to 0.5 km on a global model. This inertia–gravity regime is a good test of the accuracy of the penalized approximation of the reflecting boundary conditions for height since reflection off coastlines and islands is an essential component of tsunami dynamics. Note that because of the sensitivity of the results on the precise choice of initial condition, bathymetry and coastline geometry a precise measure of the error is not possible although the results are qualitatively in good agreement with the observations and other simulations.

We simulate the 2004 tsunami generated by the Sumatra–Andaman Earthquake. The initial condition is based on the seismic data calculated by Fujii and Satake (2007) from available tide gauge and satellite altimetry data. This initial condition is given in the form of complete seismic data on 22 separate square geographic regions, as shown in Fig. 4 of Fujii and Satake (2007). These 22 separate sets of seismic data are used to find the perturbed surface height using the Okada (1985) method with `matlab` software written by Beauducel (2012). (Note that each of the 22 regions provides a separate sea surface height perturbation.) The initial velocity is taken to be zero.

The degree of mesh refinement is controlled by an overall non-dimensional tolerance ε (not to be confused with the relaxation time ϵ of the penalization), from which thresholds for height and velocity are deduced (Dubos and Kevlahan, 2013). The simulation was run with an overall tolerance of $\varepsilon = 0.05$, and the thresholds for height and velocity were $\varepsilon_h = H_{max}\varepsilon^{3/2}$ and $\varepsilon_u = H_{max}g/c\varepsilon^{3/2}$ where H_{max} is the maximum height perturbation at any given time step. This allows the adaptation to accurately track the waves even though after several hours their characteristic height H_{max} is only 10 % of its initial value. This modification is important for cases where the flow field is not statistically

stationary in time. Note that we have deliberately chosen a relatively high tolerance value to demonstrate that the code can provide qualitatively good results even for grid compression ratios of $O(10^3)$.

The coarsest level is $J = 9$ with 5 levels of refinement to give a maximum scale of $J = 14$ corresponding to a minimum average resolution of about $\langle \Delta x_{min} \rangle = 475\,\text{m}$. Note that a non-adaptive simulation at this resolution would require about 2.68×10^9 height nodes (hexagonal cells), while the initial condition requires only about 3.09×10^6 height nodes in the adaptive simulation corresponding to a grid compression ratio of 867.

The penalization parameters are $\alpha = 8 \times 10^{-3}$ and $\eta = 5 \times 10^{-5}$ and the minimum bathymetry depth is $H_{min} = 50\,\text{m}$. The adaptive wavelet code was run on 256 cores on the `Scinet` supercomputer.

The first arrival time of a 5 cm wave and the maximum wave height over all times up to 16 h at all positions are shown in Fig. 6. The maximum wave height results show the focusing effect of bathymetry features (particularly the Southwest Indian Ridge) and agree qualitatively with both observations and simulations using the MOST model (Titov et al., 2005). Detailed quantitative verification is not possible due to sensitive dependence of the results on details of the initial conditions, bathymetry and coastline modelling (including run-up, not included in this model).

The ability of the code to track an evolving localized tsunami wave over long times and through reflection and focusing events is illustrated in Figs. 7, 9 and 10. The actual tolerances are scaled dynamically to take into account the decreasing maximum wave height over time. Note that the finest $J = 14$ (500 m) resolution is only needed very locally along some parts of the coastline and where the wavefront is very steep or focusing. Figure 8 uses a zoomed view to show precisely where the finest resolution is required in the interior of a focusing wave packet. As mentioned above, we have deliberately chosen a relatively large tolerance since we are interested in the propagation of the wavefront (and to illustrate the extreme adaptivity potential of the method). If we were interested in accurate simulation of the entire wavefront (e.g. the residual wave motion shown in Fig. 10 at 16 h) we could select a smaller tolerance.

This simulation has demonstrated the potential of the dynamically adaptive wavelet method with penalization for high resolution simulation of tsunami propagation. Local resolutions of less than 500 m have been achieved on a global model with modest consumptions of computational resources: the simulation until the arrival at the African coast requires only two to three days on 256 cores of a computing cluster. Because of the localization of the wavefronts, tsunami propagation is particularly well-suited to adaptive simulation. The plot of the grid compression ratio shown in Fig. 11 shows that the code achieves very high grid compression ratios, ranging from 936 at 40 min to 400 at 16 h when the wave has entered the Atlantic ocean. When all potential degrees of freedom are included (height and velocity nodes) the grid compression ratio varies from 1240 to 455. Since Aechtner et al. (2014) found that the adaptive wavelet code is about three times slower per active height node than the non-adaptive TRiSK code we expect the tsunami simulation to be between 130 and 300 times faster than the non-adaptive code for a $J = 14$ resolution. Compared to a similar spectral code the adaptive simulation should be about 248 to 91 times faster.

