Response to Reviewer

Reviewer :

❖ The authors thus found a long path to reinvent something we all knew and that the authors did not introduce upfront in their analysis: resistance to flow equations.channels from braided channels. As we all know, the width/depth ratio B/h does separate meandering.So, we have gained the grand illusion of rediscovering something that has been hidden all along: resistance to flow.

➢ We do not think that our work is just to be found a long path to reinvent something we all knew.In our study, combined with the dissipative structure, we bring the concept of entry and presents a cusp catastrophe model which enables discrimination and prediction of the stability of the channel patterns. We do not study the alluvial channel regime by combining the four fundamental relationships, so do not refer to the resistance to flow equations in our paper.

Behaviors of dynamical systems can usually be described by differential equations called entropy function. In applying science it is customary to divide it into two groups; first, the space is of finite dimension (LPS), and second, it is of infinite dimension (DPS). In this paper, it is adopted the first type, called lumped-parameter systems (LPS). The phase's space of LPS is a finite-dimensional space. Let us consider n-dimensional Euclidean space. The LPS' can usually be written in the form:

$$dS = \frac{dz_i}{dt} = f(z_1, z_2 \cdots z_n, \alpha_1, \alpha_2 \cdots \alpha_m), i = 1,2 \cdots n \qquad (1)$$

Here z_i denote state variables, t is time, and α_i is generally a system parameter.

In the above system, we shall assume the system is autonomous, i.e., The time variable does not occur on the right-hand sides (RHS) explicitly. The systems should be assumed to be continuous and continuously differentiable functions of their arguments. The steady-state (stationary) solution z of the system satisfies the set of nonlinear equations as follows:

$$f(z_1, z_2 \cdots z_n, \alpha_1, \alpha_2 \cdots z_m) = 0, i = 1,2 \cdots n \qquad (2)$$

In the study of dynamic systems, there are many types of approaches. The most general and qualitative, topological techniques are used to classify topologically distinct types of behavior possible in the given model. Catastrophe theory belongs to this category and used in this paper.

Catastrophe theory is a relatively new mathematical method for describing system behavior where a gradually changing force can produce a sudden dramatic effect. It was created by Rene Thom. The theory is based on new theorems in multi-dimensional geometry, which classify the way that the discontinuities can occur in terms of a few archetypal forms; of the seven identified forms the cusp catastrophe is the most common. It is used to describe the behavior of a system which depends upon two control factors. A full description of the mathematics underpinning the cusp catastrophe is in the form :

$$E = \frac{1}{4}z^4 + \frac{1}{2}xz^2 - yz \tag{3}$$

$$\frac{dE}{dz} = z^3 + xz - y = 0 \text{，} \tag{4}$$

here E is the potential function; x, y are defined as the control parameters, and z is the state parameter of the system.

Based on the similarity between the entropy function of dissipative structures and potential function in the Catastrophe theory, it is can be described as the steady-state equations to express the state of a system with the cusp catastrophe model:

$$dS = \frac{dE}{dz} = z^3 + xz - y = 0 \tag{5}$$

From the equation, it is known that the gradient of E is zero if there is no external forcing on the structure. A three-dimensional surface, which represents equilibrium states, is established. All the points on the surface are in a steady state and satisfy the Eq. (5).

The self-formation of an alluvial channel is not only to satisfy the request of equilibrium, but also distribute the internal energy according to certain principles. There are so many factors influencing the transformation of channel patterns that it can be difficult to establish equations of mathematical physics. The research remains to be systematically understood in the future.

The characteristic of cusp catastrophes; namely bifurcation, sudden-jumps, hysteresis, inaccessibility, and divergence can be seen from the natural river. According to the catastrophe theory, it can be used the theory based on the phenomenon to research the river through the model of the cusp catastrophe theory.With the development of nonlinear mathematics, some researchers adopted catastrophe theory to describe the formation of river channels. Thornes (1980) studied the characteristics of Spanish rivers based on theories of nonlinear mathematics; Richards (1982) brought catastrophe theory into the mechanism of fluvial processes; Graf (1988) described the transformation among straight, meandering and wandering river patterns with the cusp catastrophe theory and pointed out that rivers may result in different patterns although under the same silt-discharge and boundary condition. Nevertheless, the catastrophe theory has just explained the phenomenon, and lacked quantitative results on the mechanism of fluvial processes.

