

# Response to reviewer #1 of “A new sub-grid surface mass balance and flux model for continental-scale ice sheet modelling: validation and last glacial cycle” by K. Le Morzadec et al.

July 28, 2015

## 1 General comments

**I am really ashamed that I submitted the previous review before careful checks. Sorry to confusing. Here, I submit the revised review. Please forget about the first one. Thanks. This paper presents the potential impact of sub-grid scale processes such as surface mass balance and the ice transport to large-scale ice-sheet evolution. The method is an extension of previous work by Marshall et al (1993), with more topographic information and using two-way coupling between sub-grid and course grid model. The performance of the sub-grid model is evaluated using idealized and realistic regimes by a higher-order ice-sheet model ISSM. The sub-grid model is installed in a large-scale ice sheet model GSM and tested for simulation of the last glacial cycle.**

**I think this paper is fairly well written, but description of the model is lacked or left ambiguous. The point which were most unclear to me on first reading is the relation of and structure of SG, CG, ISSM and GSM. A CG cell is in a sense equivalent with a gridcell of GSM in the manuscript, not a model. CG model (e.g., p3049, L20) corresponds to GSM (but not explicit explanation). ISSM is just a reference model to be compared with the hypsometric flow-line model (SG model).**

We clarify in the introduction that the GSM is our CG model: "the Glacial Systems Model (GSM, formerly the MUNGSM), our coarse grid model,..."

The abstract now states "We develop a new flowline SG model for embedding in coarse resolution models." In the introduction of section 2.3 we now state "In this section, we describe how the SG model is embedded in the GSM and the conditions applied to activate or deactivate the SG model in each CG cell."

**Surface mass balance computation is performed with the same equation over all the three (SG, CG/GSM, ISSM) models. Such a rough picture may not easily be obtained. It might be better to extract the surface mass balance section 2.1.3 as a common aspect.**

And the main text now states that this method is used in the three models at the end of section 2.1.3. "The GSM and ISSM compute the surface mass balance using the same PDD method."

**Design of coupling between SG and GSM is also difficult to understand on first reading. Also as far as I understand, when perform coupling, the whole domain is computed by coarse grid (1 × 0.5 degree)**

**GSM. Some coarse gridcells (cell? synoptic grid? please unify the terms) are activated as SG mode when some condition is satisfied. Each gridcell SG activated has own (prescribed) hypsometric bins and other parameters. Thickness evolution of corresponding SG model is computed for each activated coarse grid cell. There are two way interaction between the activated coarse grid cell and corresponding SG model, where SG model information modifies corresponding coarse grid information. These rough structure is extracted by reading through section 2.4 in this manuscript. Rather, a flow chart or brief summary of the design may help.**

A diagram (figure 2) has now been added depicting the relationship/coupling between CG and SG. "Synoptic grid" is changed to "CG" in the revised manuscript.

**Detail methods are also bit hard to understand on first reading. Schematic figures to describe, for example, the redistribution of CG flux to SG levels and its opposite may also help.**  
as above, with the new diagram.

**Next thing I am curious is that an extension to the alternative parameterization in Section 3.2. At an extreme end, we can compute the same computation as ISSM does for the same domain but with SIA model (e.g., GSM core) with the same flow parameters (in this case, rate factor at 0 degree). It corresponds to include all the topographic characteristics to the SG model. If it is not deviate from ISSM results, then an adaptive model with light SIA model, not heavy higher-order models may be practical for long-term simulations. It is beyond the scope of this paper, I do not require to include, but still happy to see.**

This comparison would be worth investigating, but as it is beyond the scope of this paper, we now include this idea in the conclusion as ideas for future work: "Other alternatives to the hypsometric parameterization, such as running a high resolution SIA model in the region of rough topography, could be considered."

## **2 specific comments**

**Abstract, first sentence I would not write like this in the abstract. Although I agree that typical grid resolution at the moment is around 10 to 50km for long-term computation, this is not always a necessary condition. Rather, I would state simply that this resolution is a current typical configuration (instead of 'need to be run...').**

"need to be run" was modified to "are typically run".

**p3038, L26. better to delete 'coarse' (I feel it a bit subjective) as the same reason above. I would just state the fact simply, at this stage. The following sentences naturally drive us this resolution as 'coarse' one.**

"coarse" has been removed here. But we have added it in quotations in the abstract to help define what we mean by coarse.

**p3039, L6 'the mean surface elevation' of a coarse grid?**

"the mean surface elevation" has been changed to "the mean surface elevation of a coarse grid cell".

**p3039, L8, citing Abe-Ouchi et al.: The first part is somewhat misleading and confusing. Van den Berg et al explicitly discuss the sensitivity of ice-sheet evolution to the grid resolution, while Abe-Ouchi et**

al. (I am the second author) do not explicitly discuss the errors due to a lower grid resolution, although one can lead such point from the paper. Dr. Abe-Ouchi and I both agree that the lower grid resolution in that paper leads to such errors as the author mentioned, but it seems to be an overstatement only by citing this paper. Instead I suggest to include, in addition, the paper Abe-Ouchi and Blatter (1993), *Ann. Glaciol.* 18, 203–207. which is relevant for this context.

This reference was added to the revised manuscript.

**p3940 L10 ‘the size of these bins’ the total area of these bins?**

"the size of these bins" was replaced by "the thickness of these bins".

**p3940 L11. What the CG level means?**

To avoid confusion the word level is not used anymore to refer to the hypsometric levels (bins is used instead in that context). SG and CG level and defined at their first occurrence in a footnote: " SG level represents the hypsometric curve while CG level correspond to a GSM cell." Figure 3, 5 and 8 had to be modified to change "level" with "bin" in the legend.

**p3041 L17 ‘cubic dependence of ice flow on surface slope’ This statement requires the explanation of the shallow ice approximation under Glen’s flow law with exponent 3 beforehand, or at least refer equation 3 in advance and postpone the meaning of the cubic dependence etc.**

This statement is now referred to eq.3.

**p3041 L23 ‘from 1 to N’ better to write 10, or N(=10) instead of N, or define value of the N beforehand.**

"divided into 10 bins (or bins)" has been replaced by "divided into N bins".

**Equation 1. Please define which corresponds the lower level, 1 or N (I expect it is N).**

This sentence has been changed using: "from 1 (highest) to N (lowest)."

**p3041 L16 and after. This block is somewhat unclear to me and I am still puzzled what the authors do with the following equations. How to compute slope<sub>k</sub>, the denominator of Eq. (1)? I read three or more times and finally I suppose that when ice starts to build up, there is no ice and the surface slope is the same as basal slope, which means slope<sub>k</sub> is computed by GEBCO 1km DEM averaged over the same bin, and prescribed through the simulation. Is it correct? I suggest to reformulate this part to separate the definition of variables and their explanation. For example, The sentence ‘The effective length, L, ...’ may be ‘The effective length, L is computed for each level as: .... Eq (1). Using the effective length L, slope<sub>t</sub> is updated as .... Eq.(2). As no information is ....’**

This paragraph has been updated to take these suggestions into account.

**p3042 L5. ‘To compute the slope at the lowest level...’ Is this same meaning with ‘ice cliffs boundary conditions’ (p3046, L10)?**

This clarification has been added.

**Equation (3)  $\bar{u}$  is computed at each SG levels? If so, better to write  $\bar{u}_k$ ,  $H_k$ ,  $hd_k$  etc, or mention to omit before the equation. And what is the relation of  $\partial h d$  and the slope<sub>k</sub> in  $\partial x$  Eq. (2)? The same quantity?**

subscript k and superscript t have been added.

**Equation (6) the same as Equation (3).**

Eq.3 (ice velocity) has been removed as it is already defined in the description of the effective diffusivity term of Equation 6 (now Equation 3).

**Equation (8) Please define  $\Delta x_k$  and  $\Delta y_k$ . I suppose  $\Delta x_k$  proportional  $L_k$  and  $\Delta y_k$  is the width defined in p3042.**

$\Delta x_k$  and  $\Delta y_k$  have been defined.

**p3048 L9. I do not understand the method here. The condition is ‘Lowest hypsometric level surface elevation’ reaches the bedrock elevation of the highest level. To obtain surface of lowest hypsometric level, we need computation of thickness by SG model equations, which means the SG model is turned on. Is this surface elevation computed using CG level thickness and SG level bedrock? p3049 not p3048**

This method is used to deactivate the SG model so the SG information at that stage is available. The above described added clarification of the usage of turned on/off and (de)activated should address this.

**Section 2.4 about coupling. The coordinates of GSM (degree) and SG (Cartesian) are different. I am curious about the way how to convert the information from one to the other and/or the effective length computation.**

The effective length is used only at the SG level and is computed using the high resolution data in km. Only ice volumes are exchanged between the CG and the SG cells. A clarification of how the Cartesian coordinated are converted to degrees is done in section 2.1.1 Hypsometric curves. " To select a region fitting the coarse resolution grid cell of the GSM (degrees), the GEBCO Cartesian coordinates are converted in degrees assuming the earth as a perfect sphere of radius 6370 km."

**p3049 L26. Does it means that CG ice volume is replaced by sum of the volume of SG levels below the lowest unfilled level? On L20 above it is said that CG ice is added to the SG levels. I am afraid that this loop makes the SG ice volume infinite by this procedure.**

CG ice is added to the SG bins only when the SG model switches from deactivated to activated. This has been clarified in the revised manuscript. "... when the SG model switches from deactivated to activated."

**p3050 L15, adjacent CG flux into SG model. Is this procedure done after the computation of equation (6)?**

We have now clarified that eq.6 is not used at the lowest bin of the sub-grid level and that instead, the CG fluxes are used to remove ice at the lowest bins. "When coupled to the GSM, the SG model does not compute flux out of the lowest bin through Eq.3."

**p3051 comparison. I am curious how much is the difference in the computation time between SG and ISSM, just for information.**

"The SG model computation time for 3000 years simulation, using 10 hypsometric bins, is about 0.02 seconds. At a resolution of 1 km and using 10 cpus, ISSM run time is about 2 to 5 hours (depending of the topographic region used). The sub-grid model adds 3 to 6 hours (depending of the parameter vector used) to the glacial cycle run-time over North America." This information has been added at the beginning of section 3.

**Table 1. Caption, ‘At least half of the area’: ‘half of the area’ of what? coarse grid?**

That sentence has been replace by: "At least half of the CG basal elevation is above sea level".

**Same, ‘HCG = Volume of lowest SG levels’: Confusing. Thickness of lowest SG levels? or Volume divided by the areas?**

That sentence has been replaced by: " While SG is activated,  $H_{CG}$  is set to the  $\frac{totalicevolume}{totalarea}$  of the filled SG bins at each CG timestep (the total SG ice volume is used during the deactivation timestep)".

**Same, ‘HSG ’: difficult to understand what it means.**

That sentence has been replaced by: "When SG switches to activated, the CG ice volume is redistributed over the SG bins using the mean between two methods: equal redistribution over all bins and redistribution of ice over the lowest bins"

### **3 technical corrections**

**Section 2.2. This section should be move to the end of section 2, or before 3.1.**

This change was made to the revised manuscript.

**Section 2.4 and after. The terms (De)activation and turn-on(off) are sometimes mixed up. In Fig.8 caption ‘turn on/off’ are used in terms of coupling/decoupling, while in section 2.4.1 are used in terms of activation/deactivation. I would keep them consistent.**

In the revised manuscript, Turned on/off is kept for coupling/decoupling.

**Section 3. ‘Sub-grid model performance’, or ‘Sub-grid surface mass balance and flux model performance’ is proper.**

Section 3 title was modified to "Sub-grid model performance and tests".

**Table 1. Rough topography  $\Delta hb$  ... Better to separate by some ways, e.g., Rough topography ( $\Delta hb > 500m$ ).**

This change was made to the revised manuscript.

**(total volume when SG is turned off) Not necessary, because SG is not activated**

The difference between turn off and deactivated was made clearer in a previous comment. "total volume when SG is turned off" was modified to "the total SG ice volume is used during the deactivation timestep".

**Supplementary Figure S1. Define NHYPS. The lines of ISSM and NHYPS=5 are hardly distinguished. The five point line of NHYPS=5 may easily be regarded as ISSM line.**

The line style was changed and "NHYPS=5" was replaced with "5 bins" to keep a consistent legend with the other plots.

# Response to reviewer #2 of “A new sub-grid surface mass balance and flux model for continental-scale ice sheet modelling: validation and last glacial cycle” by K. Le Morzadec et al.

July 28, 2015

The paper is about a revised version of the hypsometric approach by Marshall and Clarke (1999), which is thought to improve representation of topography in coarse resolution ice sheet models. For the longer time-scales of glacial cycles, modellers rely on coarse spatial resolution due to limitation in computational resources. Including hypsometric curves can better resolve accumulation of ice in higher mountainous region as well as melting of ice in lower valleys and, at the same time, preserving coarse resolution. A sub-grid scale (SG) model operating on these hypsometric levels is coupled with a coarse resolution ice sheet model in shallow ice approximation. While Marshall and Clarke used synthetic curves, the present paper uses a digital elevation model to create hypsometric curves for representative regions over North America. For parameterization of flux between hypsometric levels besides effective lengths, a slope parameter is used. Different parameterizations for the sub-grid scale flux are tested. The SG model is validated using a higher order ice sheet model of the Blatter-Pattyn type, although agreement of results between both of the models appears rather poor. Finally, the importance of the SG model for simulations of the last glacial cycle with the GSM (formerly MUNGSM) model is demonstrated.

"Validation" in the title was a problematic choice of word. We meant validation in the sense of testing and quantifying misfits not in the simplistic sense of proving the SG model to be a valid replacement for a high resolution model. We modified the word validation by testing in the title.

