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The Explicit Wake Parametrisation V1.0: a wind farm parametrisation in the mesoscale model WRF

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Abstract

We describe the theoretical basis, implementation and validation of a new parametrisation that accounts for the effect of large offshore wind farms on the atmosphere and can be used in mesoscale and large-scale atmospheric models. This new parametrisation, referred to as the Explicit Wake Parametrisation (EWP), uses classical wake theory to describe the unresolved wake expansion. The EWP scheme is validated against filtered in situ measurements from two meteorological masts situated a few kilometres away from the Danish offshore wind farm Horns Rev I. The simulated velocity deficit in the wake of the wind farm compares well to that observed in the measurements and the velocity profile is qualitatively similar to that simulated with large eddy simulation models and from wind tunnel studies. At the same time, the validation process highlights the challenges in verifying such models with real observations.

1 Introduction

Wind turbines capture the kinetic energy of the wind with their turning blades, which transfer the energy to a transmission system that drives an electric generator. In this process the flow in front and behind a wind turbine is decelerated by the forces acting on the rotating blades. In large wind farms, the interaction of the flow and the wind turbines is further complicated by the interaction of the wake of one wind turbine with neighbouring turbines. Besides the changed velocity field around the turbines, there is also evidence that wind turbines affect planetary boundary layer (PBL) processes due to the changed turbulence (Baidya Roy et al., 2004; Baidya Roy and Traiteur, 2010; Barrie and Kirk-Davidoff, 2010; Wang and Prinn, 2010; Hasager et al., 2013).

Coastal regions in Northern Europe and China are expected to become major areas for wind energy production, since winds there are generally strong, steady, and less turbulent. To obtain an optimal yield, which is among others a function of power production, electrical cabling, and installation costs, it is often convenient to group wind

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farms together. Examples are the Danish Rødsand 2 and Nysted or the Belgian Belwind, Northwind, and Thornton wind farms, in which the wind farm separation is only a few kilometres. In planning new wind farms near existing ones it is important to estimate the velocity perturbation from the nearby farm as accurately as possible, because the power production is highly sensitive to the wind speed. Velocity deficits behind wind farms can be considerable. Christiansen and Hasager (2005), for example, found for near neutral atmospheric stability, velocity deficits of 2 % up-to 20 km downstream of the Horns Rev I (25 km²) wind farm. The accurate measurement of wakes from nearby farms becomes even more important in light of the fact that as of 2015 large offshore wind farms cover areas of up-to 100 km². Mesoscale models are suitable tools to estimate wind energy resources in these sea areas (Hahmann et al., 2014). However, the collective effect of the wind turbines to the flow needs to be included in these models and because these occur at scales smaller than the model's grid-size and remain unresolved, the effects have to be parametrised.

The parametrisation of the effect of wind turbines largely depends on the model and its mesh size. For instance Large Eddy Simulation (LES) models, while resolving the local flow around wind turbines, need to parametrise the local drag forces on the turbine blade (Porté-Agel et al., 2011; Wu and Porté-Agel, 2013). The grid spacing of mesoscale models, on the other hand, is on the order of kilometres and tens of metres in the horizontal and vertical direction, respectively. This means that the profile of the turbine-induced velocity deficit can be captured to some extent in the vertical direction. However, the downstream development of the velocity remains completely unresolved for scales smaller than the grid size. The major challenge in the parametrisation is to account for the unresolved relevant processes in agreement with the flow equations of the model. In recent years, steady progress has been made in the parametrisation of the effect of wind farms in global and mesoscale models: from the representation of wind farms by an increased roughness length in Keith et al. (2004) and Frandsen et al. (2009), up-to the more advanced drag approaches in Adams and Keith (2007); Blahak et al. (2010); Baidya Roy (2011); Jacobson and Archer (2012); Fitch et al. (2012), and

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Abkar and Porté-Agel (2015). Apart from a local drag force, an additional turbulence Kinetic Energy (TKE) source term is assumed in the schemes proposed by Adams and Keith (2007); Blahak et al. (2010); Fitch et al. (2012), and Abkar and Porté-Agel (2015).

In this article we develop a new approach, which is hereafter referred to as Explicit Wake Parametrisation (EWP). We define the TKE from random fluctuations around the ensemble-averaged velocity, instead of around the grid-cell averaged velocity as done in the previous parametrisations. Therefore, to be consistent with the flow equations of the model, we apply a grid-cell averaged drag force and additional TKE is only provided by the PBL scheme where there is an increased vertical shear in horizontal velocity compared to the free-stream velocity profile.

We implemented the EWP scheme in the open source Weather Research and Forecast (WRF) model (Skamarock et al., 2008). The WRF model already includes a wind farm parametrisation option, WRF-WF (Fitch et al., 2012; Jiménez et al., 2014). We validated the WRF-WF and EWP parametrisations against long-term meteorological (met) mast measurements in the wake of an offshore wind farm. To our knowledge, measurements in the wake of a wind farm have not been used for the validation of a wind farm parametrisation model in previous literature.

In Sect. 2, we explain the theoretical basis of the EWP scheme and its implementation in the WRF model. Section 3 describes the measurements, whereas Sect. 4 introduces the WRF model set-up and the configuration of the EWP and WRF-WF scheme. In Sect. 5 both wind farm parametrisations are validated against the met mast measurements in the wake of the wind farm. A discussion of the results finalises the article in Sect. 6, followed by the conclusions in Sect. 7.

2 The Explicit Wake Parametrisation

We start by introducing the relevant mesoscale model equations. Thereafter, the additional source terms that represent the effect of the the wind turbines are derived and

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added to the model equations. At the end of the section their implementation in the mesoscale model is described.

2.1 The mesoscale model framework

Wind turbines are well described by drag devices that slow down the wind velocity from a free-stream value U . Downstream, due to mixing of fluid particles from within and outside the wake, the velocity deficit is gradually reduced to the point at which the background conditions are restored.

We use a mesoscale model for the simulation of the wind farm wake and its recovery. It uses the Reynolds Averaged Navier–Stokes (RANS) equations,

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial \overline{U_i U_j}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} - \varepsilon_{ijk} f U_k - \delta_{i3} g + F_{D_i}, \quad (1)$$

to describe the flow evolution. We will use the notation and symbols of Wyngaard (2010). The upper-case letters refer to ensemble-averaged quantities, whereas lower-case letters refer to fluctuations. The exception is the average air density $\rho(\mathbf{x}, t)$, where t denotes the time and \mathbf{x} the position. In Eq. (1), $U_i(\mathbf{x}, t)$ and $P(\mathbf{x}, t)$ represent the mean velocity components and the pressure, whereas f and g are the Coriolis frequency and the gravitational acceleration constant. Furthermore, the right most term $F_{D_i}(\mathbf{x}, t)$ is the ensemble-averaged horizontal forcing due to the action of wind turbines ($F_{D_3} = 0$).

Mesoscale models generally simulate the effects of turbulence in the vertical direction only. The components of the Reynolds stress are parametrised in a 1.5-order PBL scheme as

$$\overline{U_i U_3} = -K_m \frac{\partial U_i}{\partial x_3}, \quad (2)$$

where the turbulence diffusion coefficient for momentum, $K_m(\mathbf{x}, t) = S_m \ell \sqrt{2e}$, depends on the stability function $S_m(\mathbf{x}, t)$, the turbulence length scale $\ell(\mathbf{x}, t)$, and the TKE per

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unit mass $e = \overline{U_i U_i}/2$. We write the most general form of the TKE equation:

$$\frac{\partial E}{\partial t} + T = P_s + P_b + P_t - \epsilon, \quad (3)$$

where on the left hand side (l.h.s.) E denotes the TKE and T the transport, which includes the advection by the mean flow, turbulence transport of TKE and the divergence of the pressure correlation. On the right hand side (r.h.s.), P_s represents the turbulence production from the vertical shear in the horizontal velocity (shear production), P_b the turbulence production or destruction related to buoyancy forces, P_t the turbulence induced by the turbine rotor, and ϵ the dissipation.

