

# ***Interactive comment on “Non-singular spherical harmonic expressions of geomagnetic vector and gradient tensor fields in the local north-oriented reference frame” by J. Du et al.***

**J. Du et al.**

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Dear Prof. Mehdi Eshagh,

Thank you for your constructive comments. We have revised the manuscript according to your comments and here replied each comment bellow. The original comments are in plain text and the replies in italics. The revised manuscript is attached as the Supplement.

Referee #1 by Prof. Mehdi Eshagh (Received and published: 10 December 2014) A. General comments The paper deals with non-singular formulation of the elements of the vector and tensor of the Earth’s magnetic field similar to the works done by Petro-

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vskaya and Vershkov (2006) and Eshagh (2008, 2009). The main difference is related to the normalization factor as in the geomagnetism the semi-normalised associated Legendre functions (ALFs) are used, but in the gravity field studies the fully-normalised ones. The developments are very trivial, but can be useful. In addition, the authors provide the non-singular formulae for the third-order derivatives of the geomagnetic field. The paper is recommended for publication in Geosciences Model Development after a major revision. The following general and specific comments are provided for improving the paper.

B. Specific comments

1. The authors are asked to write some words about the differences between the works done by Petrovskaya and Vershkov (2006) and Eshagh (2008, 2009) and to explain why semi-normalised ALFs are used for the geomagnetic field. >Jinsong Du et al.: Thank you. In geomagnetic field studies, the Schmidt semi-normalized associated Legendre functions (SSALFs) is usually used (e.g. Blakely, 1995; Langel and Hinze, 1998). As for the differences between the works done in gravity field studies by Petrovskaya and Vershkov (2006) and Eshagh (2008, 2009), we have added the corresponding content in the end of section 2.1 in the revised manuscript, which are as following: It should be stated that our work differs from those presented by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the LNORF and also the associated Legendre functions (ALFs). Nonetheless, the following mathematical derivations are carried out based on their studies in gravity field.
2. In the abstract, it is written higher-order derivatives, whilst the paper considers the third-order ones. It should be revised. >Jinsong Du et al.: Thank you for pointing this out. We have changed the 'higher-order derivatives' to 'third-order derivatives'.
3. According to the reference system theory, the local north-oriented frame is defined as a frame whose z-axis is radially upward and the system is left handed. The equations that e.g. Eshagh (2009) has used are based on such a frame. Please explain why this frame is defined differently in the paper. >Jinsong Du et al.: Thank you. For the geomagnetic fields modeling and their applications, it is usual to utilize a local topocentric coordinate system (please see the page 113 in the chapter '5 Sources of the Geomagnetic Field and the Modern Data That Enable Their Investigation' by Nils Olsen et al. (2010) in 'Hand-

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book of Geomathematics' edited by W. Freedon et al.). In the local reference frame, the X axis points toward geographic North and the Y axis geographic East and the Z axis vertically down. This reference frame is an orthogonal right-handed coordinate system. We have added the corresponding reference to the revised manuscript in section 2.1. 4. The paper presents the mathematical derivations in 7 subsections, but the problem is that the reader cannot find the connection with these mathematical proofs and the traditional expressions. It is recommended that the authors start with the traditional expressions of the vector and tensor of the geomagnetic field as well as the third-order derivatives, and discuss about their importance and roles in geomagnetic studies, and in the mathematical derivations they refer to the traditional formulae so that the reader can see the connections between the new and old formulae. For example, see the Eshagh (2009) that you have referred to. >Jinsong Du et al.: Thank you very much. According to your suggestion, we have adjusted this part and stated the connection with the studies by Petrovskaya and Vershkov (2006) and Eshagh (2009) in the revised manuscript. Based on these connections, our mathematical derivations are clearer than those in the discussion paper. 5. The appendix repeats the things that have been already presented in the paper by Eshagh (2009). Please remove it! Those coefficients related to the third-order derivatives can simply be moved into the text. >Jinsong Du et al.: Thank you. In fact, because of the differences in the local-north-oriented reference frame and also the normalized associated Legendre functions, some coefficients in the Appendix are different with those presented in the paper by Eshagh (2008, 2009). Therefore, we have added the coefficients into the text in the revised manuscript. 6. The purpose of the numerical investigation is not clear. If the goal is just to present the maps of the vector and tensor quantities based on the new formulae, then what will be the role of considering two geomagnetic models? One of them should be enough, otherwise the author should discuss about the discrepancies between the models. In addition, the maps of the third-order derivatives are missing, and this could be a good contribution, which the paper deals with improperly. >Jinsong Du et al.: Thank you for your suggestion. The two models are different. The one is the core field, which is

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dominated by the spherical harmonic degrees/orders from 1 to 12~20. Another one is the lithospheric field, which is dominated by the spherical harmonic degrees/orders higher than ~16. Originally, we want to use these two models to test the correctness of the formulae in the full range of the spherical harmonic degrees/orders. In the revised manuscript, we have used only the GRIMM\_L120 v0.0 (Lesur et al., 2013) with degrees and orders of 16~90 to illustrate the purpose. At the same time, a core field model with spherical harmonic degrees/orders 1~15 is also used to test and the results not shown here indicate the correctness of the formulae in the full range of the spherical harmonic degrees/orders, where the computational stability of the Legendre function with ultrahigh-order is not considered. Meanwhile, in the revised manuscript, we only show the results near the two poles. The third-order derivatives are also presented aiming to further interpretations of the lithospheric magnetic field models in the future.

C. Technical comments

1. All abbreviations should be defined properly in the introduction even if they are well known and they should be given some reference, e.g. ESA, GOCE, CHAMP, SAC-C, ST-5, Ørsted. . . >Jinsong Du et al.: We have defined all abbreviations in the revised manuscript or added the corresponding references.
2. The abbreviation 'SHA' has been defined but never used. Please remove it! >Jinsong Du et al.: Thank you for pointing out this abbreviation and we have removed it.
3. In Section 2, above Eq. (1), it is written that '... at point P' whilst P will be introduced later as the ALF. Simply write any point with the geocentric distance  $r$ , co-latitude  $\theta$  and longitude  $\varphi$ . The same holds for the text above Eq. (2a). >Jinsong Du et al.: We have added some corresponding descriptions about the  $P(r,\theta,\varphi)$  when appearing first time in the text.
4. Below Eq. (44), the abbreviation SH has not been defined already. Please write the full name! >Jinsong Du et al.: We have changed this abbreviation and used its full name.
5. The sentence above '2-derivation of ...' write: 'the Kronecker delta'. >Jinsong Du et al.: Thank you for pointing this out and we have corrected it.
6. The article 'the' should not be used when an equation is referred by its number. For example, write: Eq. (1) and NOT 'the Eq. (1)'. The same holds for 'Lemma 3'. >Jinsong Du et al.: We have removed the corresponding expression 'the' in the revised manuscript and thank you.

Best regards, Jinsong Du et al. 5 May 2015

Please also note the supplement to this comment:

<http://www.geosci-model-dev-discuss.net/7/C3700/2015/gmdd-7-C3700-2015-supplement.pdf>

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Interactive comment on Geosci. Model Dev. Discuss., 7, 8477, 2014.

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