General comments on corrections done

We would like to thank all five reviewers for their valuable input. A major revision of the structure of the document has been done as suggested by most. The technical details related to the code were moved to the Appendix.
Also, their remarks lead to the rephrasing of three paragraphs and the discussion part in the conclusions was extended. More details have been provided on the setup of the experiments with different modeling of B. Some plots and captions have been corrected and completed. The first part of this document gives an overview of the modifications done, followed by the answers to the comments of each reviewer.

(1) Modifications of the structure of the document.

Reviewers asked modifications of the structure of the document:

In Sect. 2.0, Sect. 2.2.3 doesn't exist anymore.

As asked by different reviewers, the technical details of Sect 3. has been moved in Appendices. Thus, Sect 3. is renamed "Five stages to generate the background error covariance statistics (GEN_BE code version 2.0). and subsections from 3.1.1 to 3.1.4 renumbered from 3.1 to 3.4.
The previous Sect. 3.2 does not exist anymore:

- Sect. 3.2.1 have been included in the Appendix A (FORTRAN code and input/output description)
- The first part of the Sect. 3.2.2 has been merged to the new Sect 3.2.
- Section 3.2.3 becomes Appendix C (Installation, compilation, set up and visualization).
- The description of the namelist options goes in Appendix B (Description of the namelist options)

Section 5.0 includes now the results related to chemistry data assimilation previously shown in Appendix A. Sect. 5.0 is renamed "Cloud and chemistry variational data assimilation"

- Sect. 5.1 is named "Generation of a multivariate background error covariance for hydrometeors.
- Sect. 5.1.1 is added and is composed by the part related to the balance operator previously presented in Sect. 3.2.2. Section 5.1.1 is named "Generation of a multivariate background error covariance for hydrometeors.
- Previous Sect 5.1, and 5.2 becomes 5.1.2 and 5.1.3
- Previous Appendix B becomes Sect. 5.2 and is named "Background Error for Chemical Species"

(2) Modification Equations

Some Equations has been corrected, added and renumbered. We give an update below of the different modifications done.

- Eq. (1) J_b and J_o terms are added
- Eq. (2) new equation added to present a general definition of B
- Eq. (3) the definition of $\delta x=(x_b-x)$ added and renumbered
- Eq. (4) renumbered
- Eq. (5) $B^{1/2}$ is presented instead of δx
- Eq. (6) new equation to present the calculation of the regression coefficient.
- Eq. (7) presents of the calculation of the unbalanced part of the perturbations δt_u
- Eq. (8a) presentation of the Daley's formula that define the vertical length scale for one dimension along the vertical (z).
- Eq. (8b) presentation of an approximation of the formula of Daley along the vertical
- Eq. (9a) presentation of the Gaussian formula that define the vertical length scale for one dimension along the vertical (z).
- Eq. (9b) inverted expression of 9(a)
- Eq. (10a) corrected and renumbered
- Eq. (10b) corrected and renumbered
- Eq. (11) corrected and renumbered
- Eq. (12a-c) Identical

(3) Modification Figures

Previous Fig. 14, that shows the distribution of the vertical model level in function of pressure level, is presented earlier in the document (in the first paragraph of section 3.0 and becomes Fig. 3).
It allows visualizing the density of the vertical model in function of pressure and switch from vertical model level to pressure accurately when results are presented in sect 3.0, 4.0 and 5.1.

Fig. 9, 10,11,12,13,15,16,18a added right vertical axis in hPa pressure levels.

(4) Modification Tables

Table are renumbered:
Table 4 becomes Table 1
Table 2 is created to gather the setup information about the different modeling of B.
The other Tables are moved into the appendix:
- Previous Tables B1, B2, and B3 become Tables A1, A2 and A3.

- Previous Tables 1, 2, 3, 6, 7 and 5 become respectively Tables B1, B2, B3, B4, B5, and B6.

(5) Major revision in the text

Description of the experiments:

(a) The description of the D-ensemble dataset (50 members over the CONUS domain) coming from DART is done in the second paragraph of Sect. 3. : "Figures shown in … Romine et al. (2014) to generate the ensemble and … Table contains detailed information setup of the data assimilation experiment."