5.3 Wind-driven ocean circulation

The second simulation is of global wind-driven ocean circulation over several years. This tests the adaptive wavelet model with Brinkman penalization in the quasi-geostrophic regime for slow dynamics. Our goal is to qualitatively predict the structure of the main ocean gyre flows, within the limits of the rotating shallow water equation model. In this case large basin-scale circulation is driven by the applied wind stress forcing via the Sverdrup relation. Intense boundary currents are expected to form along western coastlines (e.g. the gulf stream). The shallow water equations are modified by adding a wind-stress forcing term $\tau/(\rho h)$ to the right hand side of the equation.

As for the tsunami case, the bathymetry and topographic data are from the 1 arc min NOAA ETOPO1 global relief data base (Amante and Eakins, 2009). The wind-stresses $\tau(x,y)$ are stationary in time and derived from the mean December wind stresses from the NCAR Hellerman and Rosenstein Global Wind Stress Data set (Hellerman and

Rosenstein, 1980, 1983) shown in Fig. 12. This data set consists of monthly averaged wind stress over the global ocean for the years 1870 through 1976 on a two degree latitude–longitude grid. The wind stress data is evaluated on the adapted grid using bilinear interpolation.

⁵ The numerical experiment is characterized by a few independent dimensional parameters: τ/ρ with τ the mean wind stress and ρ the density of water, the planetary rotation rate $\Omega \sim f$ and radius R, the basin scale $L \sim R$, the mean ocean depth H, gravity g, the Reynolds number Re, the width of the boundary layer δ_M, and the Froude number of the boundary layer Fr_{BC}. Once these are defined a few other scales emerge ¹⁰ following Sverdrup balance in the ocean interior and balance between viscous friction and meridional transport of planetary vorticity in the western boundary current. The kinematic viscosity is therefore $\nu = U_{BC}\delta_M/Re$, and $\beta = \nu/\delta_M^3$. The gyre velocity U_{Sv} set by Sverdrup balance is

$$U_{Sv} \sim \frac{1}{\beta H L}\frac{\tau}{\rho}. \tag{34}$$

¹⁵ The gyre is characterized by its Rossby number $Ro = U_{Sv}/fL$ which should be small.

The dimensional and non-dimensional parameters are fully summarized in Table 1. Given the limitations of the shallow-water model, we have sacrificed the realism of some of the dimensional parameters, while preserving the main scales of the gyres and boundary currents. We retain realistic values of R, L, H, U_{BC} and U_{Sv}. On the ²⁰ other hand we choose unrealistic values for gravity g and planetary rotation Ω, β. Indeed, a large gravity wave speed c imposes small explicit time steps which make the simulation very costly without affecting the gyre and boundary current. Hence we sacrifice the realism of c and reduce it to a minimum, i.e. Fr_{BC} is set as large as possible without producing shocks. This defines $c = U_{BC}/Fr_{BC}$ and $g = c^2/H$. The Reynolds ²⁵ number is set moderately large to permit barotropic instability and the generation of vortices.

The wavelet simulation uses a tolerance of $\varepsilon = 1.0$, and the thresholds for height and velocity are $\varepsilon_h = U_{Sv} RoR / g \varepsilon^{3/2}$ and $\varepsilon_u = U_{Sv} Ro \varepsilon^{3/2}$. The coarsest level is $J = 12$ with 3 levels of refinement to give a maximum level of $J = 12$ and a minimum average resolution of about $\Delta x_{min} = 1.9$ km or $1/64°$. The penalization parameters are $\alpha = 10^{-2}$ and

5   $\eta = 10^{-4}$. The minimum bathymetry depth is limited to $H_{min} = 50$ m. The initial conditions are zero velocity and zero sea surface height perturbation. The adaptive wavelet code was run on 256 cores on the `Scinet` supercomputer.