An equation of equilibrium state of alluvial channel patterns is established based on the model of the cusp-catastrophe surface. A three-dimensional critical surface is obtained from this model to identify the transformation of channel patterns in a direct way. An equation is obtained from the equilibrium state of channel regime, which is a cusp catastrophe surface in a translated three dimensional coordinate. The stability of channel patterns can be identified by such a model in a direct way. Based on the cusp model of alluvial channel regime, a new channel pattern classification is presented according to the channel stability. Describing the critical lines in the two-dimensional plane and a control plane, which splits it into three zones was established, and then put the field data set of natural rivers in the plane,the 2D projection of the cusp catastrophe surface can be used to classify alluvial channel patterns as Fig.7.

The reviewer pointed out that the width/depth ratio B/h does separate meandering. However, in our study, we do not use the traditional methods to classify the channel pattern, we adopted the cusp catastrophe model to establish the diagram classifying alluvial channel patterns, and the relative critical lines have been obtained from Eq. (14) in our paper. It can be used to judge the river pattern when the control factors can be obtained in the field. It shows a bound result with the model. Also, we can obtain some conclusions from Fig.7:

1). The meandering river belongs to the first zone(1), which correspond to the upper sheet of the model (Fig.7). In this area, the meandering rivers are in a relative stable state; the river pattern will keep and not change rapidly.

2). Most of the braided rivers belong to the second area(2), but there are still some in the first area(1), based on the model, braided river in the middle sheet may be result in two different river pattern with a small change, according to the concept of the unsteady braided rivers, it can be concluded that the chance of transformation of channel pattern may have two results.

3).Many wandering rivers are in the third zone(3), but there are still some meandering rivers nearby the origin, according to the catastrophe theory: the region around the origin of the fold line could have remarkably different outcomes of the system behavior. For although the two paths may begin close together, their paths are divergent. It is can be used to explain why it can produce the meandering river and wandering river in the nature.

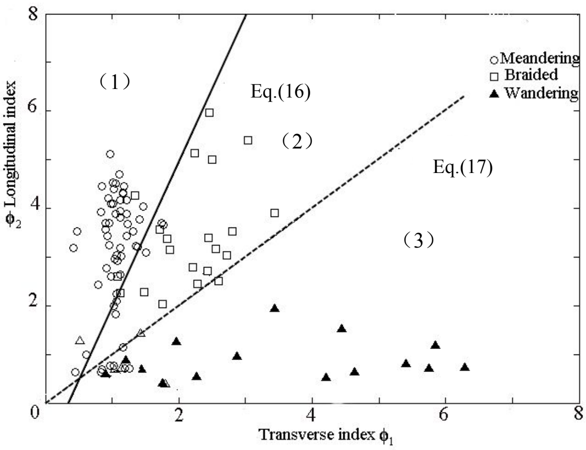

Fig. 7 The diagram classifying alluvial channel patterns in the control space of ϕ_1−ϕ_2

❖ The authors combine two parameters from Ning Chien in Eqs 3-4. One essentially rewrites the inverse of the Shields parameter and the second describes the channel width. It is worth noting that the second parameter is not even dimensionally correct.

➢ The morphology of natural river channels is determined by the interaction of fluid flow with the erodible materials in the channel boundary (Knighton, 1984). We choose the sinuosity l as the state parameter describing the transformation of channel patterns. The formation of a river is accomplished through the processes of erosion, transportation and deposition of sediment under the flow of actions. It is well known that the interrelationships between the (1) oncoming bed material load and the sediment carrying capacity of the flow, and (2) the erodibility of the river banks and the erosive power of the water decide essentially the general direction of the adjustment. On the basis of the concepts as depicted above, the synthetic indexes of river-bed stability determined by the longitudinal river-bed stability in contrast to the transverse river-bed stability are adopted as the control factors. The two parameters as follows :

$$\phi_b = \frac{B}{B_1} = \frac{BJ^{0.2}}{Q^{0.5}} \tag{3}$$

$$\phi_h = \frac{(\gamma_s - \gamma) D_{50}}{\gamma h J} \tag{4}$$

As we known, B_1 in the Eq.(3) refers to the bank-full width in the hydraulic geometry relationships to describe the shape of bank-full alluvial channels. Considerable progress has been achieved on the geometry of alluvial rivers under equilibrium, or in regime with field investigations. Typically, the exponent diagrams illustrate the good agreement with several empirical regime equations found in the literature, which were tested with a comprehensive data set consisting of field channels; comprehensive reviews of the abundant literature on the regime approach and other methods defining the hydraulic geometry relationships are available in Yalin (1992).It is noting the relationships are not dimensionally correct. In this study, we adopt the formulation of bank-full width from C. T. Алтунин to describe B_1, which is not dimensionally correct, and then obtained the control factors including the influence of flow, sediments and the bank stability. In the conclusion, we proposed that some assumptions have proposed before the establishment of the equations and model, therefore, further studies are needed to prove the reliability of the model with more field data of rivers and improve this model.

