Response to Reviewer

Reviewer :

❖ The paper describes the use of catastrophe theory to discriminate between alluvial channel forms. Although I am eager to learn of new and unique approaches to fluvial geomorphology, the effort presented by the authors is too poorly written to allow an adequate assessment of its quality or novelty. Without an appropriate introduction into how cusp catastrophe theory can help inform understanding of river dynamics in ways that other methods or approaches can not, the average reader will struggle to appreciate what new can be learned from this.

➢ I am very precious these useful suggestions for this paper, I think this is a new way to describe the river theory.As the reviewer said: “ there is few appropriate introduction into how cusp catastrophe theory can help inform understanding of river dynamics in ways that other methods or approaches can not.” , we have done some more work to explain this theory.

In the introduction of the original paper, we have provided some information about the catastrophe theory which used in the geomorphology as : “Rene Thom published the first paper on it in 1968, the first book in 1972 (Thom 1972), and now it has found applications in a wide variety of disciplines, including physics, biology, psychology, economics, and geology (Henley, 1976;Stewart and Peregoy, 1983; Gilmore, 1993; Yi, 1995; Hartelman, 1997). With the development of nonlinear mathematics, some researchers adopted catastrophe theory to describe the formation of river channels. Thornes (1980) studied the characteristics of Spanish rivers based on theories of nonlinear mathematics; Richards (1982) brought catastrophe theory into the mechanism of fluvial processes; Graf (1988) described the transformation among straight, meandering and wandering river patterns with the cusp catastrophe theory and pointed out that rivers may result in different patterns although under the same silt-discharge and boundary condition. Nevertheless, the catastrophe theory has just explained the phenomenon, and lacked quantitative results on the mechanism of fluvial processes.”

In this introduction, it points out that the “the catastrophe theory has just explained the phenomenon, and lacked quantitative results on the mechanism of fluvial processes”, and then, we proposed the objective of this study as“ This paper established equations for the equilibrium state of channel regime based on the model of cusp-catastrophe surface, the 2D projection of the cusp catastrophe surface can be used to classify alluvial channel patterns, and the model can provide some useful suggests in practical projects.”

In order to explain this theory used in this study, we have added some sentences as a appropriate introduction to explain the relationship between the catastrophe theory and the river theory from entropy function as follows:

“ 2 THE CUSP CATASTROPHE MODEL FOR ALLUVIAL CHANNEL REGIME

The self-formation of an alluvial channel is not only to content with the request of equilibrium, but also distribute the internal energy according to certain principles. There are so many factors influencing the transformation of channel patterns that it can be difficult to establish equations of mathematical physics. The research remains to be systematically understood in the future.

The characteristic of cusp catastrophes; namely bifurcation, sudden-jumps, hysteresis, inaccessibility, and divergence can be seen from the natural river. According to the catastrophe theory, it can be used the theory based on the phenomenon to research the river through the model of the cusp catastrophe theory. Combined with the dissipative structure, this study brings the concept of entry and presents a cusp catastrophe model which enables prediction of the stability of the channel patterns. Equations of stability of alluvial channel patterns are derived based on the above assumption

2.1 The cusp catastrophe model

Behaviors of dynamical systems can usually be described by differential equations called entropy function. In applying science it is customary to divide it into two groups; first, the space is of finite dimension (LPS), and second, it is of infinite dimension (DPS). In this paper, it is adopted the first type, called lumped-parameter systems (LPS). The phase’s space of LPS is a finite-dimensional space. Let us consider n-dimensional Euclidean space. The LPS' can usually be written in the form(Kubicek and Marek,1983):

$$dS = \frac{dz_i}{dt} = f(z_1, z_2 \cdots z_n, \alpha_1, \alpha_2 \cdots \alpha_m), i = 1,2 \cdots n \tag{1}$$

here z_i denotes state variable, t is time, and α_i is generally a system parameter.

In the above system, we shall assume the system is autonomous, i.e.. The time variable does not occur on the right-hand sides (RHS) explicitly. The systems should be assumed to be continuous and continuously differentiable functions of their arguments. The steady-state (stationary) solution z of the system satisfies the set of nonlinear equations as follows (Kubicek and Marek,1983):

$$f(z_1, z_2 \cdots z_n, \alpha_1, \alpha_2 \cdots z_m) = 0, i = 1,2 \cdots n \tag{2}$$

In the study of dynamic systems, there are many types of approaches. The most general and qualitative, topological techniques are used to classify topologically distinct types of behavior possible in the given model. Catastrophe theory belongs to this category and used in this study, which was created by Rene Thom. The theory is based on new theorems in multi-dimensional geometry, which classify the way that the discontinuities can occur in terms of a few archetypal forms.