## 1 Major Points

1. In general, the description of the hypsometric parameterization needs more explanation, including more formula, a schematic figure and a flow diagram. Unfortunately, is the most known procedure – the PDD scheme – explained at great length, what is not necessary, because citation of previous work would have been sufficient. However, the hypsometric scheme, particularity your novelties, are not explain sufficiently. This is ever more important, because you do not make the code public.

As requested in the manuscript preparation guidelines, a new section called "Code availability" has been added: "The sub-grid code is available upon request from the first two authors". As detailed below and in

the response to the other reviewer comments, we have added 2 figures and revised the text to better explain the hypsometric parametrization.

**2. Page 3042, lines 10-11: “Then, the size of these bins is updated to avoid empty levels.” Is the size of the bins different for each region?**

Yes, since as stated: "First, the region is divided into N bins of equal altitude range". We also clarify how the empty levels are adjusted: "Then, to avoid empty bins, the surface elevation range of each empty bin is expanded (consequently decreasing the elevation ranges of the higher and lower adjacent bins) until these three consecutive bins represent approximately the same surface area."

**3. Section 2.1.1: It is unclear how you determine the effective length L and the slope. You wrote, “Specifically, for each hypsometric level, we compute the cube root of the mean of the cube of the magnitude of the slopes.” Which quantity do you compute? Could you write down a formula for this? How is slopek in Eq. (1) defined? Is this the surface slope length?**

The quantity we compute is the hypsometric slope (slopek updated to S). This part have been clarified without the addition of a formula: "Specifically, for each hypsometric bin we compute the slope,  $S_k^0$ , as the cube root of the mean of the cube of the magnitude of the slopes from the GEBCO data."

**You further wrote: “The effective width of each hypsometric level is set to the number of grid cells, multiplied by the spatial resolution, that are in contact with adjacent lower hypsometric levels grid cells.” What is the expression for the effective width? Is the effective width the same as the effective length? Could you please check the entire sections for error and rewrite it using some more formula in order to make the section more understandable. Could you please illustrate with a schematic figure the involved quantities?**

Section 2.1.1 has been restructured to clarify that the effective width and length are different: "The flow line model requires an effective width,  $W$ , for the representation of flux between hypsometric bins.  $W$  of each hypsometric bin is set to the total contact length of the SG cells assigned to the bin with adjacent lower hypsometric bins grid cells as detailed in Fig.1."

**4. Section 2.1.3 (Surface mass balance): This section can be shorted substantially as PDD parameterization is well know, described elsewhere and is not the topic of the paper.**

This description has been significantly shortened to: "We use the positive degree day method described in Tarasov and Peltier (1999) to compute accumulation and ablation from monthly mean temperature and precipitation. A constant environmental lapse rate adjusts the temperature to the ice surface elevation. A parameterization of the elevation-desertification effect (Budd and Smith, 1981) reduces the precipitation by a factor of two for every kilometre increase in elevation. Snow is melted first and the remaining positive degree days are used to melt ice with allowance for the formation of superimposed ice. The supplement includes a more detailed description of the surface mass balance module.

The GSM and ISSM compute the surface mass balance using the same PDD method." The detailed description has been added in the supplement.

**5. Sections 2.1.2 (Ice velocity) and 2.1.4 (Ice thickness evolution): Obviously, you use the isothermal shallow ice approximation (of order zero) to yield the ice velocity in the SG model. What is the rational to use the shallow ice approximation in the space of hypsometry, as the shallow ice approximation is formulated on the Earth’s surface? The scales and gradients on the Earth’s surface are quite different from those in the hypsometric space. Thus, immediately the question appears what are x (and**

$\delta x$ ) in Eq. (3), (6) and (7)? The coordinate  $x$  cannot be a length on the Earth's surface, because in your hypsometric model there is only sub-grid area, which is not a length. Marshall and Clarke (1999) were aware about this fact, see their Eqs. (15) and (16) wherein they clearly formulate flux in the hypsometric space. To be concrete: How do your  $L_k$  and  $slope_k$  from Section 2.1.1 relate to your formulas in Sections 2.1.2 and 2.1.4? In particular, how does your flux – in your case possibly diffusivity – relate to your  $L_k$  and  $slope_k$ ? The entire Sections 2.1.2 and 2.1.4 have to be completely revised incorporating my concerns and questions.

The scales and gradients on the Earth's surface are indeed quite different from those in the hypsometric space. That is why we define effective length and width that are a parameterization of the Earth's surface.  $\frac{\delta h_d}{\delta x}$  in Eq.3 has been replaced by  $S$ , the surface slope. We also added after eq.5 that: " $\Delta x$  is the effective length  $L$  and  $\Delta y$  is the effective width  $W$  defined in Sec.2.1.1."

The notations have been clarified so that *slope<sub>k</sub>* (now  $S$ ) from section 2.1.1 is present in Sections 2.1.3 formulas.  $L_k$  is used to updated  $S$  at every times step. The flux is proportional to  $S$  as seen in eq.4.

Sections 2.1.2 (Ice velocity) and 2.1.4 have been merged together and the velocity equation has been removed as it is already defined in the effective diffusivity term.

**6. Page 3048, lines 19-21: “The GSM has been subject to a Bayesian calibration against a large set of paleo constraints for the deglaciation of North America, as detailed in Tarasov et al. (2012). We use a high-scoring sub-ensemble of 600 runs from this calibration.”** These sentences rather belong to Section 4.

The beginning of section 4 has been modified to included these sentences: "We present results of simulations over the last glacial cycle. The 39 "ensemble parameters" of the GSM (attempting to capture the largest uncertainties in climate forcing, ice calving, and ice dynamics) have been subject to a Bayesian calibration against a large set of paleo constraints for the deglaciation of North America, as detailed in Tarasov et al. (2012). We use a high-scoring sub-ensemble of 600 parameter vectors from this calibration to compare the GSM behaviour when the SG model is turned on and off. The primary supplement of Tarasov et al. (2012) includes a tabular description of the 39 ensemble parameters as well as input data sets. For the purposes of clarity and computational cost, we examined model sensitivity to different coupling and flux parameters using five parameter vectors (of the 600 members ensemble) that gave some of the best fits to the calibration constraints. As these five parameter vectors display similar behaviour we present sensitivity results using the parameter vectors for the two runs described in detail in Tarasov et al. (2012) (identified in that paper as runs nn9894 and nn9927)."

**Do you use all 600 runs in section 4.1?**

Yes, we use all 600 runs in section 4.1 unless specified otherwise for single examples. This is clarified with the modification done in comment 10. about Section 4.

"An ensemble of simulations" has also been replace by "The ensemble of simulations".

**Corresponds the “sub-ensemble” with the five best fits?**

The five best fits are five parameter vectors from the 600 parameter vectors ensemble. It is clarified in the beginning of section 4: "For the purposes of clarity and computational cost, we examined model sensitivity to different coupling and flux parameters using five parameter vectors (of the 600 members ensemble) that gave some of the best fits to the calibration constraints."

**7. Section 3.1 (Comparison with ISSM): Could you clarify: Do you couple the SG model to the ISSM model? The SG model runs on one 30 km × 60 km rectangle. This rectangle is discretised in a**



**resolution of 1 km × 1 km for the ISSM model. Is that correct?**

The SG model is not coupled to ISSM. The results from one model are compared to the results of the other one. ISSM is run over the 30 km × 60 km region. The SG model is run over the hypsometric curve generated using the 1 km × 1 km resolution DEM data for the same region.

**Further, you write that no sliding is allowed in the ISSM model. Now, I lose understanding what you are modelling with ISSM. In mountainous regions, I would expect existence of glaciers that rapidly slide. Switching off sliding makes no sense then. Could you sharpen/explain your motivation for using ISSM and switching off sliding, what implies that mainly shear stress plays a role.**

We clarify in the text the reason for not including sliding in the ISSM and SG models at this stage of the project by adding in section 3.1: "To isolate the impact of using the SIA to represent fluxes in a mountainous region containing steep slopes in the hypsometric parameterization, our current experiments have no basal sliding. As glaciers can experience surging (via significant sliding) in this type of region, the next stage of this project will include sliding."

**8. Again Section 3.1: Why do you use only 2 kyr run time for ISSM? The application (a glacial cycle) which you are targeting operates on longer time scales.**

If the ISSM could be run on glacial cycle time scales, that model would directly be coupled to the GSM for regions with rough topography. Unfortunately, the run time of the ISSM is too long for such an application. The ISSM took 2 to 5 hours using 10 cpus to generate the 2000 model years for each of the 30 by 60 km regions.

**9. Section 3.2 (Test of alternative parameterizations): Why do you present to the reader parameterizations, which did not approve anything. These parameterizations would not help a user of your model.**

Documentation of both what works and what doesn't work is of value to modellers to avoid future repetition of exploring dead ends.

**10. Section 4: This section is incomplete, unclear and not too well organized. For example, it is unclear, whether you discuss all 600 runs or only the 5 best fits in Section 4.1. Or do you discuss the 5 best fits in the entire Section 4? For example, do you use all 600 runs or only the 5 best fits runs to determine the standard deviation shown in Fig. 8? In general, you should add a more detailed motivation, description and discussion of your experiments to Section 4. Partly, you can use sentences from your conclusions for Section 4 and erase these sentences from the conclusion section.**

The structure of Sec.4 is now clarified in the introduction of this section: "We present results of simulations over the last glacial cycle. We compare the GSM behaviour when the SG model is turned on and off for the 600 members ensemble of simulations and for one of the best runs of this ensemble. The primary supplement of Tarasov et al.(2012) includes a tabular description of the 39 ensemble parameters as well as input data sets. We also examine model sensitivity to different coupling and flux parameters using five of the best fit to calibration constraints parameter vectors of the 600 members ensemble. Tarasov et al.(2012) presents in detail two of these runs (identified in that paper as runs nn9894 and nn9927). For ease of interpretation, the ice volumes are presented as eustatic sea level (ESL) equivalent."

We also clarify in Sec.4.1 when we refer only to one parameter vector: "Fig.12 shows an example, for one of the parameter vectors of the ensemble of simulations, where..."

**I recommend adding a new subsection to the beginning of Section 4, which includes a summary of the**

**model setup for the 600 ensemble runs (climate forcing, varied parameters, constraints) and which clearly says which subset of these runs you use further on in section 4.**

We do not think that including a detailed summary of the model setup is appropriate given that it is detailed in the cited reference (what is the point of repeating the same tables from the cited reference?). We do now clarify that: "The primary supplement of Tarasov et al. (2012) includes a tabular description of the 39 ensemble parameters as well as input data sets."

We also clarify that the runs (nn9894 and nn9927) presented for sensitivity experiments are described in detail in the same paper : "Tarasov et al. (2012) presents in detail two of these run (identified in that paper as runs nn9894 and nn9927)."

**The insets of Figs. 11, 12 and 13 indicate several sensitivity tests. However, in the main text belonging these figures you leave the reader somewhat alone and miss to explain sufficiently these sensitivity tests.**

Modifications concerning fig.13 discussion: Description of the impact of turning on or off the fluxes between coarse grid cells when the SG model is activated (Fig.13, previously 11) has been modified: "To better understand the range of responses to CG ice flow between grid cells that have SG activated, three case scenarios can be considered. Case 1: ice flows out of the lowest SG bins located above the ELA into the lowest SG bins located above the ELA of another CG cell. There is limited impact of not allowing ice to flow out of the CG cell as in both cases ice accumulates. Case 2: ice flows out of the lowest SG bins located above the ELA into the lowest SG bins located below the ELA of another CG cell. In that case, turning off the fluxes between CG cells tends to reduce the total melt. Case 3: ice flows out of the lowest SG bins located below the ELA into the lowest SG bins located below the ELA of another CG cell. Ice flowing into lower SG bins generates higher melting rates so permitting fluxes between CG cells will in this case tend to increase ice mass loss. In cases 2 and 3, the combination of ice flowing below the ELA from the adjacent CG cell and from the bins above the ELA can raise the surface elevation of lower bins above the ELA and reduce the melt. Depending on the proportion of each of these cases, not allowing ice fluxes out of coarse grid cells with SG activated generates higher or lower ice volumes (Fig.13). 50 ka is an example of a 60% increase of the total ice volume when the fluxes out of coarse grid cells (with SG activated) are not allowed. As a counterpose, 35 ka presents a case where turning off the fluxes out of (SG activated) coarse grid cells decreases the total ice volume."

Modifications about fig.15 discussion: "Fig.15 shows the results of the glacial cycle simulation when the SG model is turned off and when the minimum altitude variation SG activation threshold is set to 50, 150, 300 and 500 m. A non-linear dependence on the threshold can be observed. At 50 ka, for example, setting the threshold to 50 m generates the lowest total ice volume while a threshold of 150 m lead to the highest ice volume. The difference between these two runs is 34.5 mESL at 50 ka.Threshold of 300 and 500 m generate intermediate total ice volumes. Moreover, simulations using different parameter vectors (not shown) result in different behaviours. No conclusion could be drawn about the optimal threshold."

We are not sure what is meant by "leave the reader somewhat alone and miss to explain sufficiently these sensitivity tests". If it means that more description of the different setups for each test is needed in the main text, we believe the tests/comparisons are explained in adequate detail in the figure captions which we find to be more useful as a reader. If the intent is that more discussion of the implications of the results or of the results in need, then we do not see what. We feel we've conveyed the main points we wanted to from each plot.

**Further on, you refer to Fig. S8 in the supplements. I would regard the comparison with previous**

**work as important enough to show the figure in the main paper.**

Fig.S8 has been included in the main paper (as Fig.14).