The mean ocean circulation consists of basin-scale gyres driven by the wind stress via Sverdrup balance. The rigid-wall boundary condition induces narrow and intense

10   western boundary currents dominated by advection of planetary velocity and friction. This case is therefore a good test for the penalized velocity boundary conditions. We stress again, however, that the mechanism generating meanders and vortices from the gyre circulation and the boundary currents in the shallow water equations is different from the baroclinic mechanism that is believed to be the dominant effect in the oceans

15   but cannot be captured in a one-layer shallow water model.

Figure 13 shows the vorticity after 301 days. The grid has refined only at the boundary currents and the grid compression ratio for height nodes is roughly constant at about 210 once the boundary currents have developed (after about one week). Coherent vortex shedding, similar to von Karman vortex streets is clearly visible at some

20   high wind stress locations, such as the Drake passage and southern coast of Argentina shown in Fig. 13. The zoom of the unstable boundary layer region off southern Argentina shown in Fig. 14 illustrates the complex structure of the boundary current and vortices. Note that the details of the boundary current are well-captured by the adaptive grid. Higher resolutions and Reynolds numbers would lead to more complex two-

25   dimensional turbulence like dynamics (with physics different from the actual flow due to the shallow water approximation). Despite the limitations of the experimental setup, these results give an indication of the potential performance of a multi-layer model and the ability of the method to capture boundary currents and their complex vortical structure.

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6 Conclusions

We have derived and analyzed mathematically a new volume penalization for no-slip boundary conditions for the shallow water equations. This penalization is based on the physical equations for shallow water flow in a porous medium with vanishing porosity and permeability in that part of the domain corresponding to solid regions. Mathematical analysis of the linearized one-dimensional shallow water equations shows that the solution of the penalized equations converges to exact solution in the limit as porosity α and permeability η tend to zero. The error at finite α and η is $O(\alpha\eta^{1/2})$. Unlike previous penalizations of the shallow water equation, it conserves mass and energy and the wave speed is the same in both fluid and solid regions. The convergence and error properties of the method have been verified numerically for the one-dimensional linearized equations.

The primary motivation for developing this new penalization is to extend our recent dynamically adaptive wavelet method on the sphere (Dubos and Kevlahan, 2013; Aechtner et al., 2014) to model ocean flows with coastlines. Penalization techniques are ideal for dynamically adaptive methods because they implement the coastline geometry implicitly by modifying the equations of motion rather than by explicitly changing the geometry of the computation. The resolution of the coastline is high only where required by the flow dynamics.

We have implemented the proposed penalization in the adaptive wavelet code and tested it on two typical global scale flows: long-distance tsunami propagation (i.e. the inertia-gravity wave regime with fast dynamics) and wind-driven ocean circulation (i.e. the quasi-geostrophic regime with slow dynamics). These simulations show the potential of the adaptive method combined with the penalization to drastically reduce the number of computational elements. The adaptive tsunami simulation uses between 455 and 1245 times fewer computational elements (i.e. height nodes) than an equivalent non-adaptive simulation, while the wind-driven ocean circulation simulation uses around 210 times fewer elements.

Although the shallow water equations are considered quite accurate for tsunami calculations (and are used in many operational models) they are clearly physically insufficient for calculating ocean circulation. The next step in the development of the adaptive wavelet model for ocean circulation is to add vertical layers and temperature and density equations. The grid adaptation will only be done in the horizontal plane and so the three-dimensional model should actually have better parallel performance than the model on the sphere since the computational load will be better balanced. We expect to also use penalization to model bathymetry, as well as coastlines, in the three-dimensional model, following Reckinger et al. (2012).

The penalization method presented here should aid in the development of fully dynamically adaptive ocean global models for tsunami propagation and ocean circulation.

Code availability

The complete adaptive wavelet code used to generate the results in this paper is available at bitbucket.org/kevlahan/wavetrisk.

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Table 1. Physical parameters used for the reduced gravity simulation of wind-driven ocean circulation.