2.2 The mesoscale model grid

The previously defined variables in the Eqs. (1) and (3) have to be redefined on the three-dimensional model grid. A new volume-averaged velocity equation is derived by integrating Eq. (1) over the grid-cell volume. For Eq. (3), we define a (random) velocity fluctuation at any hypothetical measurement point within a grid-cell volume to be the difference between the ensemble-averaged and the instantaneous velocity. This is illustrated schematically in Fig. 1a. Then, the grid-cell averaged TKE can be thought of as the variance of the velocity fluctuations around the ensemble-averaged velocity and not around the grid-cell averaged velocity as shown in Fig. 1b.

The aim is to obtain expressions for the terms P_t (Eq. 3) and F_D (Eq. 1) that are consistent with the mesoscale model flow equations. The expression for P_t can be found by multiplying the Navier–Stokes equations with the velocity fluctuation and then applying Reynolds averaging. This gives for the additional source term:

$$P_t = \overline{\rho A_r c_T (U_i + u_i)^2 u_i / 2} \sim \rho A_r c_T \overline{U u_i^2}, \quad (4)$$

where $\rho A_r c_T (U_i + u_i)^2$ is the instantaneous forcing due to the action of the wind turbine, the lower case c_T the instantaneous thrust coefficient, the upper case C_T the

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averaged thrust coefficient, and A_r the rotor area. This additional turbulence occurs directly behind the turbine blades. Its turbulence length scale ℓ is expected to scale with the blade's cord length, which is on the order of a few metres and it is therefore much smaller than that of the atmospheric flow. The smaller length scale implies a significant higher dissipation rate $\epsilon \propto (1/\ell) \overline{u_i u_i}^{3/2}$. Therefore, we assume the source term in Eq. (4) to dissipate within a mesoscale model grid-cell and neglect P_1 of Eq. (4) on the grid-cell average. Additional turbulence is generated by shear production, which we assume to be the dominant mechanism on the grid-cell average. In the following sections, we derive the expression for the grid-cell averaged F_D .

2.2.1 The sub-grid wake expansion

The velocity deficit expansion within one grid-cell is not negligible, which leads to flow decelerations that extend beyond the rotor swept area. This part of the wake expansion is not accounted for in the mesoscale model, hence we estimate it explicitly with a sub-grid scale (turbulence) diffusion equation. Then, a grid-cell averaged force is determined and added to the model RANS equations.

The sub-grid-scale model describes the unresolved turbulence diffusion process that results from turbulence shear production. In this model, we assume the horizontal advection of velocity and the turbulence diffusion to dominate in Eq. (1). Considering first only the flow behind the turbine rotor, we obtain from Eqs. (1) and (2) the diffusion equation:

$$U_0 \frac{\partial}{\partial x} (U_0 - \hat{U}) = K \frac{\partial^2}{\partial z^2} (U_0 - \hat{U}) + K \frac{\partial^2}{\partial y^2} (U_0 - \hat{U}) \quad (5)$$

that describes the expansion of the velocity deficit, $U_0 - \hat{U}$, behind the turbine. Here, we denote the advection velocity at hub-height h by $U_0 = |U(h, t)|$ and the unresolved velocity in the stream-wise direction x in the wake of the turbine by $\hat{U}(x)$. The tur-

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lence diffusion, which causes the wake expansion, is described by a single turbulence diffusion coefficient $K = K_m(h, t)$.

2.2.2 The velocity deficit profile

We define the vertical structure of the velocity deficit as $U_d = U_s \xi$, where $U_s(x)$ is the maximum velocity deficit at the centre of the wake and $\xi(x, y, z)$ a function that determines the vertical wake extension (Tennekes and Lumley, 1972). Equation (5) can be solved for the velocity deficit profile:

$$U_d = U_s \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma} \right)^2 - \frac{1}{2} \left(\frac{y}{\sigma} \right)^2 \right], \quad (6)$$

where the length scale that determines the vertical wake extension is

$$\sigma^2 = \frac{2K}{U_0} x + \sigma_0^2. \quad (7)$$

In Eq. (7), σ_0 denotes the initial length scale. Equation (7) represents the vertical wake extension, resulting from turbulent diffusion of momentum and it is similar to the solution of Eq. (4.29) in Wyngaard (2010) for the dispersion of plumes. Equation (6) describes the ensemble-averaged profile of the velocity deficit around hub-height at a given point in the far turbine wake (Tennekes and Lumley, 1972). The turbine wake is normally divided in a near and far wake, where the far wake begins between one rotor diameter D_0 (Vermeer et al., 2003) and $3D_0$ (Crespo and Hernández, 1996).

We can find the velocity deficit profile for wind turbines by equating the total thrust to the momentum removed by the action of the wind turbine, i.e.,

$$\frac{1}{2} \rho C_T \pi R_0^2 U_0^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho U_0 U_d dz dy = \rho U_0 U_s 2\pi \sigma^2, \quad (8)$$

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where $C_T(U_0)$ is the turbine thrust coefficient and R_0 the radius of the rotor. In Eq. (8), the l.h.s. represents the local forces at the rotor swept area, whereas the r.h.s. describes the equivalent distributed force for the expanded wake at any x . From the Eqs. (8) and (6), we find the velocity deficit profile for a specific thrust force,

$$U_d = \frac{C_T R_0^2 U_0}{4 \sigma^2} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma} \right)^2 - \frac{1}{2} \left(\frac{y}{\sigma} \right)^2 \right]. \quad (9)$$

When we insert the velocity deficit of Eq. (9) in the second term of Eq. (8) and integrate in the cross-stream direction y , we have the integrated thrust profile

$$F_d = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho U_0 U_d dz dy = \int_{-\infty}^{\infty} \rho \sqrt{\frac{\pi}{8}} \frac{C_T R_0^2 U_0^2}{\sigma} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma} \right)^2 \right] dz. \quad (10)$$

The term on the r.h.s. within the integral describes the equivalent distributed thrust force in the vertical direction at any distance x . Next, we derive from Eq. (10) a single effective thrust force, which represents the average wake expansion within a grid-cell.

2.2.3 Turbine forcing in the mesoscale model

For the mesoscale model we derive an effective thrust force, which describes the average wake expansion within a grid-cell. Therefore, we first determine the effective velocity deficit profile U_e by averaging the velocity deficit of Eq. (9) over the cross-stream direction y and over a downstream distance L that the wake travelled within the grid-cell. It is convenient to approximate this area-averaged velocity deficit profile by a Gaussian-shaped profile:

$$U_e = \sqrt{\frac{\pi}{8}} \frac{C_T R_0^2 U_0^2}{\sigma_e} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma_e} \right)^2 \right] \cong \frac{1}{L} \int_0^L \int_{-\infty}^{\infty} U_d dy dx, \quad (11)$$

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see Appendix. Here σ_e is the effective length scale that is related to the model grid-size,

$$\sigma_e = \frac{1}{L} \int_0^L \sigma dx = \frac{U_0}{3KL} \left[\left(\frac{2K}{U_0} L + \sigma_0^2 \right)^{\frac{3}{2}} - \sigma_0^3 \right]. \quad (12)$$

From the definition of the effective length scale, we can obtain the total effective thrust force F_e by substituting the length scale σ in Eq. (10), with the effective length scale σ_e .