Reference about DC3 experiment of Romine et al. 2012 is replaced by: Romine G., S., Schwartz C., S., Berner J., Fossell, R., K., Snyder C., Anderson J. and Weisman M., L.: Representing forecast error in a convection-permitting ensemble system, Mon. Weather Rev., doi: http://dx.doi.org/10.1175/MWR-D-14-00100.1, 2014.

(b) A new table 2 is presented Section 4.0, to give details about the benchmark performed.

Table 2: Description of the setup of the background error matrix modeling diagnosed over the CONUS Domain. $\mathbf{B}_{eof}$ and $\mathbf{B}_{rcf}$ are diagnosed using GEN_BE code version 2.0 and the D-Ensemble method while $\mathbf{B}_{nam}$ is performed by NCEP using the NMC method.

Paragraphs rephrased:

(a) In the introduction, the first paragraph has been corrected, the second and the third rephrased following the remarks of the different reviewers.
(b) Section 2.2.2, the order of the description of the different transform match the Eq. 5:
- The $\mathbf{U}_p$ matrix, called physical transform or balance operator, …
- The $\mathbf{S}$ matrix is …
- The $\mathbf{U}_v$ matrix, called vertical transform, …
- The $\mathbf{U}_h$ matrix, called horizontal transform, …

(c) First paragraph of Section 3.0 has been rephrased.
(d) Section 3.2 has been rephrased (merge of previous sections).
(e) First paragraph of Sect 4.0 is rephrased and additional information is given to the general setup of the different modeling of B ($\mathbf{B}_{eof}$, $\mathbf{B}_{rcf}$ and $\mathbf{B}_{nam}$). References have been added: Romine et al. 2014, Rogers et al. 2009 and Wu 2005.
(f) Section 4.2 has been rephrased
(g) Section 5.1.1 coming from the previous Sect. 3.2.2 is partially rephrased to become independent.

(h) The discussion has been extended in Section 6, which is partially rephrased.

(6) Direct answers are given on the different referee below, in the following document.

Corrections Referee 4

General comments

This paper gives a detailed description of the GEN_BE 2.0 system including theoretical discussion, equations, options and the code. I found this paper is very useful for the readers who wants to learn and use this tool and for readers just want to learn more details on how to model a BE in variational analysis. Those details are necessary information when apply this tool in data assimilation but, on other hands, I think those details make this paper hard to follow and read through. My suggest is to do a major revision of the paper structure: put all the details on code and namelist options to the appendix as reference for readers who want to apply the tools but leave the theoretical analysis, practical discussions, and test results of modeling BE in the paper.

(1) Answers to the general comments

We want to thank Referee 4 for his or her valuable advice to modify the structure of the code and to improve the introduction of the manuscript.

A major revision of the structure of the paper has been done to present the GEN_BE code Version 2.0 and its application. The details on code and namelist options have been moved in 3 appendices. (See general comments at the beginning of the document)

(2) Answers to the detailed comments

1. Page2, Line14-16: the statement on a“multivariate approach” Is not clear. It can refer to adding new control variables for the cloud analysis, or to the GEN-BE for providing covariance among all the analysis variables.

It has been rephrased:
“Different choices of control variables and their correlated errors used to mimic general physical balance (geostrophic, hydrostatic, ...) in the atmosphere have been largely investigated by different operational centers and referenced in Banister (2008b). Since then, such multivariate relationship approaches has been studied …”

2. Page3, Line2: change “or the UK Met office ”to“ and the UK Met office”.
Done

3. Page3, Line3: add “,” after“ techniques”.

Done

4 Page3, Line6: change “that minimize” to “, to minimize”
This part of the sentence has been removed.
More details have been written in the paragraph 2.2.2 that describes Up transform.

5. Page3, Line9-12: More available observations are not the only reason why cloud and chemical data analysis are needed. I think the needs of improving the cloud forecast and chemical (pollution) forecast are major drivers of the development of the cloud and chemical data assimilation.

This part has been rephrased in the first paragraph of the introduction.

6. Page3, Line27:“the two first sections”. This is confusion.
Each section is described separately now.

7. Page4, Line14: Please give more details on which kind of “results” author will give in the Appendix to give reader an idea what in the Appendix.

The document has been restructured to gather all the technical details in the appendices and are not anymore presented in the introduction. They are presented for the first time in the first paragraph sect. 3.0.