Non-dimensional parameters of boundary layer determining simulation		
Reynolds number	Re	10^4
Froude number of boundary layer	Fr_{BC}	0.3
Non-dimensional boundary layer width	δ_M^*	0.0125
Unconstrained parameters		
Radius of Earth	R	6.3710×10^6 m
Reference length scale (radius of North Atlantic)	L	3.0000×10^6 m
Mean ocean depth	H	3.5729×10^3 m
Velocity of boundary layer	U_{BC}	1.8000×10^0 m s^{-1}
Rotation rate	Ω	5.7664×10^{-7} s^{-1}
Wind stress	τ	7.1592×10^{-2} N m^{-2}
Density	ρ	1.0270×10^3 kg m^{-3}
Quantities determined by above choices		
Boundary layer width	$\delta_M = \delta_M^* L$	3.7500×10^4 m
Kinematic viscosity	$\nu = U_{BC}\delta_M/Re$	6.7500×10^0 m^2 s^{-1}
Effective β parameter	$\beta = \nu/\delta_M^3$	1.2800×10^{-13} m^{-1} s^{-1}
Sverdrup (gyre) velocity	$U_{Sv} \sim \frac{1}{\beta HL}\frac{\tau}{\rho}$	5.2875×10^{-2} m s
Wave speed	$c = U_{BC}/Fr_{BC}$	6.0000×10^0 m s^{-1}
Gravitational acceleration	$g = c^2/H$	1.0076×10^{-2} m s^{-2}
Coriolis parameter	$f \sim \Omega$	5.7664×10^{-7} s^{-1}
Rossby radius of deformation	$R_d = c/f$	1.0405×10^7 m
Rossby number	$Ro = U_{Sv}/(Lf)$	3.0565×10^{-2}

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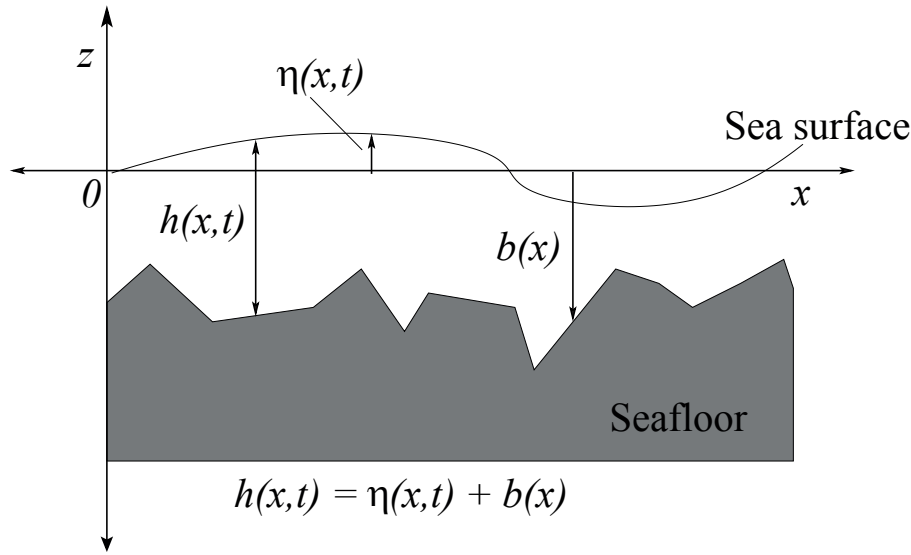
Figure 1. Shallow water geometry. The perturbation of the sea surface from equilibrium sea surface $z = 0$ is $\eta(\boldsymbol{x})$ and the sea depth is given by the bathymetry $b(\boldsymbol{x}) \geq 0$, which is the depth of the seafloor below the equilibrium sea surface. The total height of the fluid is then $h(\boldsymbol{x}) = \eta(\boldsymbol{x}) + b(\boldsymbol{x})$. In the shallow water approximation the wavelength of the perturbations of the sea surface is much greater than the depth, and the amplitude of the perturbations is much less than the depth.