Of the seven elementary catastrophes in catastrophe theory, the cusp catastrophe model is the most widely applied. It is often used to model the behavior of a system with two control parameters and a non-linear state parameter. The potential function E for the cusp model is (Stewart, 1975):

$$E = \frac{1}{4}z^4 + \frac{1}{2}xz^2 - yz \tag{3}$$

Its equilibrium points, as a function of the control parameters x and y, are solutions to the equation as follows:

$$\frac{dE}{dz} = f(x, y, z) = z^3 + xz - y = 0 \quad (4)$$

where x, y are defined as the control parameters, and z is the state parameter of the system.

Based on the similarity between the entropy function of dissipative structures and potential function in the Catastrophe theory, it is can be described as the steady-state equations to express the state of a system with the cusp catastrophe model:

$$dS = \frac{dE}{dz} = f(x, y, z) = z^3 + xz - y = 0 \quad (5)$$

The Cardan's discriminant is written as: $\triangle=4x^3+27y^3$; Eq. (5) has one solution if $\triangle>0$, and has three solutions if $\triangle<0$; The set of values of x and y for which $\triangle= 0$ demarcates the bifurcation set, and these solutions are depicted as a two dimensional surface living in three dimensional space, the floor of which is the two dimensional (x, y) coordinates system called the "control plane"."

❖ Further, the authors fail to convince the reader of their understanding of fluvial geomorphology. No synthesis of the science problem is offered, and the authors fail to provide any physical basis of statements and equations that seem integral to their work. For instance, the authors suggest that rivers with a sinuosity greater than 1.5 are braided, whereas rivers with a sinuosity less than this are meandering. This is profoundly incorrect.

➢ This paper presents a cusp catastrophe model which enables discrimination and prediction of the equilibrium state for the channel patterns. Equations of equilibrium state of alluvial channel patterns and the transformation of channel patterns are established based on the model of cusp-catastrophe surface. The basic equations have been introduced in the revised paper, it is a key to choose the suitable paraments to reflect the state of the system.

After the establishment of the equations,combined with the entropy function of the dissipative structures, we added some sentences to supply this paper :

"Figure 3 shows the distribution of these rivers in the equilibrium state surface:

(1) Most points are on the other side of the surface, the structures are far from the equilibrium. They do not reach the steady state in the dynamic system; the internal adjustments are underway with different factors (Fig.3).

(2) Combined with the dissipative structure theory, we can obtain the entry function $dS < 0$ based on Eq. (17); the system should achieve the stable surface at the expense of energy flowing into the system from the outside. From the Fig.4, the sinuosity will increase with the system reaching the surface. It can be concluded that the meandering river is a relative stable pattern in the natural, and this conclusion is proved and accepted by most of the researchers.

(3) According to the dissipative structure theory, if $dS > 0$, the system should be in a chaotireacheste, as the second area, the system reach the lower sheets with decreasing the sinuosity of rivers, this is can be defined to transfer to wandering or unsteady braid river patterns at the expense of energy flowing out of the system in order to reach the equilibrium (Fig.5).

(4) On the basis of Eq. (17), a few points lie in the middle sheet, which is often referred as the inaccessible region, represents an unstable maximum. If the system's behavior does occupy this area then any slight alteration by an affecting factor will result in the behavior being transferred to either the upper or lower sheet. All points along the fold curve are said to be semi-stable points of inflection. The sinuosity may be either increase or decrease to reach the upper or lower sheet; these rivers may be transferred to the meandering or the wandering river pattern in the slight alteration in system behavior. It can result in two different stable states (Fig.6).