**11. Again Section 4: I find it interesting that there is such a strong sensitivity of ice volume to the SG parameters at about 60 to 50 kyr BP. Could you add further discussion and explanation about this?**

We looked at glacial initiation prior to 50ka with and without the SG model and we could not identify a reason for that strong sensitivity of ice volume around 50 ka.

This has been clarified in the revised manuscript: "Looking at the simulation used in Fig.12, the differences in ice field distribution when the SG model is turned on and off at 60 ka are minimal. We could not identify a reason for the strong sensitivity of ice volume around 50 ka other than the inherent non-linearity of the GSM."

**12. Conclusion: The conclusions are somewhat lengthy, in particular, when you address the glacial cycle simulations. Please, shorten and revise the conclusions.**

The conclusions have been shortened.

## 2 Minor Points

**13. Page 3038, lines 13-14: How do you know? Have you tried all possible parameterizations?**

That sentence was changed to: "Results show that none of the alternative parameterizations explored were able to adequately capture SG surface mass balance and flux processes."

**15. Page 3042, Eq. (2): What denote  $h_d$ ,  $k$ ? Please, explain that here either.**

Then sentence before Eq. 1 was changed to: "At any time step,  $t$ , the surface slopes,  $S_k^t$ , for SG bin  $k$ , from 1 (highest) to N (lowest), are computed from the surface elevation  $h_{d,k}^t$  and an effective length  $L_k$ :"

**14. Page 3042, Eq. (1): What denote  $h_b$ ,  $k$ ? Please, explain that here.**

$h_b$  is the basal elevation. The  $k$  is the subscript representing the bins as explain in the previous comment.

**16. Page 3042, Eq. (1) and Eq. (2) Could you eventually use for slopek a decent mathematical symbol  $s_k$ ?**

The *slopek* symbol has been replaced by  $S$ .

**17. Page 3048, line 24: "synoptic cell" I think this terminology is misleading, because the issues presented in this paper are not related with synoptic. Could you please use the terminology "coarse grid cell" instead here and for the other appearance of "synoptic cell" in the paper?**

"Synoptic grid" has been changed to "coarse grid".

**18. Section 3.1: Possibly, you can say a bit more explicit that your SG model is applied the 30 km × 60 km region.**

The beginning of Sec.3.1 was changed to: "We compare 2 kyr ISSM and SG simulations, applying constant sea level temperature and precipitation over an inclined bed and 21 different test regions in the Canadian Rockies. These regions, for both the ISSM and SG simulations, have a dimension of 30 km by 60 km and we use a DEM of 1 km resolution."

**19. Page 3056, lines 23, “setting the surface elevation”: do you mean “setting the surface elevation of the coarse resolution grid”?**

Yes, this has been clarified. “setting the surface elevation” was replaced by "setting the CG surface elevation".

**20. Page 3056, lines 26-27, “using the maximum of the two former methods”: What is the maximum of a method? To which physical quantity you applies the maximum? Please, be more precise.**

It has been clarified that the physical quantity is the surface elevation. "using the maximum surface elevation generated by the two former methods)".

**21. Page 3056, lines 25, “SC, method”: the comma should be erased.**

SC was placed between parentheses.

**22. Page 3056, lines 27, “MC, method”: the comma should be erased.**

MC was placed between parentheses.

**23. Page 3058, lines 3-26: Could you check what you wish to include in the itemized list and what not. Does the paragraph starting at line 20 belong to the itemized list too?**

Yes it does belong to the itemized list and has been fixed.

**24. Page 3059, lines 16-17, “...the installation of ISSM and helped including the new module in ISSM.”: Which module do you mean? As far as I understand the idea of Section 3.1, the ISSM model runs without the SG model and is used to assess the performance of the SG model.**

Page 3047, line 3-5: In that short section describing ISSM it is stated: "For this study, a new surface mass balance module identical to the one present in the sub-grid model, and detailed in Sec. 2.1.2, has been incorporated into ISSM." This has been clarified in the Author contribution section. "... supported ISSM installation and helped build a new surface mass balance module for the ISSM."

# A new sub-grid surface mass balance and flux model for continental-scale ice-sheet modelling: **validation** testing and last glacial cycle

Le Morzadec K.<sup>1</sup>, Tarasov L.<sup>1</sup>, Morlighem M.<sup>2</sup>, and Seroussi H.<sup>3</sup>

<sup>1</sup>Department of Physics and Physical Oceanography, Memorial University of Newfoundland, St. John's, Newfoundland, Canada

<sup>2</sup>Department of Earth System Science, University of California, Irvine, California, USA

<sup>3</sup>Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, USA

Correspondence to: L. Tarasov (lev@mun.ca)

## Abstract

To investigate ice sheet evolution over the time scale of a glacial cycle, 3D ice sheet models (ISMs) ~~need to be run at~~ are typically run at "coarse" grid resolutions (10 to 50 km) that do not resolve individual mountains. This will introduce to-date unquantified errors in sub-grid (SG) transport, accumulation and ablation for regions of rough topography. In the past, synthetic hypsometric curves, a statistical summary of the topography, have been used in ISMs to describe the variability of these processes. However, there has yet to be detailed uncertainty analysis of this approach.

We develop a new ~~SG model using a flowline SG model for embedding in coarse resolution models. A~~ 1 km resolution digital elevation model ~~to compute each was used to compute the~~ local hypsometric curve for each CG cell and to determine local parameters to represent the hypsometric ~~levels~~ bins' slopes and widths. 1D mass-transport for the SG model is computed with the shallow ice approximation. We test this model against simulations ~~produced by from~~ the 3D Ice Sheet System Model (ISSM) run at 1 km grid resolution. Results show that ~~no simple parameterization can totally~~ none of the alternative parameterizations explored were able to adequately capture SG surface mass balance and flux processes. Via glacial cycle ensemble results for North America, we quantify the impact of SG model coupling in an ISM ~~and the associated parametric uncertainties related to the exchange of ice between the SG and coarse grid levels. Via glacial cycle ensemble results for North America, we quantify the impact of SG model coupling in an ISM.~~ We show that SG process representation and associated parametric uncertainties, related to the exchange of ice between the SG and ~~coarse grid levels~~ CG cells, can have significant (up to 35 m eustatic sea level equivalent for the North American ice complex) impact on modelled ice sheet evolution.

## 1 Introduction

The resolution used in any model of complex environmental systems (e.g. Ice Sheet Models (ISMs), general circulation models or hydrological models) limits the processes that can be represented. For continental scale glacial cycle contexts, ISMs are currently run at ~~coarse~~-resolutions of about 10 to 50 kilometres (Pollard and DeConto, 2012; Tarasov et al., 2012; Colleoni et al., 2014). Processes such as surface mass balance on mountain peaks, iceberg calving, and ice dynamics in fjords are sensitive to scales of about 100 metres to a few kilometres, and therefore have to be parametrized. For example, even at 10 km grid resolution, mountain peaks are smoothed to bumps in a plateau (Payne and Sugden, 1990), inducing errors in computed surface mass balance (Marshall and Clarke, 1999; Franco et al., 2012). If the mean surface elevation of a coarse grid cell is below the equilibrium line altitude (ELA), ice ablation is overestimated (e.g. Tarasov and Peltier, 1997). Thus, ~~lower~~ coarser grid resolution can lead to temporal and spatial errors in ice sheet inception (Abe-Ouchi and Blatter, 1993; Abe-Ouchi et al., 2013) and subsequent evolution (Van den Berg et al., 2006; Durand et al., 2011).

Any model of complex environmental systems will have sub-grid (SG) processes that are, by definition, not dynamically resolved. Accurate modelling of such systems must therefore determine whether SG processes variability is relevant for the given context. If it is, some of the impact of this SG variability may be captured in a parametrized form (Seth et al., 1994; Leung and Ghan, 1995; Marshall and Clarke, 1999; Giorgi et al., 2003; Ke et al., 2013). For example, to improve surface mass balance in continental scale ice sheet models, Marshall and Clarke (1999) used hypsometric curves, which represent the cumulative distribution function of the surface elevation. In this method, each individual glacier is not explicitly represented. Instead, 2D topographic regions are parametrized with different hypsometric levelsbins, representing a discrete number of elevations and their associated area. In addition to ablation and accumulation at each SG levelbin, there is SG ice transport from high elevation regions to valleys where the average altitude is below the ELA. Starting with ice free conditions, Marshall and Clarke (1999) found an increase in the

total ice volume over North America after [a 3 kyr](#)<sup>1</sup> simulation when this hypsometric parameterization is coupled to an ice sheet model. The impact and accuracy of this SG model have yet to be quantified. The model was only validated against observations of a glacier located in the region used for tuning the parameterization (Marshall et al., 2011). Moreover, the communication between the SG and coarse grid (CG) [cells-models](#) was identified as a potentially important source of error (Marshall and Clarke, 1999), but its impact has not been documented.

In this paper, we develop a new SG model extending Marshall and Clarke (1999) and Marshall et al.'s (2011) approach. We use hypsometric curves that account for a much larger set of topographic information than just the maximum, minimum and median elevation. We present a new slope parameterization to compute the velocities that accounts for SG slope statistics. An effective width is added for the representation of the ice fluxes between SG [levelsbins](#). In contrast to the one way communication used in the past, another modification to the original model is a two way exchange of ice between the SG and CG cells. The CG ice thickness updating accounts for SG ice thickness, and the SG model accounts for ice flux out of the CG [levelcell](#). For the first time, we evaluate the accuracy of the SG model against high resolution simulations by a higher order ice sheet model (ISSM, Larour et al., 2012). Sensitivities to the SG model configuration, such as the number of hypsometric [levelsbins](#), are assessed. We examine the extent to which the inclusion of further topographic statistics (e.g. the peak density in a region or the variance of the slopes) can improve computed sub-grid fluxes. We also evaluate the impact of [embedding](#) the SG model [on-in](#) the Glacial Systems Model (GSM, formerly the MUNGSM), [our coarse grid model](#), for last glacial cycle simulations of the North American ice complex, using an ensemble of parameter vectors from a past calibration of the GSM (Tarasov et al., 2012). Special attention is given to the impact of the coupling between the SG model and the GSM.

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<sup>1</sup>In this paper, "kyr" is used to represent time intervals and "ka" for time before present day.



## 2 Model description

### 2.1 Sub-grid model

~~This model expands the approach of and~~—The sub-grid model is a finite difference flow line model composed of a diagnostic equation for the ice velocities and a prognostic equation for ice thickness evolution. The surface mass balance is calculated using a Positive Degree Day (PDD) method. The elevation of a 3D region is parametrized using a hypsometric curve. Differences between the new SG model and the Marshall et al. (2011) approach are summarized in Table 1.

#### 2.1.1 Hypsometric curves

Marshall and Clarke (1999) built their hypsometric curves, representing the basal elevation of a region, synthetically from the minimum, maximum and median elevation of the topography. We generate the hypsometric curves from the GEBCO 1 km resolution digital elevation model (DEM) (BODC, 2010). To select a region fitting the coarse resolution grid cell of the GSM (degrees), the GEBCO Cartesian coordinates are converted in degrees assuming the earth as a perfect sphere of radius 6370 km. The curves are obtained in a two-step process. First, the region is divided into ~~10 levels (or bins)~~ N bins of equal altitude range ~~and the number of grid cells present in each of these bins is computed. Then, the size of these bins is updated to avoid empty levels.~~ Then, to avoid empty bins, the surface elevation range of each empty bin is expanded (consequently decreasing the elevation ranges of the higher and lower adjacent bins) until these three consecutive bins represent approximately the same surface area.

We use 1 km resolution gridded data, so that the area of each bin is proportional to the number of high resolution grid cells assigned to that bin. The alternative of using equal areas in each ~~level bin~~ has been discarded as it smooths the results in regions of low peak density. ~~10 levels bins~~ have been selected in this study, based on the comparison against

high resolution modelling (see Sec.3.1). Marshall and Clarke (1999) and Marshall (2002) used, respectively, 10 and 16 hypsometric levels-bins in their hypsometric curves.

At any time step,  $t$ , the surface slopes,  $S_k^t$ , for SG bin  $k$ , from 1 (highest) to N (lowest), are computed from the surface elevation  $h_{d,k}^t$ , and an effective length  $L_k$ :

$$S_k^t = \frac{h_{d,k}^t - h_{d,k+1}^t}{L_k} \quad (1)$$

To compute the slope at the lowest bin we assume an ice cliff boundary condition. The surface elevation  $h_{d,N+1}$  is set to the basal elevation of the lowest hypsometric bin  $h_{b,N}$ . Instead of setting the hypsometric slopes with an effective length proportional to the horizontal extent of the CG cell, ~~the surface slope length accounts~~ (Marshall and Clarke, 1999), we account for the cubic dependence of ice flow on surface slope (see Eq.3 and 4). Specifically, for each hypsometric level, bin we compute the slope,  $S_k^0$ , as the cube root of the mean of the cube of the magnitude of the slopes from the GEBCO data. The effective length,  ~~$L$ , used to update these slopes when ice starts to build up, is computed for each level as  $L_k$ ,~~ for SG bin  $k$  are computed from the basal elevation  $h_{b,k}$ :

$$L_k = \frac{(h_{b,k} - h_{b,k+1})}{\text{slope}_k} \frac{(h_{b,k} - h_{b,k+1})}{S_k^0} \quad (2)$$

where  ~~$k$  represents the different hypsometric levels, from 1 to N, and  $h_b$  is the basal elevation.~~ As no information is extracted about the basal elevation downstream of the terminal SG cell, the effective length at the first upstream level bin is used at the lowest hypsometric level bin. A small effective length can generate unrealistically high velocities at that level in that bin. To avoid this, the lowest level bin effective length is set to the mean effective length of all the hypsometric levels-bins when the altitude difference between the two lowest levels bins is less than 50 m. ~~At any time step,  $t$ , the surface slopes are updated using:~~

$$\text{slope}_k^t = \frac{h_{d,k}^t - h_{d,k+1}^t}{L_k}$$

To compute the slope at the lowest level, the surface elevation  $h_{d,N+1}$  is set to the basal elevation of the lowest hypsometric level  $h_{b,N}$  m.