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Figure 2. Height initial conditions for the wave packet and Gaussian test cases and porosity $\phi(x)$ with $\alpha = 0.1$. The velocity is initially zero. Note the smoothing of the indicator function over about four grid points at the left and right solid boundaries with $\Delta = \Delta x$.
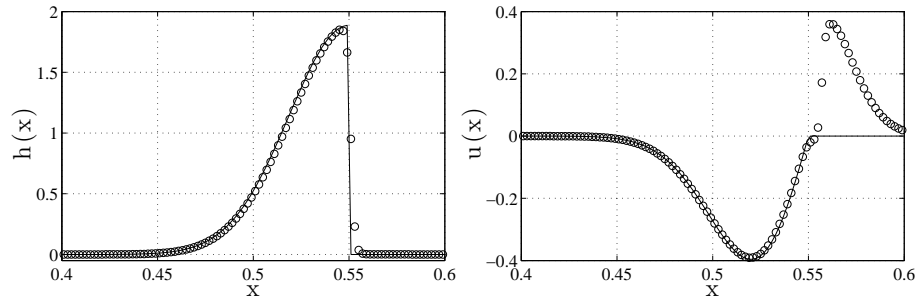
Figure 3. Solution at $t = 0.26$ just after the reflection when the wave is still interacting strongly with the wall (circles) compared with the exact solution (line). Parameters $\alpha = 10^{-3}$ and $K = 4 \times 10^{-6}$. The resolution $N = 300$ is low to clearly illustrate the internal boundary layer and the differences between the exact and penalized solution near the boundaries. Note the boundary layer in the penalized solid region for the velocity and the fact the height drops slightly inside the fluid due the smoothing of the porosity $\phi(x)$. The error in the velocity boundary condition is $0.03 \approx e^{1/2}$, as expected.
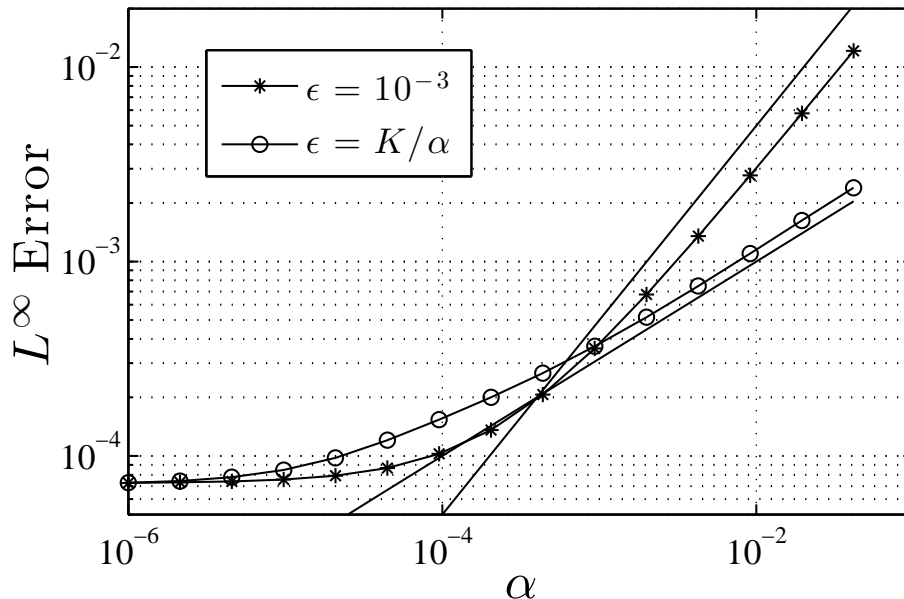
Figure 4. Control of L^∞ height penalization error by the porosity parameter α for the Gaussian wave test case compared with predicted scaling α (straight line) when $\epsilon = 10^{-3}$ is fixed, and scaling $\alpha^{1/2}$ when $\epsilon = K/\alpha$ as in the porous medium equations. The permeability is fixed at $K = (4\Delta x)^2$ and the resolution is $N = 2400$. Note that at this resolution the error of the second-order finite volume method saturates at 7.7×10^{-5}. Note that velocity results have exactly the same error as the height results.