8. Page5, Line9: “nor be stored” changes to “and to”
Done

9. Page5, Line12: I think “parameterized” has the same meaning as “Modelling” in the next line. If this is the case, please use the “modeling” just as other part of the paper.
Replaced “parameterized” by “modeled”

10.Page 5, last line” “linear operator” changes to “ linear observation operator”.
Corrected, the sentence begins now as follow:
“H is the linearized observation operator ….”

11.Page 6, equation 4. This equation can be expressed as square root of B equals to ...
replaced the equation 4 by

$$B^{1/2} = U_p S U_v U_h$$

12.Page 7, Line 7: “and make to” changes to “ and to make”
This paragraph doesn’t exist anymore.

13.Page 7, Line 8-10: “the new version ... a new model of B”. I don't understand what this sentence means.
This paragraph doesn’t exist anymore

14.Page 7, Line 20: add ‘the’ before ‘modeling’. The same line: add “background” before “error covariance” and change “become” to “becoming’.
The sentence is rephrased: “The five steps, from stage 0 to 4, that model a background error covariance matrix, become independent of the choice of control variables and model input, which allows for more flexibility (Fig. 2).”

15.Page 8 Line 4-8: please list the functions of each stage more clear the specific to help readers go through the details of each stage smoothly.

The first paragraph of section 3.0 has been rewritten to take into account this remark : “The general structure of the GEN_BE code … Appendix C explains how to compile and run the code.”

16.Page 8, Line 10-11: “sample of model forecasts” changes to “ sample of perturbations”
We changed to “sample of perturbations”.

17.Page 8, Line 20: “ an ensemble of ” changes to “ ensemble perturbations of”
We replaced “an ensemble of perturbations of previous forecasts valid at the same time” by “an ensemble of perturbations valid at the same time”.

18.Page 11, Line 2: “After”, should be “when”
Two successive steps are necessary to estimate the vertical auto-correlation parameters. First, the vertical auto-covariance matrix averaged by vertical levels is computed. Then, two different techniques can be used to diagonalize this matrix.

19.Page 12, Line 24: “we estimate length scales” means horizontal or vertical or both. Needs to clear define which part of length scales here and in other parts of the paper.
We replaced “In stage 4, we estimate length scales averaged” By “In stage 4, we estimate horizontal length scales in a 2-D plan defined by vertical level or EOF mode.”

20.Page 13, equation 8. Please define “r”
r has been defined in the sentence that follow the Equation:
“where $\rho(r)$ is the correlation calculated for a distance r between two grid points”

21.Page 13, the paragraph starts from “Usually, ...”: This is paragraph is very helpful for readers to understand the advantage and disadvantage of the each option in global_bin but it also mixed with too much details on exact number of the option. The other parts of the paper also have the same issue as I described in summary. Please think how to keep the useful discussion of the BE option in paper but leave the details to the appendix.

The last paragraph of section 3.4 has been rewritten: “The horizontal length scale ... normalization issues (Michel and Auligne 2010).”
The technical details about global_bin flag is a part of appendix B (namelist section “&gen_be_lenscale”).

22.Page 15, Line 12-15: please revise this sentence to make it easy read and understand.
The all section 3.2.2 has been merged to section 3.1.2 and rephrased.

23.Page 15, Line 25-26: please give more explanation on the purpose of showing the correlation of T with both specific humidity and relative humidity.
The all section 3.2.2 has been merged to section 3.1.2
A sentence have been added to explain the purpose of such diagnostic:”
Diagnostics such as vertical cross-covariance (Fig. 4) or vertical cross-correlation are helpful to analyze the relationship between variables and can be done by using stage 2.”

24.Page 19, Line 2-3: RAP is not using NAM BE directly. The BE for RAP is a combination of the global BE and NAM BE with some tunings. Also, NAM BE should be 1 degree of the resolution instead of 0.1 degree.
The general setup is described in Table 2 (added): forecast used to construct B, in this case, is at a 12 km resolution. Statistics of B are also binned within a latitude band of 1 degree.

25.Page 20, last line: “ the pseudo observation of 1K”: should be “the innovation of 1K.
Done

26.Figure 1 caption: what is “DC3”? Please give explanation or delete it.
We removed it.

27.Figure 8 and 9 and 10 are whole domain results, right?
Yes, for the entire domain, by vertical levels or EOF modes.

It appears now in the new Table 2 that describes the different modeling of B