This gives

$$F_e = \int_{-\infty}^{\infty} \rho \sqrt{\frac{\pi}{8}} \frac{C_T R_0^2 U_0^2}{\sigma_e} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma_e} \right)^2 \right] dz. \quad (13)$$

The grid-cell averaged acceleration for the model RANS equations, Eq. (1), is now obtained, when we divide Eq. (13) by the mass and apply the grid-cell volume. For every vertical model layer k , we obtain then from Eq. (13) the grid-cell averaged acceleration components,

$$\langle F_{D_1}(k) \rangle = -\sqrt{\frac{\pi}{8}} \frac{C_T R_0^2 U_0^2}{\Delta x \Delta y \sigma_e} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma_e} \right)^2 \right] \cos[\varphi(k)] \quad (14)$$

and

$$\langle F_{D_2}(k) \rangle = -\sqrt{\frac{\pi}{8}} \frac{C_T R_0^2 U_0^2}{\Delta x \Delta y \sigma_e} \exp \left[-\frac{1}{2} \left(\frac{z-h}{\sigma_e} \right)^2 \right] \sin[\varphi(k)] \quad (15)$$

in the x and y direction, respectively. In Eqs. (14) and (15), we use Δx and Δy for the horizontal grid-spacing in the x and y direction, $\varphi(k)$ for the wind direction, and the angular brackets for the spatial averaging. For the height $z(k)$, we use the height at the centre of layer k .

2.3 Implementation in the WRF model

We assume every turbine within a grid-cell to be positioned at its centre and use $L = \Delta x/2$ in Eq. (12). For each turbine a thrust force is calculated with Eqs. (14) and (15). The total thrust force for a turbine containing grid-cell is then obtained from a superposition of the single turbine thrust forces and is added to the numerical approximation of Eq. (1).

We use the grid-cell averaged velocity as the upstream velocity U_0 to the wind turbine (see Sect. 6). The turbulence diffusion coefficient for momentum, K in Eq. (12), comes from the selected PBL scheme. The parametrisation, therefore, can be used with any PBL scheme. However, to be consistent with the assumptions used in the derivation of the wind farm parametrisation, a second order closure scheme with a turbulence shear production term is recommended. A practical description of how to use the EWP scheme in the WRF model is given in Sect. .

3 Wind farm and measurements

3.1 The Horns Rev I wind farm and met masts

The parallelogram-shaped Horns Rev I wind farm is situated in the North Sea about 15 km West of the Danish coast (Fig. 2). It is made up of 80 Vestas V80 (2 MW) pitch controlled, variable speed turbines, resulting in a total rated wind farm capacity of 160 MW. The turbines have a swept area diameter of 80 m with the hub mounted at 70 m a.m.s.l. The equally-spaced turbines are placed in 10 columns from West to East and 8 rows from North to South, labelled C1–C10 and R1–R8 in Fig. 2b. The spacing between columns and rows is 560 m, which is equivalent to 7 turbine diameters. Hansen et al. (2012) describes in detail the various production data available at the wind farm, their quality filtering, and processing.

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Turbine (C1,R7) is used as a reference turbine, since it is not affected by the wind farm wake under westerly flows; in Fig. 2b it is marked with the solid bullet. The wind speed for each turbine is estimated from the power production data, using the turbine specific power curve and the wind direction is obtained from the adjusted yaw angle (Hansen et al., 2012).

For the validation, we use data from two met masts, M6 and M7, whose positions are shown in Fig. 2b. The masts are located 2 and 6 km to the east of the eastern edge of the wind farm and are thus directly in its wake for westerly winds. The 70 m tall masts are identically equipped and their instrumentation includes high quality Risø cup anemometers for measuring wind speeds. The data from the two masts, which are independent of the wind farm data, is used to validate the modelled wind speed reductions downstream from the wind farm.

3.2 Data selection and averaging for the validation

For the validation of the results of the model simulations, and especially because of the relatively large area-averaged fields in the mesoscale model, it is very important to ensure that measurements and model fields are comparable. This can be achieved by properly selecting the wind direction and wind speed interval at the reference turbine.

Regarding the wind direction selection, Gaumond et al. (2013) found at Horns Rev I, for narrow wind direction sectors, a higher measured turbine production compared to that estimated by wake models. Most likely this is related to wind direction variations, which, for small wind direction bins, exceed the bin size within the 10 min averaging period. Since these variations are not accounted for in the model, we follow the recommendation to average over a relatively large wind direction interval. Furthermore, it is important that the flow reaching the mast anemometers has passed through the wind farm, otherwise it does not characterise the wind farm wake. Therefore, we select velocities whose directions are at the reference turbine between 255 and 285°. As demonstrated in Fig. 2b, the flow from this sector is influenced by the wind farm wake.

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In the period 2005 to 2009, we select wind speeds from 6.5 to 11.5 m s^{-1} at the reference turbine and bin them in 1 m s^{-1} intervals. In this range, we guarantee to be above the cut-in wind speed of the turbines (4 m s^{-1}) and below the wind speed at which the control system starts to pitch the blades around (13 m s^{-1}). To obtain as much data as possible, we do not filter the measurements on stability. For strong westerly winds,

we expect the lower atmospheric stability on average to be neutral (Sathe et al., 2011). For every instance that the wind direction and wind speed at the reference turbine was within the above described range, the wind speeds at M6 and M7 were accepted. Afterwards, the selected wind speeds were averaged for each wind speed bin and normalised by the average wind speed at the reference turbine \bar{U}_{ref} (Table 1).

4 WRF model configuration and averaging

In the simulations we used the WRF model V3.4. However, the WRF-WF parametrisation has been updated to the version available in WRFV3.6. The model domain is set-up with 80×40 grid-cells, with a horizontal grid-spacing $\Delta x = \Delta y = 1120 \text{ m}$. Equally to the Horns Rev I wind farm, the model wind farm contains 80 V80 turbines and extends over 5 grid-cells in the West–East and North–South direction (Fig. 3a). The Vestas V80 thrust and power curves were used for the turbine parameters.

The model was run in an idealised case mode with open lateral boundary conditions (Skamarock et al., 2008, p. 51), Coriolis forcing and zero heat fluxes from the lower boundary. At the surface, the no slip condition holds. The surface roughness was constant in time and set to $z_0 = 2 \times 10^{-4} \text{ m}$ for the entire domain. The model simulations were run with the MYNN 1.5 order Level 2.5 PBL scheme (Nakanishi and Niino, 2009), on which the WRF-WF scheme is dependent. These and other details of the model configuration are summarised in Table 2.

We ran simulations for 5 wind speed intervals and for 9 wind directions, ranging from 255 to 285° . Additionally, to investigate a sensitivity to the vertical resolution, we set-up three experiments with 40 (L40), 60 (L60), and 80 (L80) layers in the vertical direction.

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With these vertical resolutions there are 5, 7, and 10 grid-cell volumes intersecting with the turbine rotor as shown in Fig. 3b.

All 135 simulations (5 wind speeds, 9 wind directions, and 3 vertical resolutions) were initialised with a constant geostrophic wind in height in a dry and slightly stable atmosphere. After a four day integration period, the wind converged in the whole domain to a logarithmic neutral profile within a 650 m deep boundary layer and remained independent of height above the inversion layer. The atmospheric state of each of these 135 simulations was used to drive: a control simulation without wind farm parametrisation, a simulation with the WRF-WF scheme, and a simulation with the EWP scheme. Each control or wind farm simulation lasts one day, resulting in a total simulation length of five days. The wind speeds in the control simulations were 7, 8, 9, 10, and 11 m s^{-1} at 70 m (hub-height) after 5 days of simulation.

To summarise, we performed simulations for 9 wind directions, 5 wind speeds, and 3 vertical resolutions, with and without wind farm parametrisation, which gives a total of 405 (3 times 135) simulations as outlined in Table 3. For the validation against the mast measurements, we averaged the model wind speeds over the 9 wind directions. We used the instantaneous wind speeds at the end of the simulation period. Similarly to the observations, we normalise the wind direction averaged wind speeds and hereafter simply refer to them as “velocity”. The model wind speeds from the simulations without the wind farm were used to normalise the wind farm flow.