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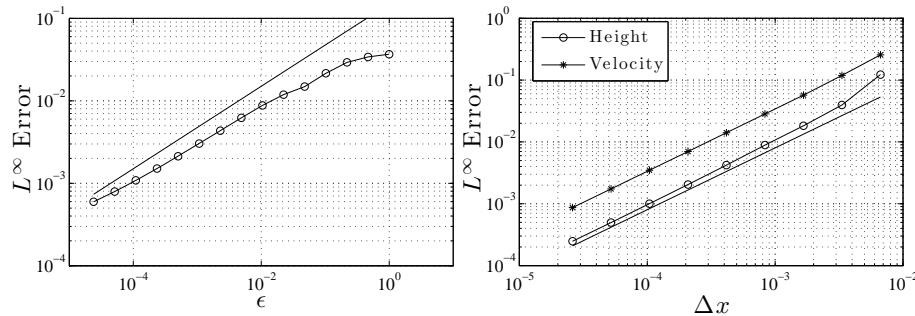
Figure 5. Left: control of L^∞ height penalization error by the permeability parameter ϵ for the Gaussian wave test case compared with predicted scaling $\epsilon^{1/2}$ (straight line). Right: convergence of L^∞ error with grid size Δx for the Gaussian wave test case compared with predicted first-order scaling (straight line). The porosity is fixed at $\alpha = 10^{-2}$. The resolution is $N = 2400$ for both results.
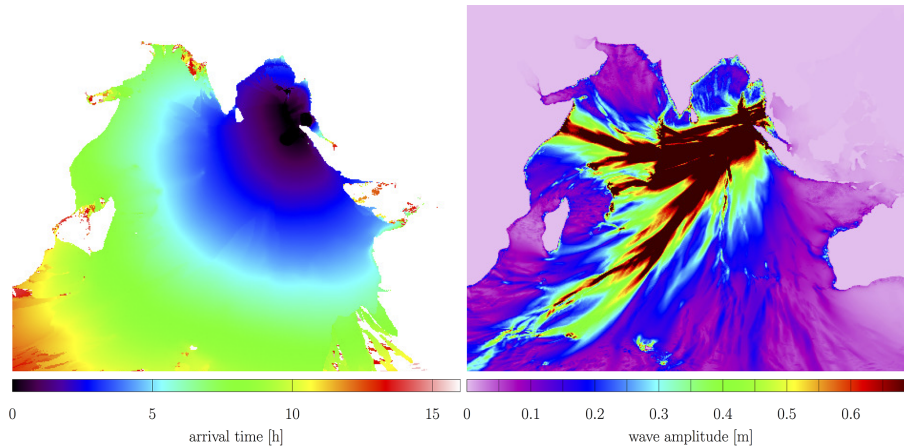
Figure 6. First arrival time (of a wave with height at least 5 cm) and maximum wave height for simulation of 2004 Indonesian tsunami.
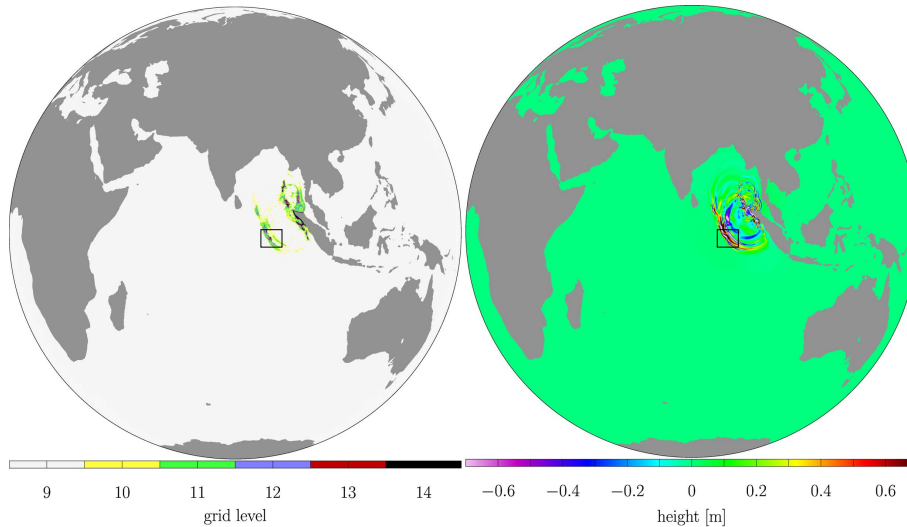
Figure 7. Tsunami after 70 min. The grid compression ratio is 930 and the finest $J = 14$ resolution is required only near the coasts where the tsunami has hit and very locally in the propagating wavefront. The black boxes indicate the zoomed regions shown in Fig. 8.