We include a new width parameterization to improve the flow line model requires an effective width,  $W$ , for the representation of flux between hypsometric levels. The effective width bins,  $W$  of each hypsometric level bin is set to the number of grid cells, multiplied by the spatial resolution, that are in contact total contact length of the SG cells assigned to the bin with adjacent lower hypsometric levels grid cells bins grid cells as detailed in Fig.1.

### 2.1.2 Ice velocity Surface mass balance

We use the positive degree day method described in Tarasov and Peltier (1999) to compute accumulation and ablation from monthly mean temperature and precipitation. A constant environmental lapse rate adjusts the temperature to the ice surface elevation. A parameterization of the elevation-desertification effect (Budd and Smith, 1981) reduces the precipitation by a factor of two for every kilometre increase in elevation. Snow is melted first and the remaining positive degree days are used to melt ice with allowance for the formation of superimposed ice. The supplement includes a more detailed description of the surface mass balance module.

The GSM and ISSM compute the surface mass balance using the same PDD method.

### 2.1.3 Ice thickness evolution

The prognostic equation for the ice thickness ( $H$ ) is computed, at each hypsometric bin, from the vertically integrated continuity equation as:

$$\frac{\partial h_d}{\partial t} = \dot{M}_s - \nabla \cdot (\bar{u}H) = \dot{M}_s - \nabla \cdot (d S) \quad (3)$$

$S$  is the surface slope and  $\dot{M}_s$  is the surface mass balance rate (basal melt is computed in the CG GSM but ignored in the SG model).  $\bar{u}$  is the vertically integrated ice velocity of the SG model,  $\bar{u}$ , is computed derived using the shallow ice approximation (SIA). To solve the

equation semi-implicitly, we decompose the flux  $\bar{u}H$  to  $d S$  where  $d$  is the effective diffusivity given by:

$$d = \frac{2}{n+2} (\rho g)^n A_0 H^{n+1} \left( \frac{\partial h_d}{\partial x} S \right)^{n-1} \quad (4)$$

~~where  $H$  is the ice thickness and~~ The creep exponent  $n$  represents the creep exponent parameter of Glen's flow law ~~and~~ is set to 3.  $A_0$  is the creep parameter in  $\text{Pa}^{-3}\text{s}^{-1}$ ,  $\rho = 910 \text{ kg/m}^3$  and  $g = 9.81 \text{ m/s}^2$ . Ice flow is insignificant when the ice thickness is on the order of 10 meters. To avoid potential numerical instabilities, velocity is set to 0 if ice thickness is less than 20 m.

In their most recent experiments, Marshall et al. (2011) tuned their revised model against the present day total ice volume (encompassing 27% uncertainties) in the eastern slopes of the Canadian Rockies. This tuning sets the ice rheology parameter for an ice temperature equivalence of approximately  $-40^\circ\text{C}$ . As the SG model is used for regions that are either starting to accumulate ice or else deglaciating, basal ice temperature (where most deformation occurs) is likely close to freezing. The creep parameter is therefore fixed to a value corresponding to an ice temperature of  $0^\circ\text{C}$  using the Arrhenius relation from the EISMINT project (Payne et al., 2000).

#### 2.1.4 Surface mass balance

~~Positive degree day methods have been widely used in surface mass balance models. Here, we use the PDD method described in to compute the ice ablation and accumulation from the temperature and precipitation fields. Ablation rates are derived from monthly mean temperature ( $T_m$ ). To increase the accuracy, hourly temperatures are considered normally distributed, with a standard deviation ( $\sigma_{PDD}$ ) of  $5.5^\circ\text{C}$ , around the monthly mean. A lapse rate is also used to adjust the temperature forcing to the ice surface elevation. The number~~

of days where the temperature is above  $0^{\circ}\text{C}$  in a year is computed as:-

$$\text{PDD} = \frac{1}{\sigma_{PDD}\sqrt{2\pi}} \int_0^{1\text{year}} \int_{0^{\circ}\text{C}}^{T_m+2.5\sigma_{PDD}} T \exp\left[-\frac{(T - T_m)^2}{2\sigma_{PDD}^2}\right] dT dt$$

The amount of snow and ice are assumed to melt proportionally to the number of positive degree-days. Snow is melted first and the remaining positive degree-days are used to melt ice. The ablation rate factors for snow ( $\gamma_{\text{snow}}$ ) and ice ( $\gamma_{\text{ice}}$ ) have a mean June/July/August temperature ( $T_{jja}$ ) dependence extracted from energy balance modelling :-

$$\gamma_{\text{ice}} = \frac{17.22T_{jja}-1^{\circ}}{}$$

$$\frac{0.0067 \times (10 - T_{jja})^3 + 8.3-1^{\circ}}{< T_{jja} < 10^{\circ}}$$

$$\frac{8.310^{\circ}T_{jja}}{}$$

and

$$\gamma_{\text{snow}} = \frac{2.65T_{jja}-1^{\circ}}{}$$

$$\frac{0.15 \times T_{jja} + 2.8-1^{\circ}}{< T_{jja} < 10^{\circ}}$$

$$\frac{4.310^{\circ}T_{jja}}{}$$

In addition, the amount of superimposed ice for a year is computed as per :-

$$\min[P_r + M, 2.2 \times (P_s - M) - d \times c_i/L \times \min(T_{\text{surf}}, 0^{\circ}\text{C})]M < P_s, \min[P_r + M, d \times c_i/L \times \min(T_{\text{surf}}, 0^{\circ}\text{C})]$$

where  $P_r$  is the rainfall in a year,  $P_s$  is the snow fall in a year,  $M$  is the snow melt in a year, 2.2 is the capillarity factor,  $d$  is the active thermodynamic layer (set to 1),  $c_i$  is the ice specific heat capacity ( $152.5 + 7.122T$ ) in  $^{-1}$ ,  $L$  is the latent heat fusion ( $3.35 \times 10^5$ ) in  $^{-1}$ , and  $T_{\text{surf}}$  is the surface temperature.

A normal distribution of the hourly temperature is also used to compute the amount of snow accumulation from the precipitation. A lower standard deviation  $\sigma_{RS} = \sigma_{PDD} - 0.5$  is assumed in that case to account for the smaller temperature variability during cloudy days. Precipitation is assumed to fall as snow when the temperature is below  $2^\circ\text{C}$ .

$$\frac{\text{accumulation}}{\text{precipitation}} = \frac{\rho_i}{\rho_w \sigma_{RS} \sqrt{2\pi}} \int_0^{1\text{year}} \int_{T_m - 2.5\sigma_{RS}}^{2^\circ\text{C}} \exp \frac{-(T - T_m)^2}{2\sigma_{RS}^2} dT dt$$

A parameterization of the elevation-desertification effect reduces the precipitation by a factor of two for every kilometre increase in elevation. This exponential reduction is a function of the surface height difference to that of present-day with an ensemble parameter threshold for activation.

#### 2.1.4 Ice thickness evolution

The ice thickness is computed from the vertically-integrated continuity equation as:-

$$\frac{\partial h_d}{\partial t} = -\nabla \cdot (H) = -\nabla \cdot \left( d \frac{\partial h_d}{\partial x} \right)$$

$\dot{M}_s$  represents the surface mass balance rate (basal melt is ignored),  $d$  represents the effective diffusivity given by:-

$$d = \frac{2}{n+2} \frac{\rho g^n A_0 H^{n+1}}{\partial x} \frac{\partial h_d}{\partial x}^{n-1}$$

This prognostic equation Eq.3 is solved semi implicitly using a central difference discretization as:

$$\begin{aligned} \frac{\Delta x_k \Delta y_k}{\Delta t} (h_{b,k}^t + H_k^{t+1} - h_{b,k}^t - H_k^t) = \\ - d_k^t (h_{b,k}^t + H_k^{t+1} - h_{b,k+1}^t - H_{k+1}^{t+1}) \frac{\Delta y_k}{\Delta x_k} + \\ d_{k-1}^t (h_{b,k-1}^t + H_{k-1}^{t+1} - h_{b,k}^t - H_k^{t+1}) \frac{\Delta y_{k-1}}{\Delta x_{k-1}} + \dot{M}_s \Delta x_k \Delta y_k \quad (5) \end{aligned}$$

The superscripts  $t$  and  $t + 1$  represent respectively the current and the subsequent time step.  $\Delta x$  is the effective length  $L$  and  $\Delta y$  is the effective width  $W$  defined in Sec.2.1.1.

At the highest level bin, we assume that no ice flows into the region. At the lowest level bin ice is allowed to flow out of the region.

#### 2.1.4 Model limitations

The shallow ice approximation, used to compute fluxes, is formally invalid for high surface slopes such as present in mountain ranges like the Rockies. Simulating ice evolution over a 3D terrain using a flow line model limits the ice flow representation. Ice flows from one SG level bin to another using an average slope. Our model configuration does not allow for ice at high elevations to flow in into an adjacent coarse grid cell, ~~or~~. Nor does it allow for ice present at low elevations, in isolated regions having a closed drainage basin, to stay in a coarse grid cell. Moreover, the Arrhenius coefficient is computed with a constant ice temperature of  $0^\circ\text{C}$ . High velocities processes, such as periodical surges (Tangborn, 2013; Clarke, 1987), cannot be represented as basal sliding is since basal sliding and basal hydrology are not present in the current study.

The hypsometric length parameterization inferred from the surface slopes are correct for ice free regions, but it is only an approximation once the ice starts building up. At the lowest hypsometric level bin, slopes are computed assuming ice cliffs boundary conditions.

For the comparison against ISSM results, the surface temperature is downscaled with a lapse rate of  $6.5^{\circ}\text{C}/\text{km}$ . This typical value used in glacial modelling represents the average free-air lapse rate observed in the troposphere which need not match the impact of changing surface elevation. Studies over Iceland, Greenland, Ellesmere Island and the Canadian high Arctic report ~~seasonally~~seasonal changes in the surface temperature lapse rates over mountain regions and in the glaciers boundary layer with a mean annual value of about  $3.7^{\circ}\text{C}/\text{km}$  to  $5.3^{\circ}\text{C}/\text{km}$  (Marshall and Losic, 2011). Rates as low as  $2^{\circ}\text{C}/\text{km}$  are measured in the summer (Gardner et al., 2009). These values are tested in the GSM ensemble simulations where the lapse rate ranges between 4 and  $8^{\circ}\text{C}/\text{km}$ .



## 2.2 Ice Sheet System Model (ISSM)

As a detailed description of ISSM is given in , only a brief description of the model components used in this study are presented here. ISSM is a finite element 3D thermomechanically coupled ice flow model. The mass transport module is computed from the depth-integrated form of the continuity equation. Using the ice constitutive equation, the conservation of momentum provides the velocities. The model offers the option of computing the velocities using full Stokes, higher-order Blatter-Pattyn, shelfy-stream or shallow ice approximation equations. The higher-order Blatter-Pattyn approximation is used in this study. As the velocity equations depend on the temperature, this field is computed from conservation of energy, including 3D advection and diffusion. For this study, a new surface mass balance module identical to the one present in the sub-grid model, and detailed in Sec.2.1.2, has been incorporated in ISSM.

## 2.2 GSM

The core of the GSM is a 3D thermomechanically coupled ice sheet model. The model incorporates sub-glacial temperatures, basal dynamics, a visco-elastic bedrock response, climate forcing, surface mass balance, a surface drainage solver, ice calving and margin forcing. The grid resolution used for this study is  $1.0^\circ$  longitude by  $0.5^\circ$  latitude.

The thermomechanically coupled ice sheet model, described in detail in Tarasov and Peltier (2002), uses the vertically integrated continuity equation and computes the three-dimensional ice temperature field from the conservation of energy, taking into account 3D advection, vertical diffusion, deformation heating, and heating due to basal motion. Velocities are derived from the SIA equations. The sub-glacial temperature field is computed with a 1D vertical heat diffusion bedrock thermal model that spans a depth of 3 km (Tarasov and Peltier, 2007). If the base of the ice is at the pressure melting point, basal motion is assumed to be proportional to a power of the driving stress. The exponent for this Weertman type power law is set to 3 for basal sliding and 1 for till-deformation (detailed description in Tarasov and Peltier (2002, 2004)). The geographic location of the

sediment cover is determined from different data sets (Laske and Masters, 1997; Fulton, 1995; Josenhans and Zevenhuizen, 1990). Ice shelf flow is approximated with a linear function of the gravitational driving stress. At the base, ice melt is also computed from the energy balance.

The visco-elastic bedrock response is asynchronously coupled to the GSM with a 100 years interval. This module is based on the complete linear visco-elastic field theory for a Maxwell model of the Earth (Tarasov and Peltier, 2002, 2007).