4.1 Wind farm parameters

4.1.1 The EWP scheme

We use the wind speeds, derived from the power production data of the turbines, to determine the initial length scale σ_0 in Eq. (12). To a first approximation the initial length scale is defined to be independent of the upstream conditions and it is therefore the same for all turbines.

For this calibration, we selected turbine production data at time stamps when the derived upstream wind speed and wind direction at the reference turbine ranged from 8.5 to 9.5 m s⁻¹ and from 255 to 285°. This wind speed bin was selected, since it contains the most turbine observations. It has a minimum of 850 observations for a turbine at the eastern edge of the wind farm and a maximum of 1612 observations for a turbine in the first wind farm column. The difference in the selected number of observations results from the additional requirement that for a given turbine the local upstream turbines have to be operational, in order to guarantee wake losses. For the power production data obtained in this way, we used the power curve to derive the wind speed. The column averaged velocity was afterwards obtained by averaging over the inner 6 rows (R2–R7). The outer rows were excluded because they experience different wake conditions under various wind directions.

Similarly to the model experiments in Sect. 4, we performed simulations for 9 wind directions between 255 to 285° for the 3 vertical resolutions with a wind speed at hub-height of 9 m s⁻¹. Additionally, we varied for the EWP simulations the initial length scale for all turbines from $\sigma_0 = R_0$ to $\sigma_0 = 2R_0$, stepping with $\Delta R_0 = 0.1$, to determine the best fitting value. For the comparison to the measurements, the column-averaged wind speed was obtained by averaging over the 3 central wind farm grid-cells in the cross-stream direction.

Figure 4 shows the results from the simulations with $\sigma_0 = 1.7R_0$, which had the smallest overall bias compared to the measurements. It shows the same velocity reduction for all 3 vertical resolutions. Consequently, the amount of energy extracted by the turbines is independent of the vertical resolution. The simulations with an initial length scale $\sigma_0 = 1.7R_0$ follow the measured velocity reduction fairly well. Therefore, we conclude that for neutral conditions the initial length scale can be assumed to be independent of the upstream conditions. We use this length scale for all simulations in Sect. 5.

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4.1.2 The WRF-WF scheme

In the validation, we also include results from the wind farm parametrisation WRF-WF from WRF-V3.6. This parametrisation was introduced in WRFV3.4 by Fitch et al. (2012). In this approach, a forcing term F_D is applied to every model level that intersects with the wind turbine blade sweep area and thus the scheme simulates the local drag forces over the turbine rotor. An additional TKE source term is parametrised as

$$P_{t,WRF-WF} = \rho A_r (C_T - C_P) U^3 / 2, \quad (16)$$

here C_T and C_P are the thrust and power coefficients of the turbine, whereas U is the grid-cell velocity. The application of one-dimensional theory (Hansen, 2003) on Eq. (16), gives $P_{t,WRF-WF} = \rho A_r C_T a U^3 / 2$ for the additional TKE term. Here, a denotes the induction factor.

The same result, $P_{t,WRF-WF} = \rho A_r C_T a U^3 / 2$, can also be obtained by defining a velocity fluctuation as the difference between the grid-cell averaged velocity and the instantaneous velocity as illustrated in Fig. 1b. This has been analytically derived by Abkar and Porté-Agel (2015) (their Eq. 21, with their $\xi = 1$). In this definition of TKE, the turbine induced velocity reduction is counted as TKE. The TKE source term from the WRF-WF scheme in Eq. (16) is much larger than the source term in Eq. (4), which comes from the different definition of TKE in the two schemes.

In WRFV3.4, the WRF-WF parametrisation the power coefficient has been obtained from the power curve information. The thrust coefficient was then determined by an empirical relationship with the power coefficient. Jiménez et al. (2014) improved the performance of the WRF-WF scheme in a comparison against turbine measurements of the Horns Rev I wind farm by using both a turbine thrust and power coefficient. This approach is used in the experiments performed in Sect. 5.

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5 Validation of the wind farm parametrisations

To obtain a complete picture of the modelled velocity field within and downstream of the Horns Rev I wind farm, we compare the hub-height velocity simulated by the WRF model using the two wind farm parametrisations for one wind speed bin. We used the second most frequently observed wind speed bin (10 m s^{-1}), which is different from that used in the calibration (Sect. 4.1.1), to be as calibration independent as possible. Afterwards, we compare the modelled velocities for all wind speed to the measurements at M6 (2 km downstream) and M7 (6 km downstream). In a qualitative validation, we examine the simulated wind farm wake characteristics, as well as the vertical profile of the wake deficits, using results from LES simulations and wind tunnel experiments as a reference.

In the validation, we use the instantaneous model outputs after the 5 days integration period. Furthermore, the velocities are normalised as described in the Sects. 3 and 4.

5.1 Velocity recovery at turbine hub-height

Figure 5 shows the wind direction averaged hub-height velocity (10 m s^{-1} bin) as a function of the downstream distance for the EWP and WRF-WF schemes and all vertical resolutions. For the EWP scheme (Fig. 5a) there is no identifiable vertical resolution dependency on the velocity, while for the WRF-WF (Fig. 5b) this is small. In the EWP scheme the velocity within the wind farm decreases almost linearly with distance, whereas for the WRF-WF scheme, after a more rapid initial decrease, the velocity becomes nearly constant with distance at the end of the wind farm. The behaviour in the EWP scheme suggests that no equilibrium has been reached within the wind farm between the extracted momentum by the wind turbines and the compensating flux of momentum from above. On the other hand, from the nearly constant velocity at the end of the wind farm for the WRF-WF scheme, it seems that the extraction of momentum by the turbines is almost balanced by the flux of momentum from aloft. At the end of the wind farm the velocity difference between the two schemes is only 0.1 %. We have

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indicated the velocity from the EWP scheme at the most easterly grid-cell of the wind farm with a dotted horizontal line in Fig. 5. This agreement is noteworthy, given the two different methods used. However, the velocity 2 km downstream of the wind farm, at M6, differs between the schemes by 4.7 %. This means that downstream of the wind farm grid-cells, where the wind farm parametrisations are not active, the velocity diverges for the two schemes and the difference becomes significant. This is important, especially if it was used to estimate the power production on a neighbouring wind farm that was located at this distance from the original wind farm. For example, the Rødsand 2 and Nysted offshore wind farm in Southern Denmark are separated by a comparable distance.

Figure 6 depicts wind direction averaged velocity recovery at the two met masts for all 5 wind speed bins and all vertical resolutions. It shows the differences in the WRF-modelled and measured wind speed at M6 (circles) and M7 (triangles), against the measured wind speed at the same masts. The WRF-modelled wind speed is obtained from linear interpolation of the wind speeds in the nearest grid-cells. The modelled recovery rate can be deduced from the slopes of the solid (between the values at M6 and M7) and dashed lines (between M7 and the free-stream velocity). A negative slope is linked to a slower modelled recovery compared to that measured, whereas a positive slope shows a faster modelled recovery. There is no vertical resolution dependency for a wind farm parametrisation when the circles or triangles for a given wind speed bin lie on top of each other.

The results show that the bias in velocity between the measurements and the EWP simulations is small ($< 0.15 \text{ m s}^{-1}$) for all wind speeds (except for the 7 m s^{-1} bin at M7). We find a positive velocity difference at M6 and a negative one at M7. Hence, the modelled wake recovery oscillates between being slightly slower from M6 to M7 and being slightly faster from the end of the wind farm to M6, as well as from M7 to the end of the wake. The velocity of the EWP scheme at M6 is nearly resolution independent and at M7 the dependency is very weak.