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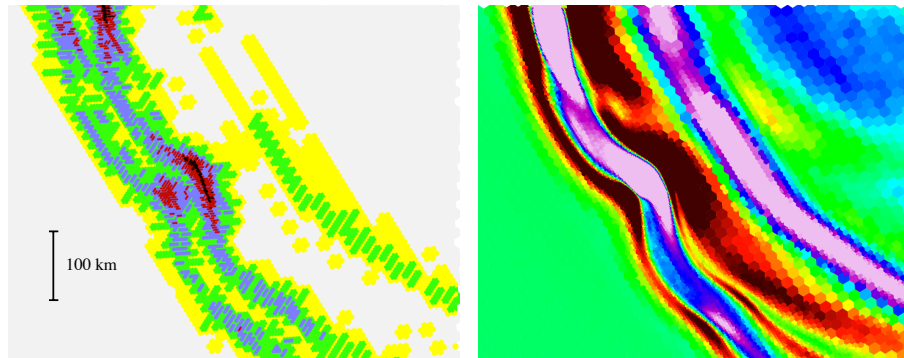
Figure 8. Tsunami: approximately 650 km × 550 km zoom of grid (left) and height (right) for results shsown in Fig. 7. Recall that in the left figure the black hexagons have size approximately 0.5 km.
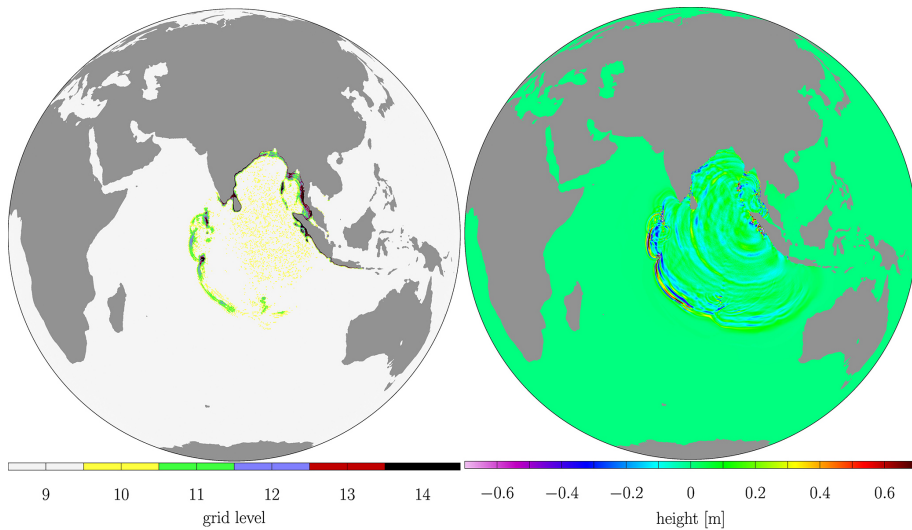
Figure 9. Tsunami: adaptive grid and wave height after 4 h. The grid compression ratio is 740.

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Figure 10. Tsunami: adaptive grid and wave height after 16 h. The grid compression ratio is 455.

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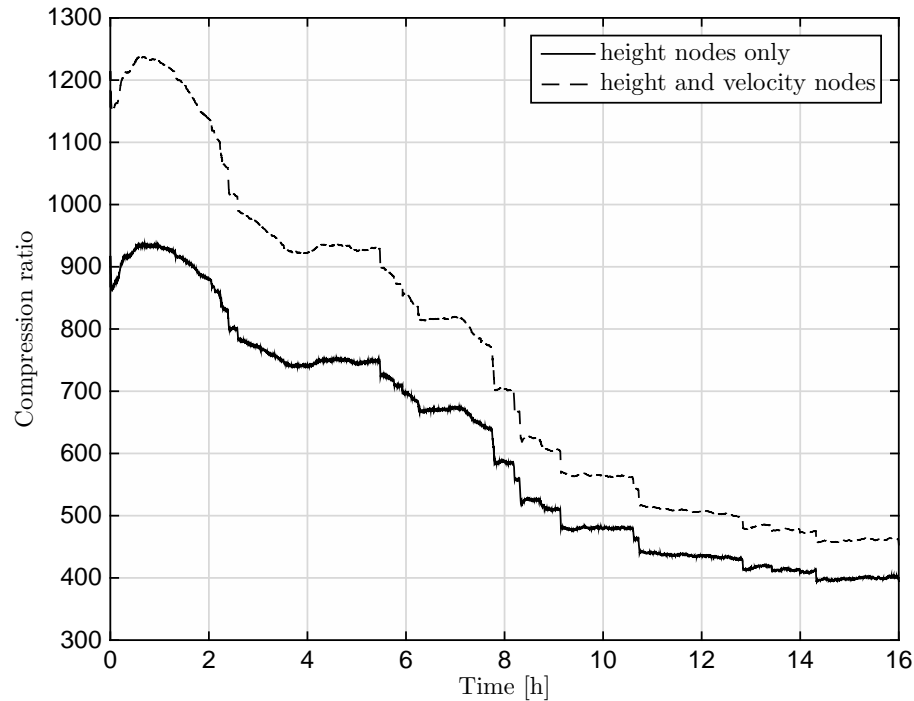
Figure 11. Grid compression ratio for tsunami simulation counting height nodes only and all degrees of freedom (i.e. height and velocity nodes).

