

Interactive comment on “Non-singular spherical harmonic expressions of geomagnetic vector and gradient tensor fields in the local north-oriented reference frame” by J. Du et al.

J. Du et al.

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Dear Anonymous Referee #2,

Thank you for your constructive comments. We have revised the manuscript according to your comments and here replied each comment bellow. The original comments are in plain text and the replies in italics. The revised manuscript is attached as the Supplement.

Referee #2 by Anonymous Referee #2 (Received and published: 8 April 2015) This paper provides new expressions for the gradient, the double-gradient, and some elements of the triple-gradient tensors that are stable at the poles in the local-north frame.

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Calculations of the gradient and double-gradient are provided for two field models. Unless one is performing a global analysis that includes data at or very near the poles, then I see the impact of this paper as limited. However, the paper still provides a useful alternative to the standard gradient and double-gradient formulae and should be published, but with more emphasis on comparison with the standard formulae. Too much effort is spent talking about the usefulness of gradients. This is not a paper about convincing people to use gradients, and it is a paper about using new, better formulae than the standard ones. General comments 1. Given that the expressions are stable at the poles, are there any other advantages in using them? I ask this because, as stated earlier, unless one is doing a global analysis that includes data at the poles, can't you just rotate the underlying spherical coordinate system such that the pole is no longer in the area of interest, which means that you can use the standard expressions? Are the new expressions less computationally intensive? Do they require less storage? >Jinsong Du et al.: Thank you very much. Our method has two main features. The one is the non-singularity at the poles. Another one is that there is no derivative of the Legendre function. Therefore, recursive calculation by the Clenshaw or Horner algorithms can be avoided. The computational efficiency can be improved and the storage is less required. Please note that we don't discuss the calculation of the Legendre function. Your suggested rotation is indeed correct and can be performed. However, compared with the rotation approach, our method doesn't need additional computation and thus reduce the complexity and also the computing time. According to this comment, we have added a sentence in the revised manuscript as following: A rotation of the coordinate system is always possible to avoid the polar singularity, but this solution is very ineffective for large data sets. 2. Even in the case where I want to compute the gradient and double-gradient at the poles, can't I rotate the coordinate system around the polar axis to eliminate the problems with $1/\sin(\theta)$? If so, why use your new expressions? >Jinsong Du et al.: Thank you. These questions are very similar with those in (1) above. We have emphasized the advantages of our method compared with the standard ones in the last paragraph of section 3 in the revised manuscript, which are as following:

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Compared with the traditional formulae in section 2.1, there are two advantages of our derived formulae in section 2.3. On the one hand, the traditional derivatives up to second-order are removed in the new formulae; therefore, the relatively complicated method by the Horner's recursive algorithm (Holmes and Featherstone, 2002b) can be avoided. On the other hand, the singular terms of $1/\sin^i \lambda$ and $1/\sin^{2i} \lambda$ are removed in the new formulae; consequently, the scale factor of e.g. 10⁻²⁸⁰ (Holmes and Featherstone, 2002a,b) is not required when the computing point approaches to the poles and the magnetic fields at the poles can also be calculated in the defined reference frame. In fact, there are differences between the results by our expressions and those by the Horner's recursive algorithm, for instance, if using the same model and the parameters as those in Fig. 1 and Fig. 2, the differences of the three components B_x , B_y and B_z are at a level of $[-3 \times 10^{-11} \text{ nT} : +3 \times 10^{-11} \text{ nT}]$. 3. Tables 1 and 2 and Figures 1 and 2 are fairly useless given that you should be showing the superiority of your new expressions over the standards. Therefore, you should have similar tables and figures for the standard expressions, being sure to show the polar neighborhoods in which the standard expressions begin to degrade. Furthermore, why have you not included polar projections in Figures 1 and 2 since this is the most important area for comparison? Also, you do not need to show two field models, just show either Figure 1 or 2. >Jinsong Du et al.: Thank you for your valuable suggestion. Our original purpose of using two models is to test the validity for the full range of the degrees and orders. In the revised manuscript, we have used only the GRIMM_L120 v0.0 (Lesur et al., 2013) with degrees and orders of 16~90 to illustrate the purpose. At the same time, a core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered. Meanwhile, in the revised manuscript, we only show the results near the two poles. The third-order derivatives are also presented aiming to further interpretations of the lithospheric magnetic field models in the future. 4. At the poles you (arbitrarily) define x_p and y_p to be aligned along some

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meridians and you show the smoothness of the functions across the poles when approached along these meridians in Figure 3. However, what happens if you approach the poles from an arbitrary meridian? Are the functions still smooth? >Jinsong Du et al.: Thank you. As shown in Figure 3 in the revised manuscript, the magnetic V , B_z and B_{zz} components at the poles are independent of the direction of the x_p and y_p axes and thus smooth cross the poles. However, while changing with the direction of the x_p and y_p axes at the poles, the B_x , B_y , B_{xz} , B_{yz} , B_{xxz} and B_{yyz} components have the periods of 360° and the B_{xx} , B_{xy} , B_{yy} , B_{xxz} , B_{xyz} and B_{yyz} components have the periods of 180° . These variations can be accurately described by sine or cosine function and the differences among these magnetic effects are magnitude, period and initial phase. Therefore, B_x , B_y , B_{xz} , B_{yz} , B_{xxz} , B_{xyz} , B_{yyz} , B_{xxz} , B_{xyz} and B_{yyz} components are not smooth cross the poles.

Best regards, Jinsong Du et al. 5 May 2015

Please also note the supplement to this comment:

<http://www.geosci-model-dev-discuss.net/7/C3705/2015/gmdd-7-C3705-2015-supplement.pdf>

Interactive comment on Geosci. Model Dev. Discuss., 7, 8477, 2014.

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