The WRF-WF scheme shows a positive bias in velocity of up-to 0.5 m s^{-1} at M6. This positive bias, between the WRF-WF scheme and the measurements, becomes larger with increasing wind speed. The bias at M6 is a consequence of the too fast modelled wake recovery from the end of the wind farm to M6. Between M6 and the point at which the free-stream velocity is reached again, the modelled wake recovery is slower than that measured. This overall positive bias implies that the modelled velocity with the WRF-WF scheme is overestimated throughout the whole wake.

Figure 7 shows the spatial structure of the modelled velocity (10 m s^{-1} bin) within and in the wake of the wind farm for the L60 simulations in the 270° wind direction. A comparison of the results from the two schemes confirms the faster initial wind farm wake recovery in the WRF-WF scheme. The 10 % velocity deficit contour, for example, extends for the EWP scheme to around $x = 15 \text{ km}$ (Fig. 7a), while for the WRF-WF scheme to $x = 8 \text{ km}$ (Fig. 7b). The difference in the distance at which a 7.5 % velocity deficit is reached is even larger: after $x = 21$ and $x = 11 \text{ km}$ for the EWP and WRF-WF scheme, respectively. Further downstream, after around 30 km , the velocity fields from the two parametrisations become similar. Possible reasons for the difference in the initial wake recovery are discussed in the next section.

Finally, Fig. 7 shows a difference in the orientation of the axis of the velocity deficit downstream from the wind farm. In both simulations the steady-state wind direction of the free-stream flow was 270° . As the velocity decreases within the wind farm, the velocity is expected to turn to the North, due to the changed Coriolis force. In the wind farm wake, where the flow accelerates again, the velocity should turn back again to the background direction. This effect is seen for the EWP scheme (Fig. 7a). Whereas, for the WRF-WF scheme (Fig. 7b) the wake turns southward behind the wind farm. A possible reason for this unexpected behaviour is the turbulence transport of momentum from aloft (Ekman spiral) within the wind farm. In the wind farm wake the flow keeps turning to the South because of the flow acceleration from the momentum transport.

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5.2 Vertical profile of TKE and velocity

To obtain a broader understanding of the mechanisms acting in the two schemes, we compare the simulated TKE (per unit mass) and the velocity deficit profiles for the 10 m s^{-1} wind speed bin in the 270° wind direction.

Figure 8 shows the difference in TKE (wind farm minus control simulation) for the L60 simulation. The cross-sections in the x, z plane are in the West–East direction and pass through the centre of the wind farm. We used different colour scales in the two plots, due to the relatively large differences in TKE production between the two schemes. However, we have kept the outermost contour ($0.02 \text{ m}^2 \text{ s}^{-2}$) the same. The maximum TKE difference was 0.30 and $1.9 \text{ m}^2 \text{ s}^{-2}$ for the EWP and WRF-WF scheme, respectively.

Compared to the environmental TKE of the pure shear flow (not shown), the TKE increases in the EWP scheme (Fig. 8a) at the end of the wind farm by a factor of 2, whereas in the WRF-WF scheme (Fig. 8b) it increases at hub-height by a factor of 5.5. The EWP scheme shows an increased and decreased TKE above and below hub-height compared to the reference simulation. The maximum increase occurs behind the wind farm, where the velocity gradients are the largest. In the WRF-WF scheme, the maximum TKE increase happens at hub-height within the wind farm. Recalling that in the WRF-WF scheme, turbulence is generated by the source term P_t and by the PBL scheme from turbulence shear production (P_s), the intensity of P_t dominates over that of the shear production. The sum of the additional source term and the turbulence shear production, causes within the wind farms an increased turbulence from the lowest model level upwards.

We use the results from the actuator disk approach without rotation from Wu and Porté-Agel (2013), to obtain a qualitative impression of the structure of the turbulence field from a LES model within a wind farm. The actuator disk approach from their LES model is most similar to the drag approach in the mesoscale model. Their Fig. 5c shows that higher and lower turbulence intensities dominate around the upper and lower tur-

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bine blade tip. Also, a positive and negative shear stress occur above and below hub-height (their Fig. 7c). This indicates that the shear in velocity dominates the turbulence production. Similar features are present in the TKE field from the EWP scheme, where the increased and decreased TKE above and below hub-height are in like manner caused by an enhanced and reduced turbulence shear production with respect to the neutral logarithmic velocity profile. Furthermore, Wu and Porté-Agel (2013) shows an upper wake edge at around 4.5 turbine hub-heights for 10 aligned wind turbines after 60 turbine diameters (their Fig. 12). They defined the wake edge at the point where the velocity reduction for a given height was 1 %. Similarly, the edge of the wake can be found from an increased TKE due to shear production. For the outermost contour in Fig. 8, we obtain a vertical wake extension of around 5 turbine diameters for the EWP scheme at 4.8 km downstream (equivalent to 60 turbine diameters). At the same distance, it is around 7 turbine hub-heights for the WRF-WF scheme.

The influence of the TKE field to the velocity profile is analysed in Fig. 9. It shows the velocity deficit profile $\Delta U_x / U_{0h}$ for both schemes and all vertical resolutions. Here U_{0h} denotes the free-stream velocity at hub-height. The velocity deficit is defined as $\Delta U_x(z) = U(z) - U_{free}(z)$, where $U_{free}(z)$ is the free-stream velocity profile from the reference simulation and $U(z)$ the velocity profile from the wind farm simulation. We choose the second and third grid-cell within the wind farm (C2 and C3) and the second point behind the wind farm (C7) that corresponds approximately to the location of M6.

Figure 9a shows the velocity deficit profiles from the EWP and WRF-WF scheme from the L60 simulation. The profiles indicate a stronger diffusion in the WRF-WF scheme compared to that in the EWP scheme. This can be recognised by the vertical extension of the vertical deficit profile at C7, behind the wind farm.

For the EWP scheme (Fig 9b), we find a maximum velocity deficit at hub-height and symmetric features around the maximum for the L40 and L80 simulations. Also, results from LES simulations, wind tunnel experiments, and measurements (Vermeer et al., 2003; Wu and Porté-Agel, 2013; Iungo et al., 2013) have a maximum velocity deficit at hub-height in the far turbine wake for neutral conditions. The profiles in Vermeer et al.

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(2003) (their Fig. 36) and in Wu and Porté-Agel (2013) (their Fig. 4) additionally show symmetric features around the maximum with a shape similar to the velocity profile of the EWP scheme. Figure 9b demonstrates the vertical resolution independence of the EWP scheme within the wind farm: the velocity deficits of the L40 and L80 simulations are almost identical. A small difference is found below hub-height in the wind farm wake.

With the WRF-WF scheme (Fig. 9c) the maximum velocity deficit is displaced vertically above hub-height, which reaches the upper wind turbine blade tip at mast M6 (C7). The dependency of WRF-WF scheme to the chosen vertical resolutions is weak; differences are found within the wind farm (C2, C3) above hub-height. Also, the profiles from the WRF-WF scheme show increased (area-averaged) velocities inside the wind farm at the lowest model level. The wind farm simulations from Wu and Porté-Agel (2013) (their Fig. 13) do not support this behaviour.

6 Discussion

We use wind farm parametrisations implemented in a mesoscale models to simulate the effect of wind farm wakes in areas on the order of hundreds of kilometres. However, the models have a limited horizontal resolution and hence the local processes within a wind farm remain unresolved. Wu and Porté-Agel (2013) found in their LES model results a sensitivity of the velocity reduction to the wind farm layout. In current implementations of wind farm parametrisations, all turbines within a grid-cell experience the same upstream velocity. Although these parametrisations are not meant to estimate the local velocity field within the wind farm, differences in velocity reduction within the wind farm could affect the velocity in the wake of it.

