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Interactive comment on "A linear algorithm for solving non-linear isothermal ice-shelf equations" *by* A. Sargent and J. L. Fastook

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We thank the referee for the constructive review and valuable comments that help to clarify and improve the paper. We made the suggested changes to the paper. Our answers to the comments are below.

Major comments:

- We agree with the referee that the applicability of the 2D model is restrictive. We made changes, reflecting this correction, in the abstract, introduction, conclusion, and the text of the paper.
- p 1845: One of the reasons why we solved the Poisson equation instead of

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solving a system of corresponding first-order elliptical PDEs, named Cauchy-Riemann equations, is the fact that the Cauchy-Riemann equations subject to certain boundary conditions are ill-posed. The following example from (course-material, Oxford) demonstrates that the solution of the Cauchy-Riemann equations could break down close to the curve where the boundary conditions are defined. The homogeneous Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0, \qquad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \tag{1}$$

for u and v in $y > 0$ subject to the Cauchy data

$$u = u_0(x) = 0, \qquad v = v_0(x) = \frac{\epsilon\delta^2}{x^2 + y^2} \quad y = 0, \tag{2}$$

where ϵ and δ are positive constants, have the solutions

$$u = \frac{2\epsilon\delta^2 xy}{\left[x^2 + (y-\delta)^2\right]\left[x^2 + (y+\delta)^2\right]}, \quad v = \frac{\epsilon\delta^2\left[x^2 - y^2 + \delta^2\right]}{\left[x^2 + (y-\delta)^2\right]\left[x^2 + (y+\delta)^2\right]}. \tag{3}$$

Both u and v blow up at the point $(0, \delta)$ regardless however small ϵ is - by choosing δ small, you can construct the solution which breaks down arbitrarily close to the initial curve $y = 0$.

On the other hand, the boundary-value problem for the Poisson equation is known to be a well-posed problem for most general boundary conditions (Barton, 1989).

The second reason for choosing to solve a linear Poisson equation instead of Cauchy-Riemann equations is the fact that because of the widespread need to solve Poisson's equation, many specialized algorithms have been developed for this problem.

On citing other works in which other elliptic equations took benefit from the same transformation - while we cannot give some specific references, the fact that

Cauchy-Riemann equations can be written as Poisson equation is a common place.

- We agree with the referee that the FFT approach can be only applied to rectangular geometries and we could have and should have used other methods for solving efficiently block tridiagonal systems.

- We agree with the referee that the absence of friction term makes the model usable only for the shelf equations. We indicated it in the title of the paper, "A linear algorithm for ... ice-shelf equations."

  We also agree with the referee that the method extension to the SSA with friction is of much higher interest. However, addition of the friction term, which is usually chosen proportional to velocity, does not allow us to reduce the non-linear equations to linear systems of equations. It could be probably done if the friction term could be chosen proportional to the stresses.

  The paper does not show that the method applies to more complex geometries than simple segments or rectangles. However the rectangular domain example for solving Poisson equation is not a limitation of the offered method. There are fast Poisson solvers for irregular geometries based, for example, on ideas of domain embedding techniques (McKenney et al., 1995), (Adelmann et al., 2012), (Lee, 1994) which, according to the researchers, are about $O(NlogN + M)$, where N is the number of interior points and M is the number of points on the boundary.

- Making the equations dimensionless was motivated by a consideration that the SSA diagnostic equations discussed in this paper could be used as a part of the prognostic-diagnostic system, where on the prognostic part the change of the ice-shelf thickness is calculated and then the diagnostic part calculates the velocity field (MacAyeal, 1997). For solving such systems, scaling analysis could be useful.

GMDD

7, C436–C442, 2014

Interactive Comment

- Definition of the mesh $\{x_i, i = 1, , N\}$ and discrete variables s_i, h_i, etc. are specified in the paper.

- We agree that Eq. (17) is a discretization of (12) but not of (7) and this is fixed in the paper.

  The scheme for τ is the second-order. While we define h and s at the nodes of the staggered grid and τ at the centroids of the cells, we define the value of τ_N at the node (the right boundary) as shown in Fig. 1. This way, the boundary condition at x_N is approximated exactly. All but one equations for τ are approximated at the nodes of the grid, while the last equation is approximated at point $x_{N-3/4}$ which is denoted by cross mark in Fig. 1. At that point, $\frac{\partial \tau}{\partial x}$ is approximated by $\frac{\tau_N - \tau_{N-1}}{(\Delta x)/2}$, h is approximated by $\frac{3}{4}h_N + \frac{1}{4}h_{N-1}$ and $\frac{\partial s}{\partial x}$ is approximated by $\frac{5s_n - 6s_{N-1} + s_{N-2}}{4\Delta x}$, all with the second order of precision at point $x_{N-3/4}$.

- The common Gauss-Seidel algorithm description is removed from the paper.

- Appendix B is removed from the paper.

- Miswritten proper names are fixed.

- page 1835: The calculation of the RHS to Eq. (16) has been removed from the paper. It had been added to the paper to ease understanding of the code provided in the supplement. Appendix A is removed as well.

- Equations (37)-(45) are removed.

- Descriptions of Figs. 4, 5, and 6 are added.

- The bibliography is extended.

Minor comments:

GMDD
7, C436–C442, 2014

- page 1831, lines 12-16: two sentences are merged into one as suggested by the referee.
- Equation (19): reference to Bueler and Brown (2009) is added.
- The word "three-diagonal" is replaced by "tridiagonal" everywhere in the text.
- p. 1837: "ice constants" and "The ice bed ..." are rephrased.
- p. 1938: "it is due accumulation" is changed to "it is due to accumulation".
- Eq. (23): we added clarification of the notations $\vec{n} = n_{\tilde{x}}$ and $\vec{n} = n_{\tilde{y}}$: in the cases when the ice front extends along the $\tilde{y}-$ and $\tilde{x}-$ axes, an ice front has an outward-pointing normals aligned with the $x-$ and $y-$ axes, $\vec{n} = n_{\tilde{x}}$ and $\vec{n} = n_{\tilde{y}}$ consequently.
- In Eqs. (29) and (30), z_s is replaced by s.
- k_1 and k_2 directly substituted in (48)

We thank the referee for thorough and valuable comments.

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7, C436–C442, 2014

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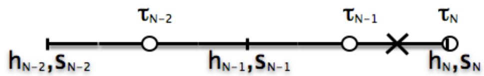

Fig. 1. Right side of 1D staggered grid: h and s are defined at the nodes, tau is defined at the centroids of the grid, except tau_N which is defined at a node.

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7, C436–C442, 2014

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