At the surface, the parametrized climate forcing (Tarasov and Peltier, 2004, 2006, 2007) is based on a linear interpolation between the present day climatology, derived from a 14 year average (1982-1995) of the 2 m monthly mean reanalysis (Kalnay et al., 1996), and a last glacial maximum (LGM) climatology. The LGM climatology field is derived from a linear combination of PMIP I ~~or~~ and II general circulation models results with weighting the linear combination dependent on the maximum elevation of the Keewatin ice dome (PMIP I boundary conditions lacked a major Keewatin ice dome, while PMIP II had a large dome). The interpolation follows a glacial index derived from the GRIP  $\delta^{18}O$  record at the summit of the Greenland ice sheet (Dansgaard et al., 1993; World Data Center-A for Paleoclimatology, 1997). The surface mass balance is derived from this climatology using the same methodology as described in Sec.2.1.2. A surface drainage solver is fully coupled asynchronously at 100 ~~years~~ year time step. It diagnostically computes downslope drainage, filling any depressions (lakes) if drainage permits (Tarasov and Peltier, 2005, 2006).

The calving module, described in detail in Tarasov and Peltier (2004), is based on a height above buoyancy criterion with added mean annual summer sea surface temperature dependence. The inhibition of calving due to the presence of landfast sea-ice is also parametrized. To reduce misfits between the model results and geological evidences of the ice configuration, mass-balance ~~forcings~~ forcing are nudged to promote compliance with geologically inferred deglacial margin chronologies (Tarasov and Peltier, 2004).

~~The GSM has been subject to a Bayesian calibration against a large set of paleo constraints for the deglaciation of North America, as detailed in . We use a high-scoring sub-ensemble of 600 runs from this calibration.~~

## 2.3 GSM and sub-grid model coupling

In this section, we describe [how the SG model is embedded in the GSM and](#) the conditions applied to activate or deactivate the SG model in each [synoptic cell and how the two models exchange information about ice thickness, surface mass balance, and surface temperature.](#)

### 2.3.1 Sub-grid model activated/deactivated

Unlike ~~the SG model is activated only in cells above sea level with rough topography. A terrain is considered rough when the differences between the maximum and minimum basal elevation is higher than 500 m. To account for regions such as the Alaskan Peninsula where synoptic cells represent regions including basal topography both above and below sea level, cells where at least half of the area is above sea level are treated at the SG level. During inception, ice accumulates and can flow into valleys, filling them and thereby reducing the surface elevation variation. The SG treatment becomes less critical and is turned off when the lowest hypsometric level surface elevation reaches the bedrock elevation of the highest level. This criterion keeps the SG model activated for a longer period of time than in CG cell.~~ [The GSM is run, at all time, over all the CG cells and the ice thickness is updated in cases where the SG model is turned off when ice reaches the lowest level. During deglaciation, mountain peaks become uncovered and surface elevation variations increase, reaching a point where both ablation and accumulation are present. The SG model is reactivated when the ice thickness at the CG level is lower than half of the difference between the basal elevation of the highest hypsometric level and the basal elevation of the CG cell. This differs from who uses only SG information to set the threshold to a fraction of the variation in SG basal elevation.](#) [activated. Fig.2 give a summary diagram of the coupling between the GSM and the SG model.](#)

### 2.3.1 Interaction between the sub-grid model and the GSM

There is two way communication between the ~~CG-GSM~~ and SG models ~~-CG ice to exchange information about ice thickness, surface mass balance, and surface temperature. Ice in a CG cell~~ is added to the SG level<sup>2</sup> when the SG model ~~is turned on, and the switches from deactivated to activated for a given CG cell.~~ The information about the ice evolution at the SG level is used to update the ice thickness, surface mass balance rate and surface temperature at the CG level<sup>2</sup>.

Marshall et al. (2011) export SG ice to the CG level only when the lowest SG ~~level bin~~ is filled and the SG model for the given CG cell is deactivated in the timestep. In our model, SG/CG ice transfer is as follows. While the SG model is ~~turned on/activated~~, CG ice volume is set to that of the ~~SG levels below the lowest unfilled level/filled SG bins~~. The rationale for this is the assumption that over a large mountainous region, such as the Rockies, an ice sheet grows by building up ice in major valleys (represented by the lowest hypsometric ~~levels bins~~) from ice accumulation and ice flowing ~~in~~ from surrounding mountain peaks. A SG ~~level bin~~ is classified as filled once its surface elevation reaches the basal elevation of the adjacent higher ~~level bin~~. The surface mass balance rate and surface temperature of the ~~synoptic CG~~ cells are updated to the new elevations. When the SG model ~~is turned off/switches from activated to deactivated~~, the total SG ice volume is transferred to the CG cell.

Once the SG model is re-activated in a ~~synoptic grid CG~~ cell during deglaciation, the ice volume present at the CG level is distributed over the different hypsometric ~~levels bins~~. To account for the higher volume of ice in valleys, represented by the lowest hypsometric ~~levels bins~~, the average of the following two mass-conserving distributions is used for SG initialization. The first is even distribution across every ~~level bin~~. The second keeps equal surface elevation for the lowest ~~levels bins~~, starting from the lowest ~~level bin~~ and using as many ~~levels bins~~ as necessary.

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<sup>2</sup>~~SG level represents the hypsometric curve while CG level correspond to a GSM cell.~~

Marshall and Clarke (1999) have no ice flux to adjacent CG cells when the SG model is active. In our model, ice transport between CG cells, computed with the GSM, is modified using SG information. We assume that only the ice present below the lowest unfilled level in the filled bins flows out of the coarse grid region; therefore, only a fraction of the CG flux is permitted. This ratio fraction is computed as the area of the SG levels below the lowest unfilled level over the total area filled SG bins divided by the total CG cell area. To avoid double counting of this inter CG flux, the SG model does not compute flux out of the lowest bin through Eq.3 when coupled to the GSM. At every iteration, the SG model accounts for the CG ice flux. For CG ice flux into a cell with active SG, the ice fills the lowest hypsometric level bin. Once that level bin reaches the elevation of the next higher level bin, the remaining ice is used to fill up the two levels bins at the same elevation. This process is repeated using as many levels bins as necessary to redistribute all the ice. For CG ice flux out of the cell, the same amount of ice is removed from all the SG levels lower than the lowest unfilled level filled SG bins. If the total volume of ice to be removed is not reached using that region of the SG cell, the excess remaining is used to empty higher levels bins one after another.

The SG model flux module is coupled asynchronously and runs at half the SG mass balance time step. Glacial isostatic adjustment from the CG model level is imposed on the SG basal topography.

### 2.3.2 Sub-grid model activation/deactivation

Unlike Marshall and Clarke (1999), the SG model is activated only in cells above sea level with rough topography. A terrain is considered rough when the differences between the maximum and minimum basal elevation is higher than 500 m. To account for regions such as the Alaskan Peninsula where CG cells represent regions including basal topography both above and below sea level, cells where at least half of the area is above sea level are treated at the SG level. During inception, ice accumulates and can flow into valleys, filling them and thereby reducing the surface elevation variation. The SG treatment becomes less critical and is deactivated when the lowest hypsometric bin surface elevation reaches the bedrock elevation of the highest bin. This criterion keeps the SG model activated

for a longer period of time than in Marshall and Clarke (1999) where the SG model is deactivated when ice reaches the lowest bin. During deglaciation, mountain peaks become uncovered and surface elevation variations increase, reaching a point where both ablation and accumulation are present. The SG model is reactivated when the ice thickness in the CG cell is lower than half of the difference between the basal elevation of the highest hypsometric bin and the basal elevation of the CG cell. This differs from Marshall and Clarke (1999) who uses only SG information to set the threshold to a fraction of the variation in SG basal elevation.

### 3 Sub-grid surface mass balance model performance

#### 2.1 Ice Sheet System Model (ISSM)

As a detailed description of the ISSM is given in Larour et al. (2012), only a brief description of the model components used in this study are presented here. The ISSM is a finite element 3D thermomechanically coupled ice flow model. The mass transport module is computed from the depth-integrated form of the continuity equation. Using the ice constitutive equation, the conservation of momentum provides the velocities. The model offers the option of computing the velocities using full stokes, higher-order Blatter-Pattyn, shelfy-stream or shallow ice approximation equations. The higher-order Blatter-Pattyn approximation is used in this study. As the velocity equations depend on the temperature, this field is computed from conservation of energy, including 3D advection and diffusion. For this study, a new surface mass balance module identical to the one present in the sub-grid model, and detailed in Sec.2.1.2, has been incorporated into the ISSM.



### 3 Sub-grid model performance and tests

~~We compare the results of the SG model and ISSM before testing alternative parameterizations.~~ The SG model computation time for 3000 years simulation, using 10 hypsometric bins, is about 0.02 seconds. At a resolution of 1 km and using 10 cpus, ISSM run time is about 2 to 5 hours (depending of the topographic region used). The sub-grid model adds 3 to 6 hours (depending of the parameter vector used) to the glacial cycle run-time over North America.

#### 3.1 Comparison with ISSM

We ~~use compare~~ use compare 2 kyr ISSM and SG simulations, applying constant sea level temperature and precipitation over an inclined bed and 21 different test regions in the Canadian Rockies. These regions, for both the ISSM and SG simulations, have a dimension of 30 km by 60 km and ~~the topographic data are available at a resolution we use a DEM~~ of 1 km resolution. To improve correspondence between ~~ISSM and the~~ ISSM and the SG model, the minimum ice thickness allowed in the SG model is set to 10 m, ~~and no basal sliding is allowed in ISSM~~. The boundary conditions at the ice margin in the ISSM are computed as an ice-air interface. To isolate the impact of using the SIA to represent fluxes in a mountainous region containing steep slopes in the hypsometric parameterization, our current experiments have no basal sliding. As glaciers can experience surging (via significant sliding) in this type of region, the next stage of this project will include sliding.

##### 3.1.1 Inclined plane test

The bed topography for this test is an inclined plane topography with a constant slope of 0.014 and a maximum basal elevation of 750 m. For this case, the accuracy of the SG model correlates with the number of hypsometric levels bins as shown in Fig. ~~??~~ 3 (ice and velocities profiles ~~shown in the supplement, are shown in supplemental~~ Fig.S1). Reducing the number of SG levels bins increases the surface gradient between two hypsometric levels bins and thereby the computed ice velocities. With 10 hypsometric levels bins, the ice vol-

ume simulated by the SG model can be as low as 40% of [the](#) ISSM prediction. The misfits are not significant in simulations where no ablation is present (e.g. for a temperature set to  $-5^{\circ}\text{C}$ ).

### 3.1.2 Rocky Mountains test

The SG model is tested on 21 regions from the Canadian Rockies, representing a wide range of topographic complexity (e.g. Fig. [??4a](#)), altitude (e.g. Fig. [??4b](#)) and slopes (e.g. Fig. [??4c](#)). The slopes of these regions are higher than in the inclined plane case. We focus on the results for simulations over the six test regions in Fig. [??4](#) forced with sea level temperature of  $0^{\circ}\text{C}$  and a desertification effect factor of 0.5. The results of other simulations, using different regions and with similar [forcings forcing](#) as used in the inclined plane experiments, are not shown as they present similar misfits [with ISSM against ISSM results](#).

In contradiction with the simplified inclined plane configuration, increasing the number of hypsometric [levels bins](#) does not reduce the misfits with ISSM simulations (Fig. [??5](#)). The SG model does not account for the build-up of ice in closed drainage basins where no flow is permitted out of the region before a threshold elevation is reached. Another complication for the "real" topography scenario comes from topographic "jumps" not addressed in the SG model. Some high resolution adjacent grid cells belong to non-adjacent hypsometric [levels bins](#). The ice flow between these two locations is not accurately captured. The number of "jumps" increases with the number of [levels bins](#) used (Fig.S2 in the supplement). 10 hypsometric [levels bins](#) are then used to limit this effect. Even so, the SG model generates 45% less to 15% more ice than ISSM simulations (25% less on average), depending on the regional topographic characteristics. No relation was found between the geographic complexity and the performance of the model, as explained in Sec.3.2.

### 3.2 Test of alternative parameterizations

We examine the impact of including more topographic characteristics such as in the velocity parameterization. Characteristics considered include: the flow direction, the terrain ruggedness (measured as the variation in three-dimensional orientation using a radius of 5 grid cells around the grid cell of interest), the sum of the squared slopes, the variance in the slopes, the number of local maxima (tested with radius sizes of 2, 6 and 10 grid cells) and the standard deviation of the surface elevation topography in the velocity parameterization.

The ISSM and the sub-grid model were run until steady state (2 kyr) for simulations with a constant precipitation rate of 1 ~~mm/yr~~-m/yr and a sea level temperature forcing of 0°C. The parameters minimizing ice volume differences were selected using a stepwise multilinear regression fit. The flow direction and the mean of the ~~slope squared~~ squared slope do not reduce the misfits. The slope variance does not improve the results when combined with the ~~other two parameters~~ remaining two parameters (elevation standard deviation and terrain ruggedness). When used alone, it does reduce the errors, but not as well as when the standard deviation of the topography is used. The terrain ruggedness and the peak density both represent the same physical characteristics and do not improve the results when used alone. Improvements are obtained when combined with the standard deviation of the topography. However, the improvement is not greater than with the standard deviation alone. The standard deviation of the topography is the parameter that correlates the most with the misfits. The average absolute value of the differences between the SG model and ISSM average ice thickness is 61 ~~mm~~. This difference is reduced to 21 m (see Fig. ~~??6~~) when the regression model generated using the standard deviation of the topography is used. More details about the results of the stepwise regression fits are provided in the supplement.

To explore potential improvement from accounting for the standard deviation of the high resolution topography,  $S_{std}$ , we test the following parameterization of the velocity,  $\bar{u}_1$ :

$$\bar{u}_1 = \frac{2}{5} (\rho g)^3 A_0 \left( P_1 H S_{std}^{P_2} \right)^{P_3} \left( \frac{\partial h_d}{\partial x} \right)^3 \quad (6)$$

This equation is used in a simulation initialized with the ice thickness, velocities and slopes of ISSM values at steady state ~~used in the stepwise regression fit section~~. The parameter  $P_1$ ,  $P_2$  and  $P_3$  (respectively 4.87, 0.016 and 2.8) are obtained using a least ~~square~~ squares approach that minimizes the differences between the velocities computed by ISSM and the SG model after one iteration (0.01 year).