In future implementations it can be possible to account for the local flow within the wind farm by using data from high resolution models, which can be input to the mesoscale model via a look-up table as suggested by Badger et al. (2013) and Abkar and Porté-Agel (2015). However, we have at the moment no measurement data in the

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wind farm wake to validate these approaches for different wind farm layouts. At the Horns Rev I wind farm, we were restricted to the geometry of met masts positions, which did not allow to study the sensitivity of the velocity field in the wind farm wake for additional wind direction sectors.

5 A fair comparison between the mesoscale model and long-term measurements can be realised in several ways. One method is to match the simulation period to that observed. By selecting corresponding time frames, one can then compare the fields of interest. The main disadvantage of this method is that errors in the background flow simulated by the mesoscale model also exist, which complicates the analysis. The
10 second method is to sample the data and the model simulations in rather strict idealised conditions of wind speed and direction. We have chosen this second method using the WRF model in idealised case mode and compare its results to properly averaged measurements. Here the equilibrium solution without the wind farm effects is purposely made to match the free stream velocity and thus background errors in the simulations
15 are absent. Besides a more detailed analyses, this method also allows the study of the vertical dependency of the velocity reductions, since the atmospheric background conditions are almost identical for the simulations with the different vertical resolution.

Before we validated the results of the schemes in the wake of the wind farm, the a priori unknown initial length scale of the EWP scheme had to be determined. We did
20 this using the turbine power measurements from the most frequently observed wind speed bin. This limitation could not be avoided, since to our knowledge no other long-term measurements from large offshore wind farms are available. This constant was then used for the validation of all wind speed bins. The difference of 0.1 % between the velocity deficit simulated by the WRF-WF and EWP scheme at the end of the wind
25 farm for the 10 m s^{-1} wind speed bin, facilitated the comparison between the schemes in the wind farm wake, where the parametrisations are not active anymore.

One major difference between the EWP and WRF-WF approach is in the treatment of the grid-cell averaged TKE budget equation. The TKE production regulates the vertical profiles of momentum, temperature, and moisture within the PBL. Differences in

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TKE production would thus affect the local weather (e.g., temperature, humidity, and possibly clouds) response to the presence of large wind farms. For the WRF-WF and EWP scheme, the PBL scheme adjusts the TKE production through the changed vertical shear in horizontal velocity. The term that represents the local turbine-induced
5 turbulence is neglected in the EWP approach, whereas it is parametrised as an additional source term in the WRF-WF scheme. The total TKE is more than 3 times larger in the WRF-WF scheme than in the EWP scheme. Future measurement campaigns around large wind farms under suitable atmospheric conditions can help to settle this issue.

10 7 Conclusions

We introduce a wind farm parametrisation for use in mesoscale models. The EWP approach is based on classical wake theory and parametrises the unresolved expansion of the turbine-induced wake explicitly in the grid-cell that contains turbines, where the largest velocity gradients occur. The associated turbulence shear production is then
15 determined by the PBL scheme in the mesoscale model. The approach has been implemented in the WRF mesoscale model and can be used with any PBL scheme. However, we recommend PBL schemes that model the TKE equation.

We analysed the results of simulations from the scheme in the wake of a wind farm. For the validation, we used the averaged wind speeds within a 30° wind direction sector at two met masts in the wake of the Horns Rev I wind farm. The model was run
20 for several wind direction bins that cover those sampled in the observations. For each wind speed bin, we compared a wind direction averaged wind speed to the similarly averaged measurements. We found that for all 5 velocity bins, the velocity modelled with the EWP scheme agreed well with the met mast measurements 2 and 6 km downstream from the edge of the wind farm. The bias was less than 0.15 m s^{-1} , except for the
25 the 7 m s^{-1} wind speed bin at mast M6, where it was around 0.23 m s^{-1} .

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To our knowledge, no long-term data-sets are available to validate the details of the vertical structure of the velocity deficit and turbulence in the wake of the wind farm. In a qualitative comparison, we found the vertical structure of the modelled TKE field to agreed with that of actuator-disk simulations by LES models (Wu and Porté-Agel, 2013), with an increased and decreased TKE around the upper and bottom rotor tip, respectively. The TKE field in the EWP scheme led to a symmetric velocity deficit profile around hub-height, similar to velocity deficit profiles in Wu and Porté-Agel (2013); Vermeer et al. (2003). Also, the vertical wind farm wake expansion in the EWP approach was similar to that described in the before mentioned studies. While validation of the WRF-WF parametrisation has before been carried out with measurements within a wind farm (Jiménez et al., 2014), this is the first time that the validation has been done in the wake of a wind farm, at the scales the mesoscale model was designed to simulate.

Appendix

In the EWP scheme, we approximate the velocity deficit profiles on the r.h.s. of Eq. (11), by a Gaussian shaped velocity profile on the l.h.s. of Eq. (11).

To show that these profiles are to a good approximation similar, we compare the difference between the average of 5000 single profile at distances $0 < x < 500$ m to the approximated Gaussian velocity deficit. We normalise both profiles by the depth, Δy , of the wake in the cross-stream direction. For the comparison we used $U_0 = 8 \text{ m s}^{-1}$, $R_0 = 40$ m, $\sigma_0 = 60$ m, $C_T = 0.8$, $K = 6 \text{ m}^2 \text{ s}^{-1}$, and $\Delta y = 1120$ m. The wake centre is defined at $z = 0$ m.

The result in Fig. 10 shows that the differences between the spatial averaged Gaussian profiles and the Gaussian profile with the space averaged spread is in the entire velocity deficit region far less than 0.001 m s^{-1} .

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Code availability

In this section a short guideline of the usage of the EWP scheme in the WRF model is given. The EWP approach can be run either in serial, shared-memory, or distributed memory options. Currently, it is not possible to run the approach with the mixed shared and distributed memory option. The scheme can be used for idealised and real case simulations. For the real case simulations, the wind farm parametrisation can be activated in any nest of the simulations. The additional namelist.input option `bl_turbine` should be used to select the wind farm parametrisation.

The EWP scheme needs additional input files in ASCII format. In the first file the positions and types of all wind turbines should be listed. The file name has to be specified as a string in the additional namelist.input option `windturbines_spec`. For every turbine used, the turbine characteristics, i.e., the hub-height and diameter, and the thrust and power coefficients as a function of wind speed, need to be listed in a file. This file name has to start with the turbine type used in the first file, followed by the extension `.turbine`.

Please contact pvol@dtu.dk to obtain the code of the EWP wind farm parametrisation.

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Table 1. Average wind speed at the reference turbine (\bar{U}_{ref}) and frequency of the measurements at the met mast for all wind speed bins within the wind direction range 255–285°.

Wind speed bin (ms^{-1})	\bar{U}_{ref} (ms^{-1})	N_{obs} at M6–M7
7 ± 0.5	7.00	887
8 ± 0.5	7.95	933
9 ± 0.5	8.95	1097
10 ± 0.5	10.05	990
11 ± 0.5	11.05	729

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Table 2. The WRF model configuration used in the simulations.

Number of grid-cells in the horizontal plane (nx, ny):	80, 40
Horizontal grid spacing (km):	1.12
Domain size in x, y, z (km):	89.6, 33.6, 15
Wind farm extension ($nx \times ny$):	5×5
Boundary condition:	OPEN
PBL scheme:	Nakanishi and Niino (2009)
Pert Coriolis:	Yes
Coriolis frequency (s^{-1})	1.2×10^{-4}

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Table 3. Details of the WRF simulations.