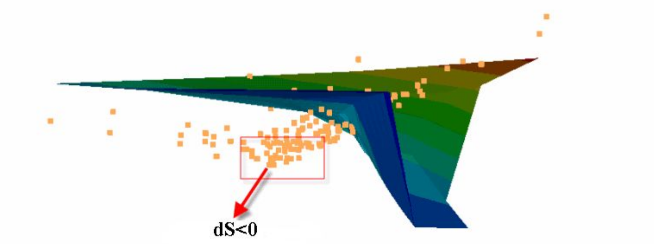

Fig. 4 The first zone judging the state of the rivers

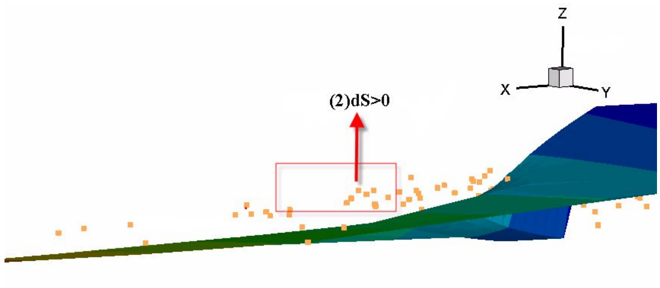

Fig.5 The second zones judging the state of the rivers

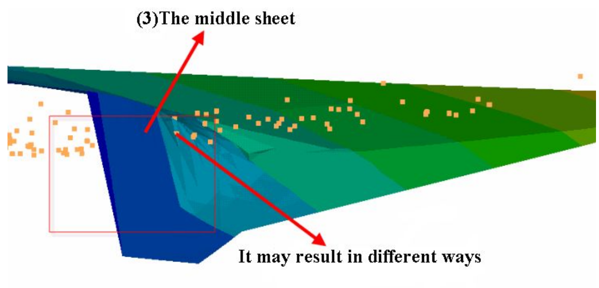

Fig.6 The third zone judging the state of the rivers

❖ Further, Eqs. (3) and (4), which describe the riverbed stability, form the framework of the analysis, but no explanations are provided of their form or derivation. Without a clear integration with existing river theory, I see no way for the current work to advance understanding or be of interest to the international river community.

➢ The morphology of natural river channels is determined by the interaction of fluid flow with the erodible materials in the channel boundary (Knighton, 1984). We choose the sinuosity l as the state parameter describing the transformation of channel patterns. The formation of a river is accomplished through the processes of erosion, transportation and deposition of sediment under the flow of actions. It is well known that the interrelationships between the (1) oncoming bed material load and the sediment carrying capacity of the flow, and (2) the erodibility of the river banks and the erosive power of the water decide essentially the general direction of the adjustment. On the basis of the concepts as depicted above, the synthetic indexes of river-bed stability determined by the longitudinal river-bed stability in contrast to the transverse river-bed stability are adopted as the control factors. here ϕ_h is used as the longitudinal index of В.и.АсграханиеВ, ϕ_b is used as the transverse index of C. T. Алтунин.The two parameters as follows :

$$\phi_b = \frac{B}{B_1} = \frac{BJ^{0.2}}{Q^{0.5}} \quad (3)$$

$$\phi_h = \frac{(\gamma_s - \gamma)D_{50}}{\gamma h J} \quad (4)$$

As we known, B_1 in the Eq.(3) refers to the bank-full width in the hydraulic geometry relationships to describe the shape of bank-full alluvial channels. Considerable progress has been achieved on the geometry of alluvial rivers under equilibrium, or in regime with field investigations. Typically, the exponent diagrams illustrate the good agreement with several empirical regime equations found in the literature, which were tested with a comprehensive data set consisting of field channels; comprehensive reviews of the abundant literature on the regime approach and other methods defining the hydraulic geometry relationships are available in Yalin (1992).It is noting the relationships are not dimensionally correct. In this study, we adopt the formulation of bank-full width from C. T. Алтунин to describe B_1, which is not dimensionally correct, and then obtained the control factors including the influence of flow, sediments and the bank stability. In the conclusion, we proposed that some assumptions have proposed before the establishment of the equations and model, therefore, further studies are needed to prove the reliability of the model with more field data of rivers and improve this model.

❖ As a result, I recommend the work to be rejected for publication by Geoscientific Model Development, but encourage the authors to redevelop their efforts to both utilize and clearly advance river theory.

➢ This is a new way to explain the river stability and the river classification, I hope that I can have the chance to modify this paper and publish it in this journal, because of its newly and further work needed, I wish it can be published as a brief communication paper to discuss this new method with other researcher to improve the future study.