The lowest hypsometric ~~level have~~ bin has the most significant misfits (e.g. Fig.S4 in the supplement). This is likely related to the margin ice cliff slope parameterization. To try to correct this, we test the following parameterization for the lowest hypsometric ~~level~~ bin velocity:

$$\bar{u}_{2,N} = \frac{2}{5} (\rho g)^3 A_0 H_N^4 \left( P_4 H_N^{P_5} \frac{\partial h_{d,N}}{\partial x} \right)^3 \quad (7)$$

Using the same least-squares approach as above, the parameters  $P_4$  and  $P_5$  are respectively set to 5924.4 and -1.6383.

These two parameterizations do not reduce the ice thickness differences with ISSM transient results (see Fig.???) . Ice thickness, velocities and slopes over the six regions analyzed are presented for the different parameterizations in Fig.S5 of the supplement. As the model is highly non-linear, the improvement generated by the least squares fit method for an initialization with ISSM steady state conditions does not persist over thousand year runs.

The following modifications of the current version of the SG model have been explored, but did not improve the model. The central difference discretization of the ice thickness in the effective diffusivity coefficient was replaced by an upwind scheme. Simulations with different values of the Arrhenius coefficient, the power of the ice thickness and the slope,

in Eq. ??, were run4}, were analyzed. An extra parameter was added in the velocity equation to account for neglected stresses. Turning off the internal SG model flux term increased significantly the misfits with ISSM simulations by a minimum of 100% (as shown in Fig. ??8). The basal elevation downstream of the terminus has been computed using a linear extrapolation of two or three upstream levelsbins. The lowest hypsometric level-bin effective length generated with these basal elevations did not reduce the misfits with ISSM results.

## 4 Behaviour of the sub-grid model in the GSM

We present results ~~from a 600 member ensemble of simulations~~ of simulations over the last glacial cycle. ~~We also examine~~ The 39 "ensemble parameters" of the GSM (attempting to capture the largest uncertainties in climate forcing, ice calving, and ice dynamics) have been subject to a Bayesian calibration against a large set of paleo constraints for the deglaciation of North America, as detailed in Tarasov et al. (2012). We use a high-scoring sub-ensemble of 600 parameter vectors from this calibration to compare the GSM behaviour when the SG model is turned on and off. The primary supplement of Tarasov et al. (2012) includes a tabular description of the 39 ensemble parameters as well as input data sets. For the purposes of clarity and computational cost, we examined model sensitivity to different coupling and flux parameters using five parameter vectors (of the 600 members ensemble) that gave some of the best fits to the calibration constraints. As these five parameter vectors display similar behaviour, we present sensitivity results using the parameter vectors for the two runs described in detail in Tarasov et al. (2012) (identified in that paper as runs nn9894 and nn9927). For ease of interpretation, the ice volumes are presented as eustatic sea level (ESL) equivalent<sup>3</sup>.

### 4.1 Last glacial cycle simulations over North America

The SG model can significantly alter the pattern of ice accumulation and loss. Fig. ~~??~~9 shows an example ~~of SG ice accumulating while melting in the CG model,~~ for one of the parameter vectors of the ensemble of simulations, where SG ice accumulates while it melts at the CG level (Fig. ~~??~~9a), and an example where CG ice is about 60% greater than the SG ice (Fig. ~~??~~9b).

~~An~~ The ensemble of simulations of the last glacial cycle over North America with the SG model activated ~~generate~~ generates, on average, between 0 and 1 mESL more ice than when the SG model is turned off (Fig. ~~??~~10).

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<sup>3</sup>using a conversion factor of  $2.519 \text{ mESL}/10^{15} \text{ m}^3$  of ice

The impact of the SG model depends, however, on the climate forcing and the ice sheet extent and elevations. During inception, when the SG model is ~~activated~~turned on, ice accumulating in higher regions, flows downhill and accumulates in regions close to the ELA and in valleys (Fig.??11). This allows, for example, ice to build up in the northern part of Alaska. For typical runs, the ice generated by the SG model in the Alaskan peninsula is, however, insufficient as compared to geological inferences (Dyke, 2004). The ensemble ~~run~~-mean and standard deviation of the differences between runs with SG on and off at 110 ka, are respectively 0.4 and 1 mESL. However, at specific timeslices, the differences can be much larger. Once the ice sheet has grown to a sizeable fraction of LGM extent, for example at 50 ka, the standard deviation of the ensemble run differences ~~can reach~~ (between SG on and off) reaches 5 mESL. Fig.??12 shows an example where ice in a region of low altitude in the centre of Canada is not allowed to grow when the SG model is used. On the other hand, a simulation using different ensemble parameters generates ice in this region only when the SG model is turned on (Fig.S6 of the supplementary material). In extreme cases, differences can reach tens of mESL (Fig.S7 in the supplement). We could not identify a reason for the strong sensitivity of ice volume around 50 ka other than the inherent non-linearity of the GSM.

## 4.2 Sensitivity of the model to different flux and coupling parameters

~~Five of the best fit to calibration constraints parameter vectors obtained from the glacial cycle calibration are used in this section. We focus on results from one of the ensemble parameter vectors as all 5 runs, unless otherwise stated, display similar behaviour.~~

~~Turning off the The accounting of SG fluxes has varying impacts over a glacial cycle simulation (Fig. ~~??~~13). At 50 ka, for example, the ~~flux module allows ice, accumulating in higher elevation regions, to flow into the ablation zone and reduces the total ice volume with parameter vector nn9894 is reduced~~ by 50%. ~~The same process is observed during deglaciation when SG fluxes are included. During inception, on the other hand, inclusion of SG fluxes increases the total amount of CG ice (Fig. S8b in supplement). During inception, the flux module transports ice to 14, again with nn9894).~~~~

To better understand the range of responses to CG ice flow between grid cells that have SG activated, three case scenarios can be considered. Case 1: ice flows out of the lowest SG bins located above the ELA into the lowest SG bins located above the ELA of another CG cell. There is limited impact of not allowing ice to flow out of the CG cell as in both cases ice accumulates. Case 2: ice flows out of the lowest SG bins located above the ELA into the lowest SG bins located below the ELA at a greater rate than it can be melted, thereby increasing the total amount of CG ice of another CG cell. In that case, turning off the fluxes between CG cells tends to reduce the total melt. Case 3: ice flows out of the lowest SG bins located below the ELA into the lowest SG bins located below the ELA of another CG cell. Ice flowing into lower SG bins generates higher melting rates so permitting fluxes between CG cells will in this case tend to increase ice mass loss. In cases 2 and 3, the combination of ice flowing below the ELA from the adjacent CG cell and from the bins above the ELA can raise the surface elevation of lower bins above the ELA and reduce the melt. Depending on the proportion of each of these cases, not allowing ice fluxes out of coarse grid cells with SG activated generates higher or lower ice volumes (Fig. S8a in supplement) – 13). 50 ka is an example of a 60% increase of the total ice volume when the fluxes out of coarse grid



cells (with SG activated) are not allowed. As a counterpose, 35 ka presents a case where turning off the fluxes out of (SG activated) coarse grid cells decreases the total ice volume.

With Marshall et al.'s (2011) flux equation, differences between runs with SG fluxes turned on versus off are **smaller (Fig.S8 in supplement)**.

**As expected from the comparison between ISSM and the SG model results, not allowing ice fluxes out of coarse grid cells with SG active generates ice volumes up to 60% higher at 50 negligible over the full glacial cycle (Fig.??14).**

As described in Sec.2.3.1, the CG ice thickness used by the GSM, conserves the ice volume of the **SG levels under the lowest unfilled level filled SG bins** (Volume Conservation, VC, method). As this ice is redistributed over the total area of the coarse grid cell, the surface elevation of the ice, and consequently the fluxes, are underestimated. The surface gradient between adjacent cells is then lower than the gradient at the SG level. We tested setting the **CG** surface elevation to the maximum value between the surface elevation of the coarse grid cell and the lowest hypsometric **level bin** (Surface Conservation, **SC**, (**SC**) method). We also implemented a method using the maximum **of surface elevation generated by** the two former methods (Maximum Conservation, **MC**, (**MC**) method). During inception, **(between 118 to 114 ka)** the VC method generates **up to between 10% and 20% (which is equivalent to 0.5 to 1 mESL)** more ice than the two other methods (Fig.??). **This 15). During the first 60 kyr of simulation, the difference in total ice volume stays under 1 mESL independently of the flux redistribution methods (Fig.S8 in the supplement). Between 60 ka and the LGM, the SC method generates between 1 and 12 mESL less ice than the two other methods (Fig.S8). The VC method was used for the ensemble runs as it generates more ice over Alaska peninsula, Northern and Southern mountain range,** thereby reducing misfits against geological inferences. **After inception, the flux redistribution methods have different impacts (Fig.S9 in the supplement) as they start from different ice configuration.**

**The impact of the Fig.16 shows the results of the glacial cycle simulation when the SG model is turned off and when the minimum altitude variation SG activation threshold is more complex. Fig. ?? shows a set to 50, 150, 300 and 500 m. A non-linear dependence on the threshold. Simulations can be observed. At 50 ka, for example, setting the threshold to 50**

m generates the lowest total ice volume while a threshold of 150 m lead to the highest ice volume. The difference between these two runs is 34.5 mESL at 50 ka. Thresholds of 300 and 500 m generate intermediate total ice volumes. Moreover, simulations using different parameter vectors (not shown) result in different behaviours. No conclusion could be drawn about the optimal threshold.

## 5 Conclusions

Our new sub-grid surface mass balance and flux model extends the initial work of Marshall and Clarke (1999) and Marshall et al. (2011). The evaluation of the model, done for the first time against results from a high resolution higher order model ~~ISSM~~ (ISSM), demonstrates that:

- ~~Accounting for accumulation and ablation at different SG levels alters the ice volume evolution in a GSM cell.~~

Depending on the regional topographic characteristics, the new SG model simulates ice volumes 45% lower to 15% higher than simulated by the ISSM (using 10 hypsometric levels bins). Increasing the number of hypsometric levels bins to more than 10 did not reduce misfits for simulation simulations over rough topographic regions extracted from the Canadian Rockies.

- Turning off the SG internal fluxes ~~significantly increase~~ increases the ice volume misfits with ISSM simulations by a minimum of 100%.
- Increasing the number of topography characteristics used in the SG model, as suggested by Marshall and Clarke (1999), did not reduce the misfits with the high resolution model during transient runs. ~~The topographic characteristics tested in the alternative parameterization were: the flow direction, the terrain ruggedness, the sum of the squared slopes, the variance in the slopes, the number of local maxima and the standard deviation of the surface elevation topography. The latter did improve the fit in the single time step tests.~~

An ensemble of simulations over the last glacial cycle of the North American ice complex shows, on average, an increase of ice generated with inclusion of the SG model. The ensemble mean for each time step is between 0 and 1 mESL (with a standard deviation up to of a minimum of twice the mean and reaching 5 mESL at 50 ka) of ice generated with inclusion of the SG model. At the end of inception, at 110 ka, the SG model increases the ice volume on average by 0.4 (with a standard deviation up to 1) but still does not generate sufficient ice in increase of ice volume from SG model inclusion is still insufficient over the Alaskan peninsula when compared to geological inferences. Over the glacial cycle, the SG model generates different patterns of ice extent. In some instances, the SG model prevents ice growth, while in others it enables extra ice build up over thousands of square kilometres.

Simulated ice evolution is sensitive to the treatment of ice fluxes within the SG model and between the SG and CG levels.

- The flux term has an important impact on the SG model. Not allowing ice to flow between hypsometric levels generates up to bins increase the total ice volume with a maximum increase of 50% more ice at 50 ka (in a glacial cycle run). During inception, however, the flux module can generate more ice. Different parameterizations of the flux term impact the results. A SG ice rheology parameter corresponding to ice at about  $-40^{\circ}\text{C}$  (as used in Marshall et al., 2011) generates the same amount of ice during inception as when the flux term is off.
- The flux term used in Marshall et al. (2011) study, with the ice rheology parameter representing ice at about  $-40^{\circ}\text{C}$ , generates an ice volume higher than when a flux parameterization with a rheology value representing ice at about  $0^{\circ}\text{C}$  is used.
- Not allowing ice to flow out of a CG cell where SG is activated increases the total amount of ice by up to 60% at or decreases the total ice volume depending of the ice configuration. At 50 ka, the total increases by 60%.
- The ice configuration from simulations over the last glacial cycle of North America is sensitive to the choice of SG to CG ice redistribution scheme.

We have identified the representation of SG fluxes between CG cells to be a challenging issue that can significantly impact modelling ice sheet evolution.

We have shown that the above geometric and ice dynamics factors can have significant impacts on modelled ice sheet evolution ([with up to a 35 mESL difference in North American ice volume at 50 ka](#)). Therefore, significant potential errors may arise if subgrid mass-balance and fluxes are not accounted for in the coarse resolutions required for glacial cycle ice-sheet models. [Other alternatives to the hypsometric parameterization, such as running a high resolution SIA model in the region of rough topography, could be considered.](#) One issue we have not examined is the downscaling of the climatic forcing. Temperature and especially precipitation can exhibit strong vertical gradients in mountainous region. Whether this can have significant impact on CG scales is unclear. Improvements of the precipitation representation are possible using, for instance, a linear model of orographic precipitation for downscaling climatic inputs Jarosch et al. (2012).