Wind direction range ($^{\circ}$):	258.75–281.25
Wind direction interval ($^{\circ}$):	3.75
Wind speeds ($m s^{-1}$):	7.0, 8.0, 9.0, 10.0 and 11.0
Number of vertical levels (nz):	40 (L40), 60 (L60), 80 (L80)
Wind farm parametrisation:	None, WRF-WF, EWP

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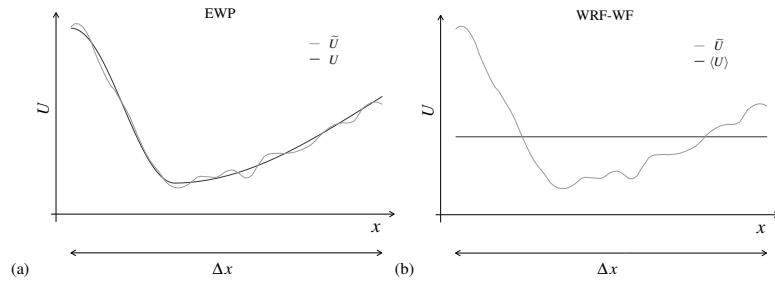


Figure 1. A sketch of the downstream development of a turbine induced velocity reduction for **(a)** the EWP scheme and **(b)** the WRF-WF scheme. Δx indicates the grid-cell size. The black lines represent the average for each approach and the grey line the instantaneous velocity \tilde{U} . We used U for the ensemble-average velocity and $\langle U \rangle$ for the grid-cell averaged velocity. The difference between the average and instantaneous velocity defines the turbulence fluctuation at each distance x .

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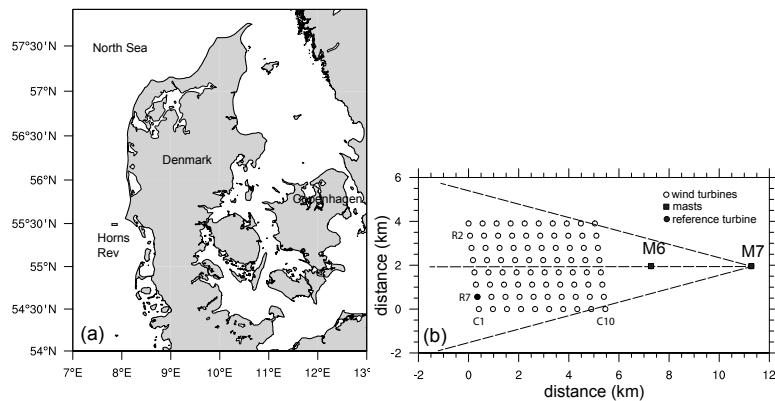


Figure 2. Location and layout of the offshore wind farm Horns Rev I including two nearby masts (M6 and M7). The centre of the wind farm and width of the sector ($\pm 15^\circ$) used for the filtering of the measurements are indicated by the dashed lines.

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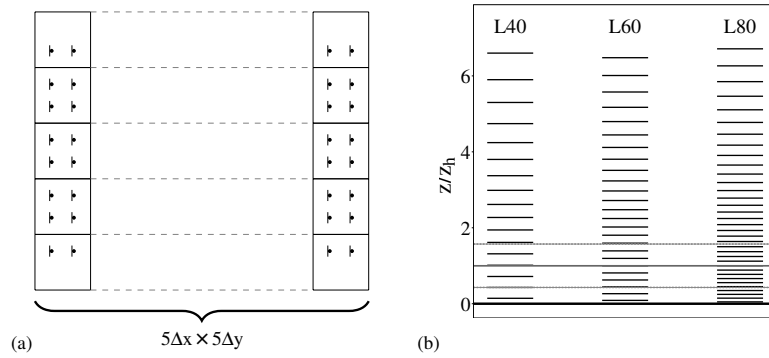


Figure 3. (a) Illustration of the model wind farm layout as applied in the mesoscale model. The squares indicate the model grid-cells and the wind turbines are marked with triangles. (b) Height (normalised by the turbine hub height) of the model mass-levels for the three simulations: L40, L60 and L80. The solid grey line and the dashed lines mark the hub-height and the upper and lower turbine blade tip, respectively.

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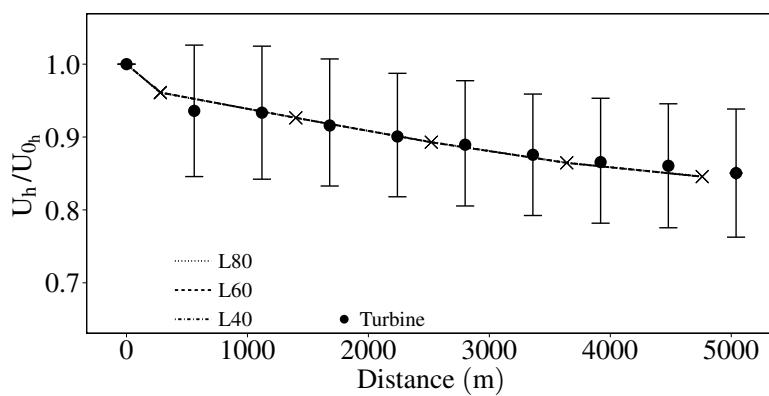


Figure 4. Measured and simulated hub-height velocity within the wind farm. The lines show the model simulated velocities averaged over wind direction with an initial length scale $\sigma_0 = 1.7 R_0$ for the 3 vertical resolutions (L40, L60, and L80). The dots represent the measured turbine velocity averaged over each row and the bars indicate their standard deviations. The crosses mark the velocity at the grid-cell centre.

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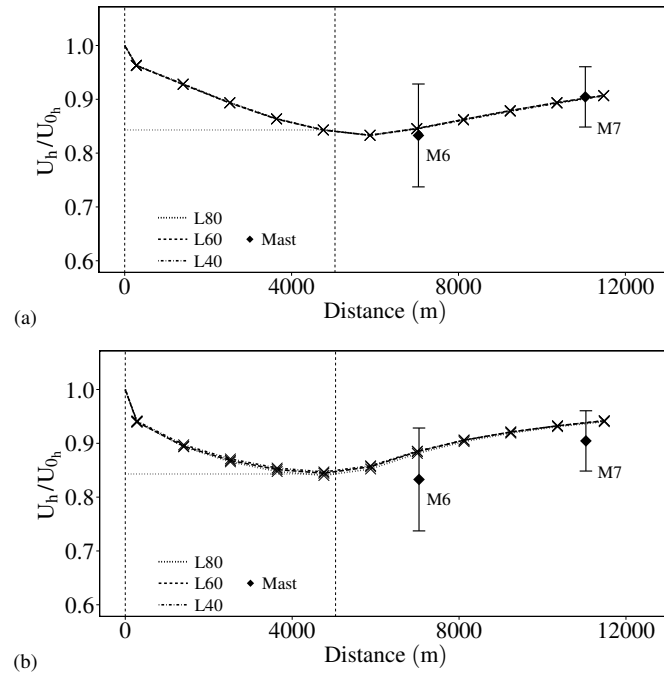


Figure 5. Hub-height velocity for the EWP (a) and WRF-WF (b) parametrisations for the L40, L60 and L80 simulations and observations as a function of distance from the western edge of the wind farm. The lines show the model simulated velocities, with the crosses showing the velocity at the grid-cell centre. The diamonds are used for the met mast measurements the bars their standard deviation. The vertical dashed lines show the wind farm extension and the horizontal dotted line the velocity with the EWP scheme at the easternmost grid-cell.