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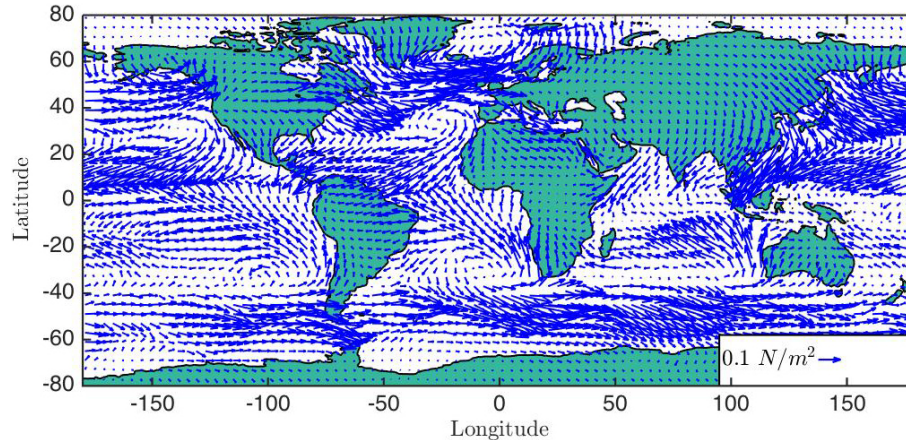
Figure 12. December wind stress field from Hellerman and Rosenstein (1980, 1983) used to force wind-driven ocean circulation shown in Fig. 13. Only every other wind stress data point is shown. The rms wind stress is $7.1592 \times 10^{-2}\,\mathrm{N\,m^{-2}}$.

Figure 13. Relative vorticity field for wind-driven ocean circulation after 301 days. Note vortex shedding from the boundary current off Argentina and in Drake's Passage.
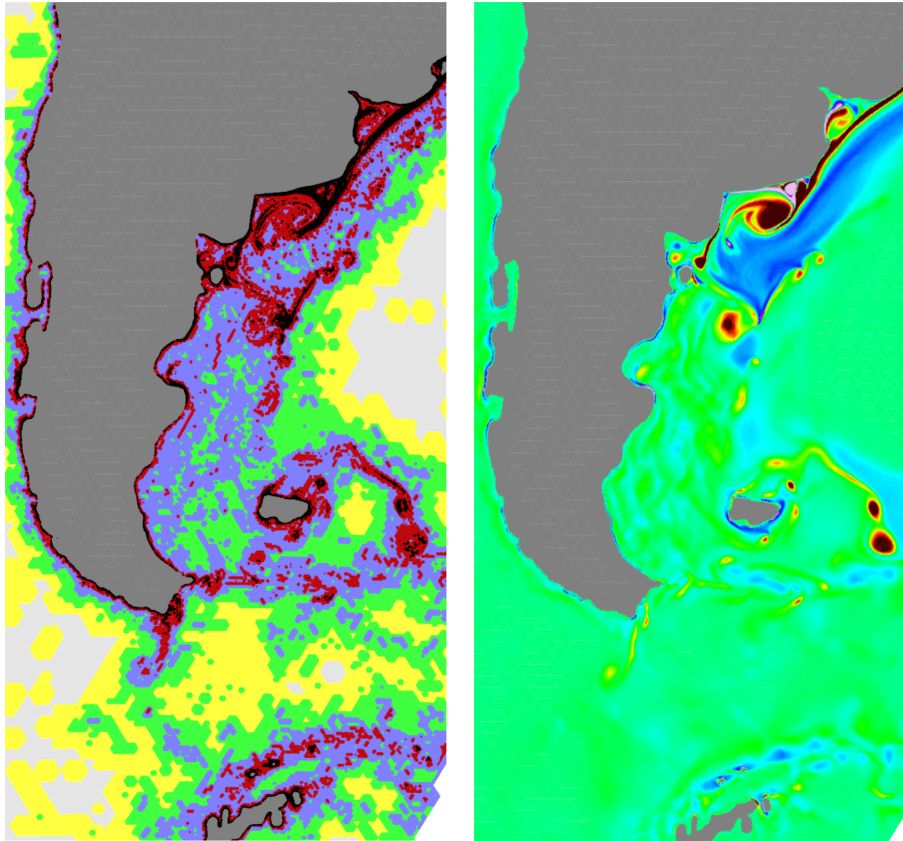
Figure 14. Zoom of vortex shedding dynamics off the southern coast of Argentina shown in Fig. 13: grid (left), relativity vorticity (right). The scales are as in Fig. 13. Note the complex boundary layer structure and vortices captured by the adaptive grid.

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