## 6 **Author contribution**Code availability

The sub-grid code is available upon request from the first two authors.

## 7 **Author contribution**

Kevin Le Morzadec and Lev Tarasov designed the experiments. Kevin Le Morzadec developed the SG model code and performed the simulations. Kevin Le Morzadec and Lev Tarasov coupled the SG model into the GSM. Mathieu Morlighem and Helene Seroussi supported ~~the installation of ISSM and helped including the new module in~~ ISSM installation and helped build a new surface mass balance module for the ISSM. Kevin Le Morzadec prepared the manuscript with contributions from Lev Tarasov and the other co-authors. Lev Tarasov heavily edited the manuscript.

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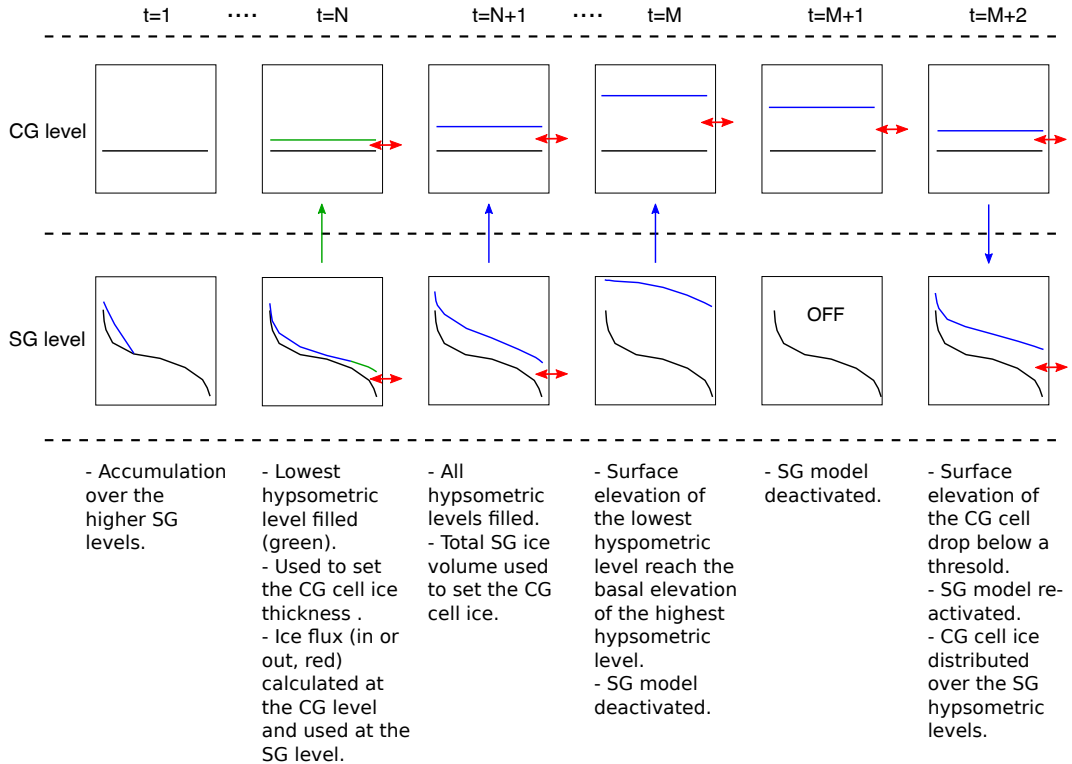
World Data Center-A for Paleoclimatology, National Geophysical Data Center, B.: The Greenland Summit Ice Cores CD-ROM, CD-ROM, 1997.

**Table 1.** Differences between our new SG model and Marshall and Clarke (1999)/Marshall et al. (2011) models

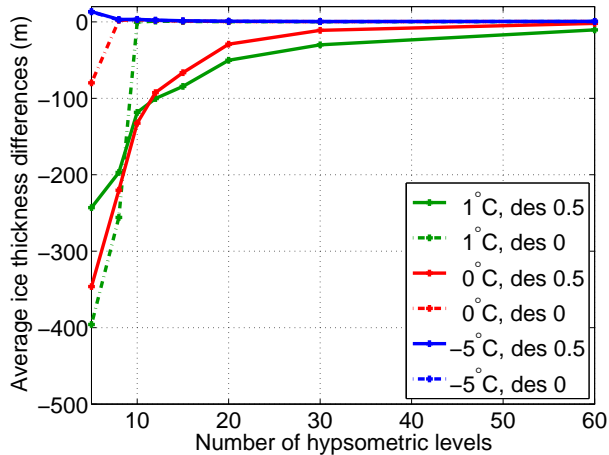
	New SG model	Marshall's model
<b>Hypsometric curves</b>		
Elevation	Computed from the DEM	Min, max and median elevation
Effective lengths	Computed from the DEM slopes	$\propto 50$ km
Effective Width	$\propto$ Number of cells in contact with adjacent bins	Not included
Number of bins (N)	10	10 to 16
<b>SG fluxes</b>		
Approximation	SIA	SIA
Ice rheology ( $T^\circ$ equivalence)	$0^\circ\text{C}$	$-40^\circ\text{C}$
<b>SG model activation</b>		
topographic mask for activation	Rough topography ( $\Delta h_b > 500\text{m}$ ) and at least half of the CG basal elevation is above sea level	Every grid cell
Deactivated	Lowest SG bin surface elevation reaches the bedrock elevation of the highest bin	Lowest SG bin filled
Activation	$H_{CG} < \frac{h_{b,SG}(top) - h_{b,CG}}{2}$	$H_{CG} < \frac{h_{b,SG}(top) - h_{b,SG}(bottom)}{\text{number of SG bins}}$
<b>SG <math>\rightleftharpoons</math> CG</b>		
Ice thickness	SG to CG: While SG is activated, $H_{CG}$ is set to the $\frac{\text{total ice volume}}{CG_{cell\ area}}$ of the filled SG bins at each CG timestep (the total SG ice volume is used during the deactivation timestep) CG to SG: When SG switches to activated, the CG ice volume is redistributed over the SG bins using the mean between two methods: equal redistribution over all bins and redistribution of ice over the lowest bins	When the lowest SG bin is filled  Not explained
Isostatic adjustment	CG elevation adjustment applied to all SG bins	Not included
<b>Flux to adjacent CG cells</b>		
	Fluxes computed with the CG model and reduced by the area fraction of the lowest filled SG bins Fluxes in or out of a CG cell redistributed over the lowest SG bins in the adjacent CG cell(s)	Fluxes computed with the CG model only No flux out of the CG cell treated at the SG level Fluxes coming from an adjacent CG cell to a SG cell redistributed over the lowest SG bin

							6	6	5
		6				8	7	7	5
5	6	7	5				8	7	6
8	7	7	6				9	9	8
8	7	7	6						
8	6	6	5						
					8	6	6		
					8	7	6		
					8	8	6		

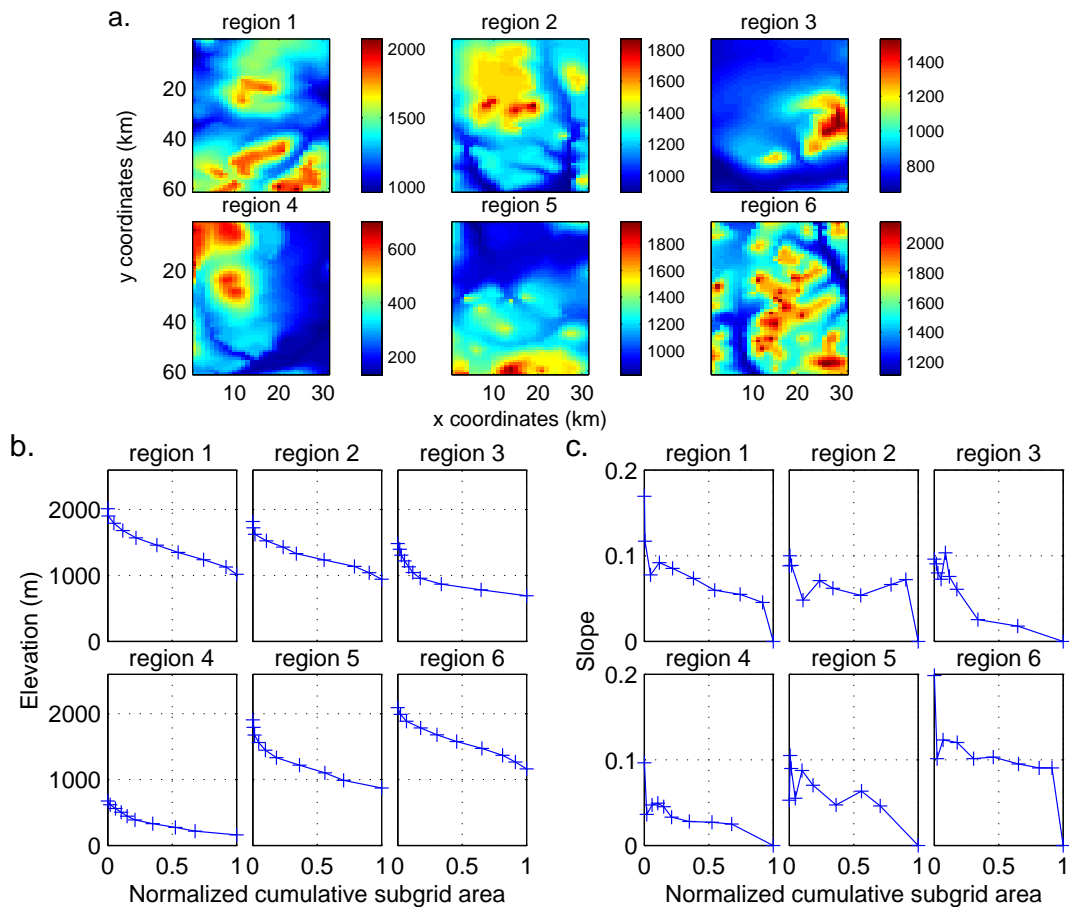
**Figure 1.** Schematic representation of the effective width of the 7<sup>th</sup> hypsometric bin for a region of 10 km by 10 km. Each square represents a high resolution (1 km) grid cell. The numbers define the hypsometric bin these SG grid cells belong to. The total length of all red lines (14 km) represents the effective width for the 7<sup>th</sup> bin.



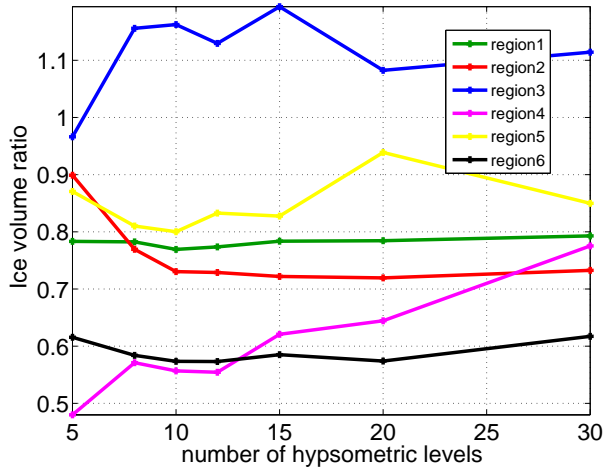
**Figure 2.** Communication between the GSM and the SG model for one CG cell.



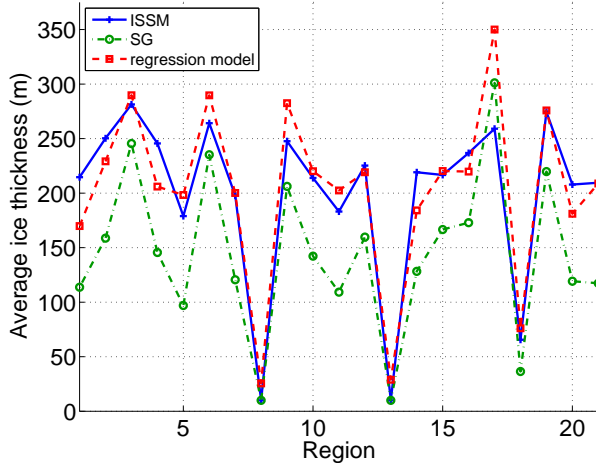
**Figure 3.** SG model vs. ISSM differences over an idealized inclined plane terrain. Average ice thickness differences (SG model - ISSM) are presented for simulations using different temperatures, desertification effect factors and hypsometric [levelsbins](#)' number.



**Figure 4.** Topography characteristics for 6 regions over the Canadian Rockies. **a.** summarizes surface elevations, **b.** the hypsometric curves, and **c.** the mean slope for each hypsometric level bin.

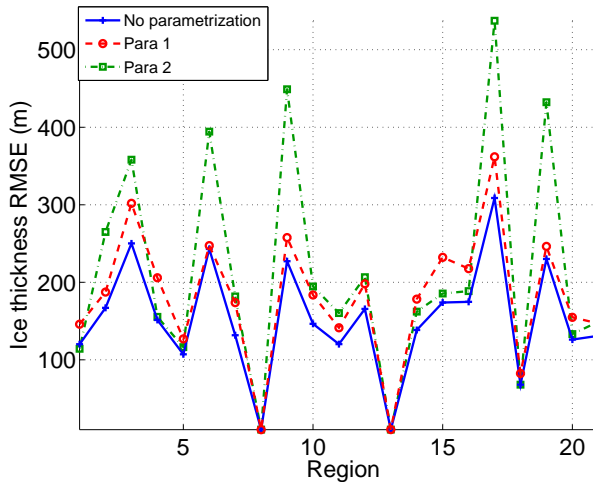


**Figure 5.** Ratio of the SG model over ISSM total ice volume for six different regions in the Rockies as a function of hypsometric [levelsbins](#). The simulations were run until steady state with a constant sea level temperature of  $0^{\circ}\text{C}$  and a desertification effect factor of 0.5. The steady state ice thicknesses, velocities and slopes from [the](#) ISSM and the SG model (using 10 hypsometric [levelsbins](#)) are presented in Fig.S3 of the supplement.