3517

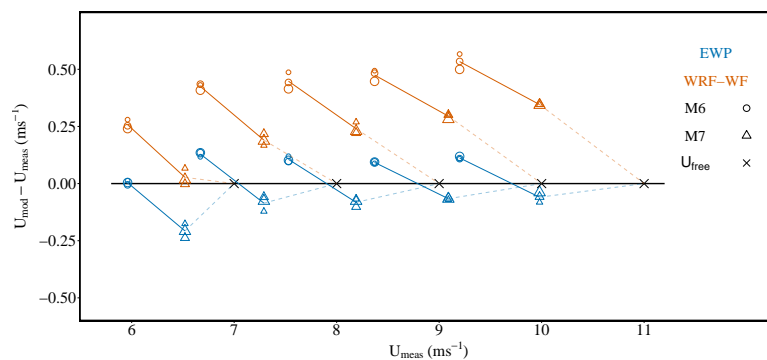


Figure 6. Modelled (U_{mod}) minus measured wind speeds (U_{meas}) at M6 (circles) and M7 (triangles) as a function of their measured wind speed, for the EWP (blue symbols) and the WRF-WF scheme (red symbols) for five wind speed bins and three vertical resolutions (symbol sizes). The coloured solid lines link the M6 and M7 values for the same wind speed bin, whereas the dashed lines the values from M7 and the free-stream velocities at the end of the wake. The crosses indicate the free-stream velocity.

3518

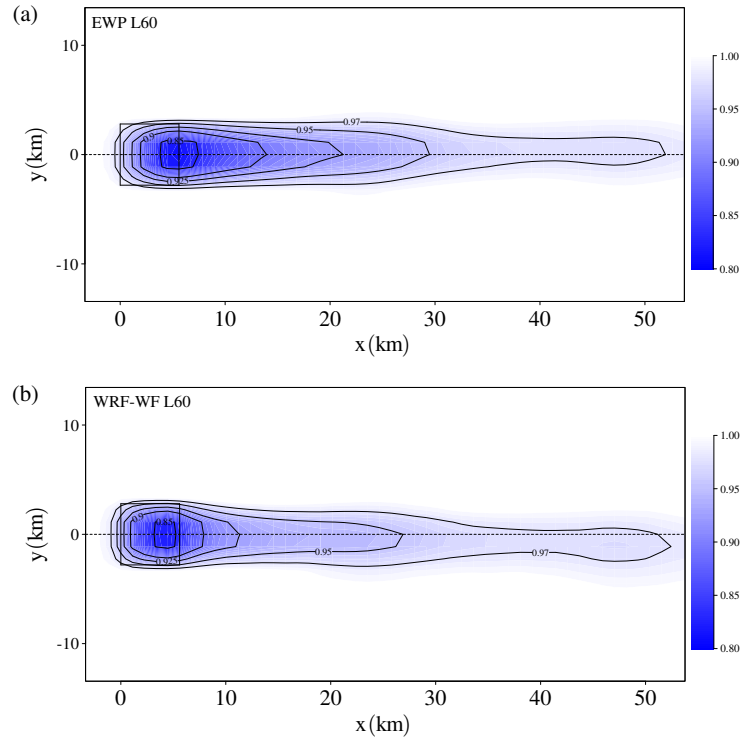


Figure 7. Horizontal view of the WRF-simulated velocity field at hub-height using the (a) EWP and (b) WRF-WF schemes. The simulations are for 10 m s^{-1} in the 270° wind direction and L60. The dotted line indicates the longitudinal centre and the solid rectangle the outer boundary of the wind farm.

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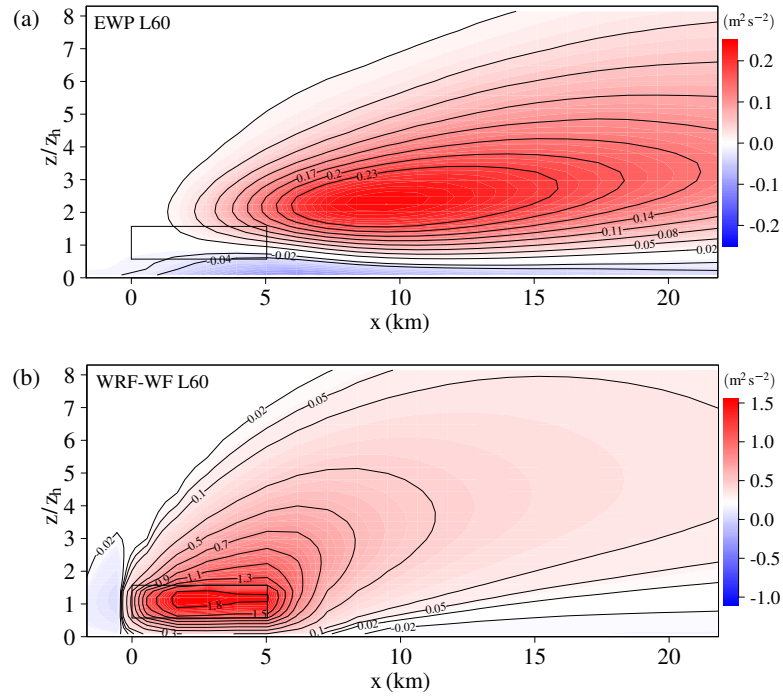


Figure 8. Vertical cross-section of the TKE difference ($e_{\text{wf}} - e_{\text{ref}}$) ($\text{m}^2 \text{s}^{-2}$) for the simulations for 10 m s^{-1} in the 270° wind direction and L60 for the (a) EWP and (b) WRF-WF scheme. The region in the model containing turbine blades is indicated by the rectangle.

3520

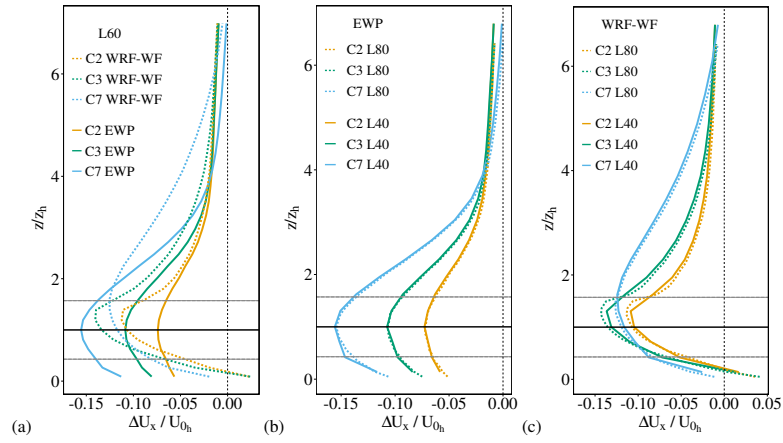


Figure 9. Comparison of the vertical profiles of simulated velocity deficit for the second (C2), third (C3) and seventh (C7) grid-cell containing wind turbines from the first westernmost turbine: **(a)** L60 simulations, **(b)** L40 and L80 simulation for the EWP, and **(c)** L40 and L80 simulation for the WRF-WF scheme. The turbine hub-height is indicated by the horizontal solid line and the turbine blade bottom and top by the dashed lines. The free-stream wind speed was 10 m s^{-1} in the 270° wind direction.

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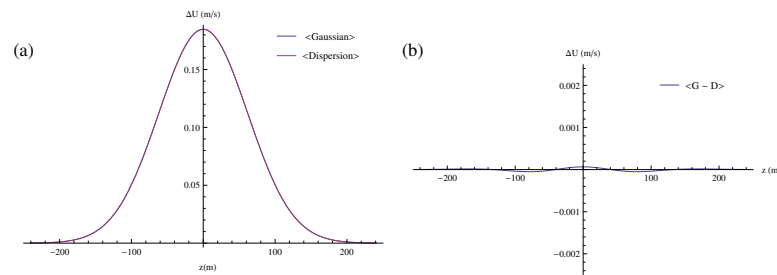


Figure 10. **(a)** shows a comparison between the distance average of the velocity deficit profiles (blue line) and the Gaussian profile with the average spread (σ) (red line). **(b)** shows the difference between the space averaged Gaussian profiles and the Gaussian profile with the average spread.

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