❖ There is actually no rationale to use a = b = 0.5 in Eqs. 5 and 6, except for reducing the variability in the data.

➢ Considering the importance of the oncoming bed material and the erodibility in the river process, the control parameters in this paper can be written as:

$$\phi_1 = (\phi_b)^a \tag{5}$$

$$\phi_2 = (\phi_h)^b \tag{6}$$

In our study, the exponents of control factors a=b=1/2. Under the advice of the reviewer, let a=b=1, and a=1/2,b=1,do the same work to consider the influence of the control factors, and then for example, we obtained the figure to classify the channel patterns as Fig.8(when a=1/2, b=1): we can see the results are in good agreement with the results in the paper as a=b=1, the pattern does not change with the exponents varying. It is a key to choose the suitable control factors to reflect the plan view of the channels as the state of the river system. The variation of the exponents of the control factors can not influence the channel pattern the channel belongs to.

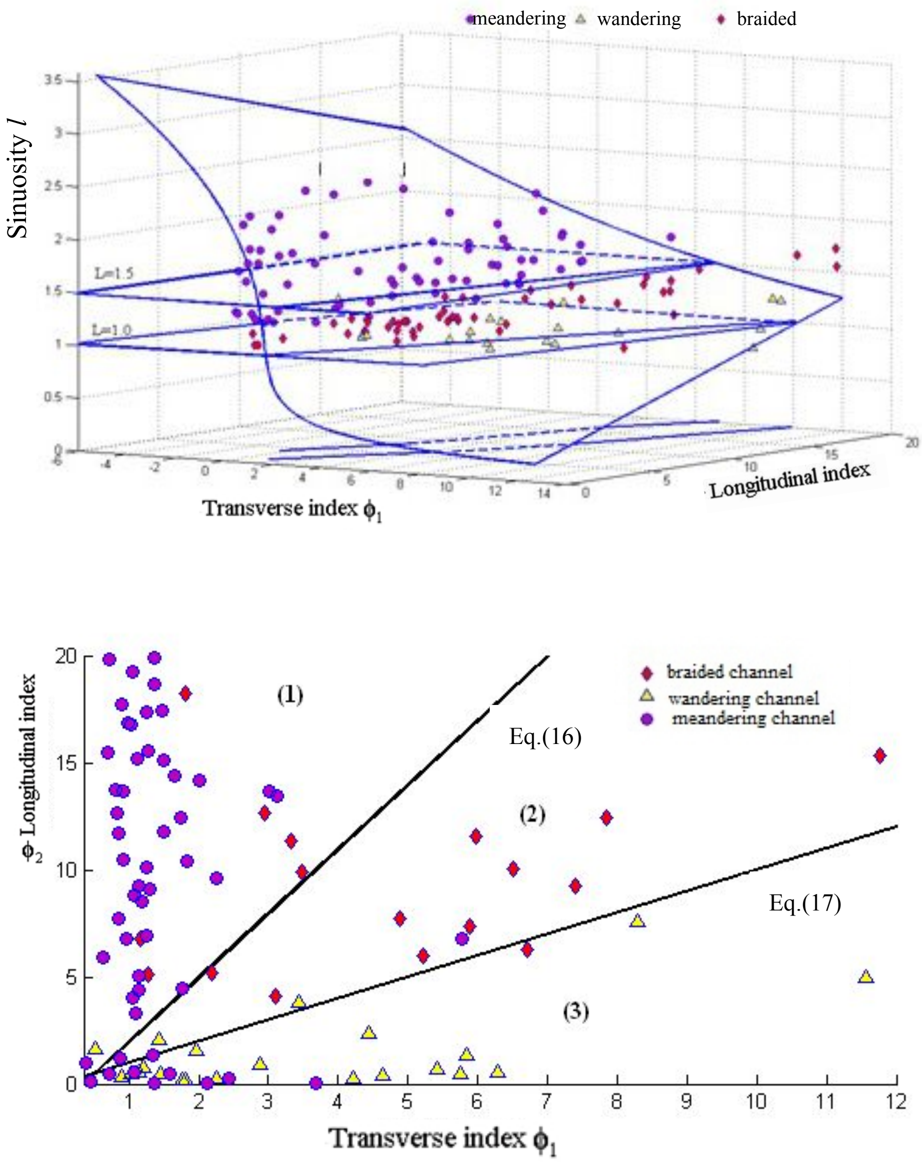

Fig. 8 The 2-D control plane judging the channel pattern when a=1/2.and b=1