Response to Reviewer #2

March 13, 2015

1 General comments

Modular software libraries are used to solve partial differential equations (PDEs) modeling ice sheets. The libraries are from the Trilinos project for solution of linear and non-linear systems of equations, for discretization of PDEs with the finite element method, and for parallelization on thousands of cores. This is a good idea and extensions of the code to include inverse modeling and sensitivity estimation are possible using other parts of the libraries. The PDEs chosen here to model the ice are the Blatter-Pattyn equations. Under simplifying assumptions they can be derived from the full Stokes equations. The accuracy of the implementation is evaluated using manufactured solutions and comparisons with another code. The solution in the Greenland ice sheet is computed in parallel in a test with a realistic geometry. The parallel scalability for the whole code is good and the convergence is as expected by theory when the mesh is refined. The paper is worth publishing if the comments below are taken into account in a revised version.

We thank the reviewer for his/her thoughtful feedback, which has been addressed in the revised manuscript, and has helped us to improve our paper. All changes to the original manuscript are marked in blue in the revision. Responses to your comments can be found below, also in blue.

2 Specific comments

1. A major contribution in the paper is the use of software libraries to build the solver. Is it possible to estimate how much time and effort

that has been saved by using these libraries? Such an estimate could encourage also similar developments for many other applications.

The FO Stokes PDEs and relevant boundary conditions were implemented and verified in *Albany* (Sections 1-5 in the paper) in approximately 6 months, with one staff member working on this task approximately half-time (0.25 FTE, where FTE stands for "full-time equivalent"). It is estimated that all the work presented in the paper (including development of the AMG preconditioner based on semi-coarsening) took approximately 1.5 FTEs worth of work. These numbers have been noted in the paper, in Section 3.2. The fact that, with software libraries, the verification, extension and maintenance of the libraries is amortized over many projects by subject-matter experts, has saved us a lot of development time, and will continue to save us development time in the future.

2. For prediction of ice flow, the Stokes equation has to be coupled to an equation for the motion of the surface and for long time intervals also for the bedrock. This coupling and the moving mesh or the ice surface cutting through a stationary background mesh are not discussed.

   The moving mesh scenario would only be encountered in a transient simulation, which goes beyond the scope of this paper. We have added to the paper (the "Conclusions" section) some discussion of how it is possible to do prognostic (transient) simulations using the *CISM-Albany* and *MPAS-Albany* dycores, which we are developing but which are not discussed specifically in the paper. The former dycore has the ability to solve an equation for the motion of the bedrock for long time intervals; however, we are in general not concerned with this scenario as we plan to use the model mainly for decadal and century scale predictions, and we expect the bedrock motion to be a small factor over those time scales.

3. If a time dependent problem is considered, the difficulty with the nonlinear solver to find an initial guess is resolved by taking the solution from the previous time step (as remarked on p 15). Then the homotopy method may be needed only in the very first step.

   The reviewer is correct that for a transient simulation, the homotopy in general would only be needed in the first time step (diagnostic solve).

Although transient runs go beyond the scope of this paper, we had actually noted this in Section 3.1.1.

4. A better explanation of the choice of manufactured solutions in ch 4 is needed. Are the analytical solutions typical for the ice solutions in the interior and at the boundaries? For a fair evaluation, they should have some relation to what can be expected from the Stokes equations.

   Our MMS test cases in Chapter 4 were based on equations obtained by neglecting the $\partial/\partial z$ terms from the FO Stokes equations. The test cases were not intended to verify the 3D FO Stokes equations: rather, they were intended to be used as part of a multi-stage code verification that includes also verification of the 3D FO Stokes equations using code-to-code comparisons and mesh convergence studies on realistic geometries (Sections 5 and 6 in the paper). We have attempted to make this more clear in the paper, and also have made it clear that our MMS problems are for a 2D version of the FO Stokes equations, not the 3D equations.

   We feel despite being simplified, our MMS test cases are nonetheless useful. The task of deriving source terms for an MMS study for the 3D FO Stokes equations is cumbersome, if not intractable. In contrast, our MMS problems are simple enough to be implemented by anyone simply by referring to the expressions in our paper.

   We do agree with the reviewer that an MMS test case whose solution is related to what is expected from the 3D FO Stokes equations is worthwhile. To address this point, we have derived an MMS test case for the FO Stokes equations in the x–z plane (obtained by neglecting the v and $\partial/\partial y$ terms in the equations), and showed some mesh convergence results for our code on this test case (see Section 4.2). The analytic solution to this test case is the sum of shallow ice approximation (SIA) and shallow shelf approximation (SSA) analytic solutions (given the geometry and boundary conditions) and therefore physical in nature.

5. On p 28, line 6, the difference between the present code and another code for the FO equations is of O(1e-10) which is remarkably small considering that at least two different finite element discretizations are used (p 25). Is there an explanation to the small difference?

   The two codes used the same meshes and same basis function with the same discretization (FEM). Therefore it is expected that the difference between solutions obtained in these codes is close to machine precision.

6. The resolution in the z-direction is studied in experiments in ch 6.2. How many layers to use depends on the required accuracy. One percent relative error in Fig 14 is probably enough (considering all the modeling and data errors). Then 10 layers should suffice. A graded or a uniform mesh does not seem to matter. An explanation could be that there is a boundary layer also at the ice surface which is not resolved by the graded mesh (see e.g. Schoof, C. and Hindmarsh, R.: Thin-film flows with wall slip: an asymptotic analysis of higher order glacier flow models, Quart. J. Mech. Appl. Math., 63, 73114, 2010). A relative accuracy less than 1e-4 can never be necessary.

   We realized after submitting the paper that the results from our z–convergence study reported originally did not give the complete picture. To give a complete picture, we have added results showing the effect of z–refinement compared to the effect of horizontal refinement (Tables 4–5). Previously, we were reporting only the last rows of these tables and the data there are somewhat misleading as the solutions are close to the reference solution. For problems with a coarser horizontal mesh resolution, there is some benefit in refining vertically over horizontally but, it appears, only up to a point. We have modified the discussion in Section 6 to explain this. Our recommendation given the new data is that it is more worthwhile to refine from a 1 km GIS mesh with 10 vertical layers to a 1 km GIS mesh with 20 vertical layers, over refining to a 500 m GIS mesh with 10 vertical layers (if one is forced to choose between refining vertically or horizontally only). We leave it up to the reader to select his/her desired level of accuracy from our data.

   We agree that there is not much value in using a graded spacing, but include those data for completeness.

7. Appendix A may be removed. The full Stokes equations can be found in many references.

   We have decided to keep Appendix A in the paper, to keep the paper self-contained.