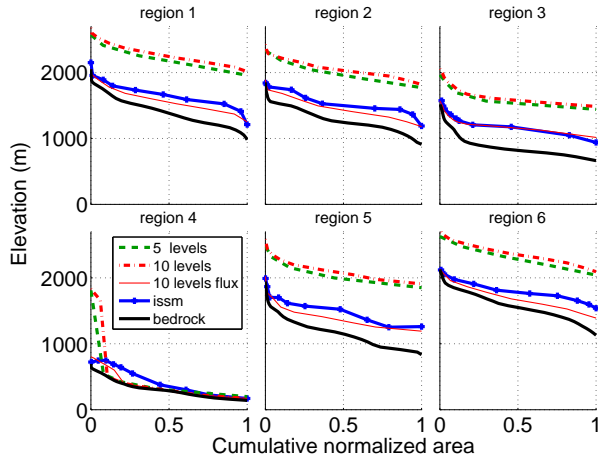


**Figure 6.** Average ice thickness in m for different topographic regions in the Rockies. Results are shown for [the](#) ISSM, the regression model (generated by the stepwise regression fit including only the standard deviation of the topography) and the SG model using 10 hypsometric [levelsbins](#).

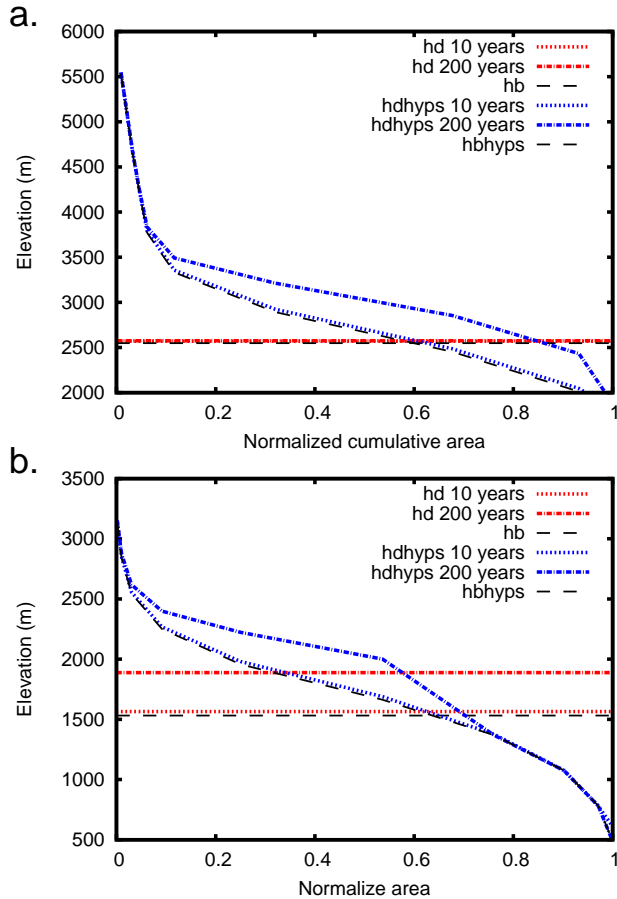




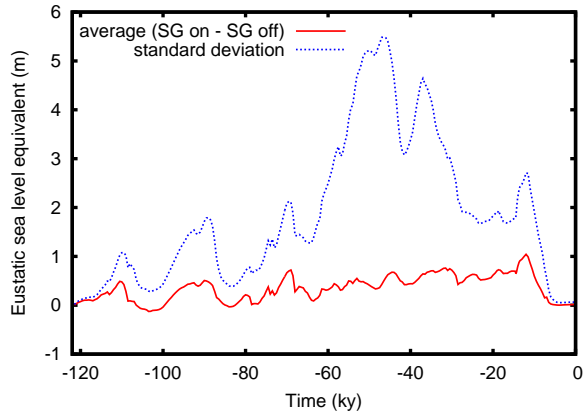
**Figure 7.** Average ice thickness root mean square error (RMSE) between [the](#) ISSM and the SG model for different topographic regions. Simulations are run over 2 kyr using a constant precipitation rate of 1 mm/yr-m/yr and a sea level temperature forcing of 0°C. Different SG parameterizations are presented. Para 1 is the standard deviation of the topography parameterization and Para 2 the lowest hypsometric slope parameterization.



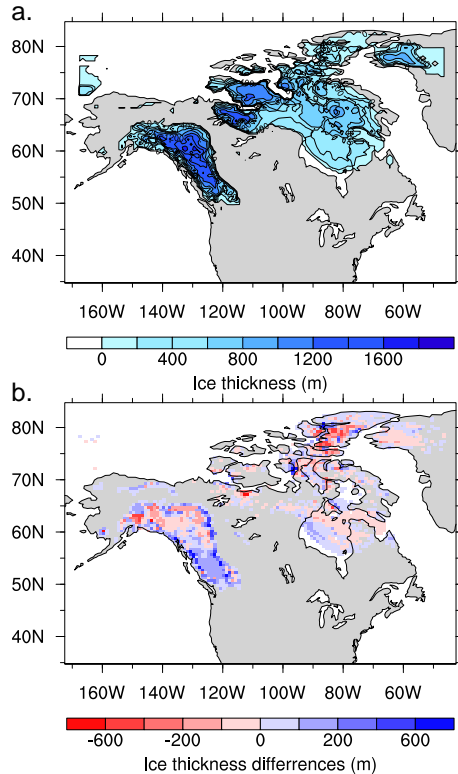
**Figure 8.** Surface elevation generated by [the](#) ISSM (solid blue line), the SG model with no flux term, using 5 and 10 hypsometric [levels](#)bins, (dotted lines) and the SG model including the flux term (solid thin red line). These simulations use a constant sea level temperature of  $0^{\circ}\text{C}$  and a desertification effect factor of 0.5. Results are shown at steady state after 2 kyr for six different regions with different topographic characteristics.



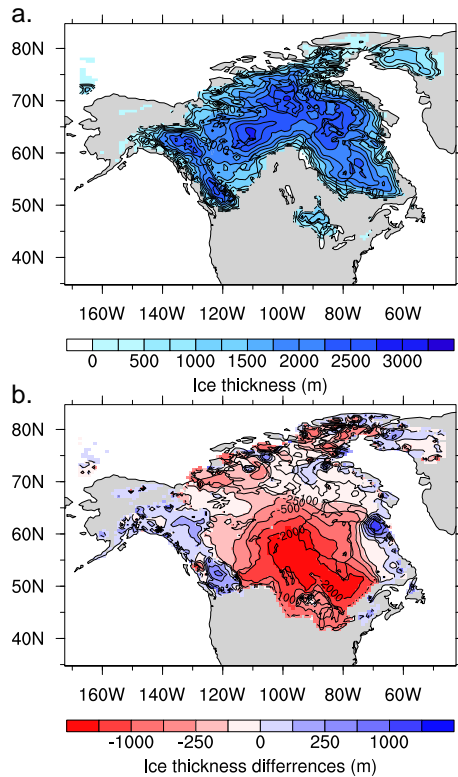
**Figure 9.** Elevations comparison when the SG model is turned on (blue) or off (red) at different time steps [using the parameter vector nn9894](#).  $h_d$  10 years is the CG surface elevation after 10 years.  $h_{dhyps}$  10 years is the SG surface elevation.  $h_b$  is the basal elevation. **a.** And **b.** represent cases where the ELA is above and below the coarse grid surface elevation.



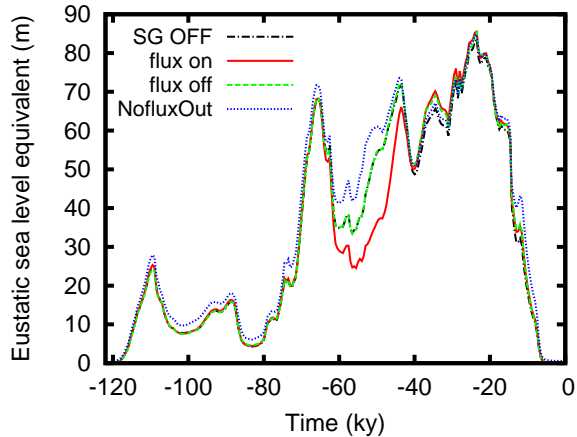
**Figure 10.** Average- Ensemble mean (solid red line) and standard deviation (dotted blue line) eustatic sea level equivalent of the total ice volume differences when the SG model is turned on and off, for an ensemble run over the last glacial cycle.



**Figure 11.** Ice field during inception at 115 ka for a simulation using one of the parameter **vector vectors** that generates **the best fit fits** to the calibration constraints ([nn9894](#)). **a.** Ice thickness **when with SG is activated turned on**. **b.** Ice thickness differences between simulations **where with** the SG model **is turned** on and off. 0 differences are presented in the same colour as the continent.

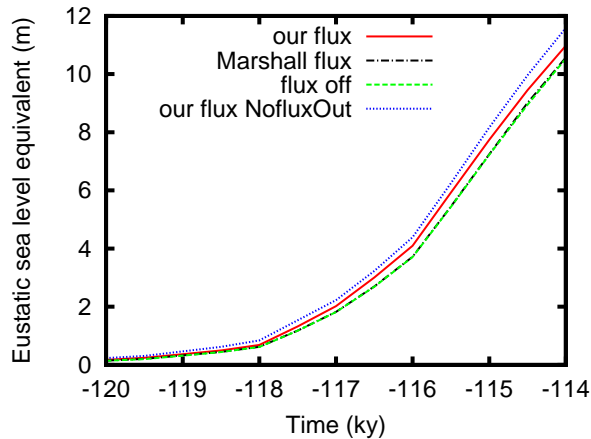


**Figure 12.** Ice field at 50 ka for a simulation using **one-of-the** parameter **vector-that-generates-the** **best-fit-to-calibration-constraints** [nn9894](#). **a.** Ice thickness **when-with** **SG** **is-activated** **turned-on**. **b.** Ice thickness differences between simulations **where-with** the **SG** model **is-turned** on and off.



**Figure 13.** Total ice volume evolution for a simulation using parameter vector nn9894. “flux on” and “flux off” both include the SG surface mass balance calculations but the latter has no SG ice fluxes. “NofluxOut” has SG on, but no SG ice flux between coarse grid cells. The “SG OFF” line is most of the time hidden under the “flux off” line.

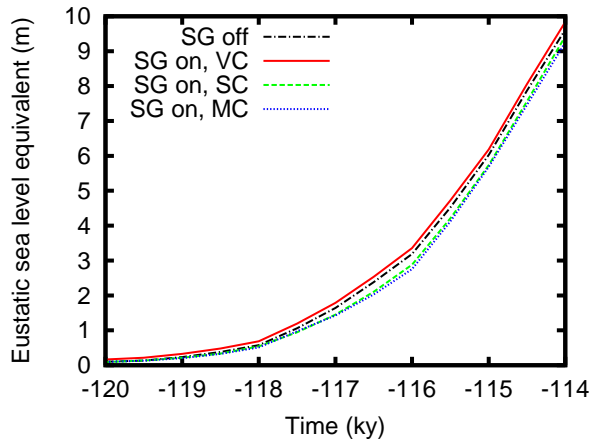
Total ice volume evolution for a simulation using a better fitting (relative to calibration constraints) parameter vector. Different curves represent simulation where the SG model and the flux code are used ("flux on") or not ("flux off"), and if the flux between coarse grid cells is turned off



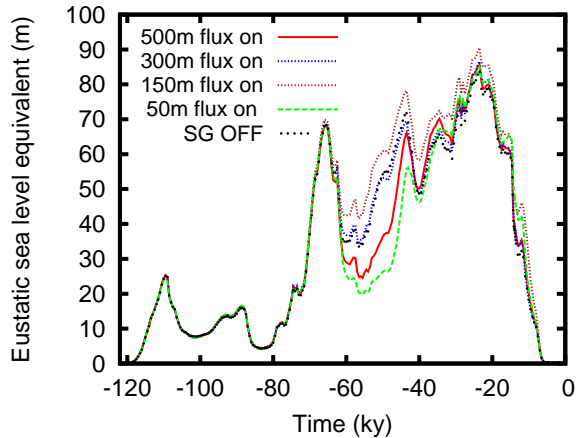
("NofluxOut"):

**Figure 14.** Ice volume evolution for a simulation over North America (parameter vector nn9894) with the SG model turned on during inception. "our flux" represents the flux code used in our SG model and "Marshall flux" the flux code used in Marshall et al. (2011) experiment. "flux off" represents the simulation with no ice flux between SG bins and "NofluxOut" has no SG flux between coarse grid cells (but SG fluxes within each coarse grid cell are still enabled).





**Figure 15.** ~~Ice~~ Total ice volume evolution for a simulation over North America during inception with the SG model turned on (SG on) using the parameter vector of run nn9927. Different methods of ice redistribution at the CG level are compared. "VC" is for ice volume conservation, "SC" for surface elevation conservation and "MC" uses the maximum of the previous two methods. "SG off" represents a run where the SG model has been turned off.



**Figure 16.** Total ice volume evolution for a simulation using a better fit parameter vector [nn9894](#). Different curves represent simulations with different minimum altitude variation thresholds used for the SG activation.