Review of "A Lagrangian Advection scheme with Shape Matrix (LASM) for solving advection problems"

by
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Conclusion

This is an interesting manuscript including new ideas. There are, however, a few important issues (plus some less important ones) that I would like the authors to address before I can accept the paper. Thus, I can accept the paper after my concerns discussed below have been addressed.

Concerning principal evaluation criteria:

- **Scientific Significance:** Good/Excellent
- **Scientific Quality:** Good
- **Scientific Reproducibility:** Good
- **Presentation Quality:** Good

Summary and general comments

This paper introduces a novel fully Lagrangian scheme (LASM) designed to solve the transport problem in GFD and air pollution models. The main idea is obviously inspired by other similar schemes that have been proposed over the last 5 to 10 years, yet it includes a new way of treating the deformation and mixing of Lagrangian parcels: the shape of Lagrangian parcels is simulated by a linear transformation matrix in such a way that the degree of deformation and the direction of the parcel streching is memorised. The parcel streching and the degree of disorder, in terms of the local spread in the direction of the parcel strechings, determine if mixing should be active or not.

My main concerns with the manuscript regard the inter-parcel mixing and the discussion of aliasing.

The interparcel mixing between neighboring parcels takes place according to Eq. (18) in the manuscript where the parameters β_1 and β_2 controls the relative mixing weight along the major parcel axis and along lateral axis. The mixing is activated when a certain threshold of parcel deformation is γ_m is reached, and only then. In most of the simulations γ_m is set to 100 but it is reduced to e.g. 5 if the degree of disorder $\mathcal{D}_i$ is high. It is not clear what physical principles were used to set the specific cut-off value of $\mathcal{D}_i$, and the actual reduced value of γ_m. "5" seems as a completely ad hoc choise. Nature is not discontinous in terms of mixing. Parcels in the real world mix gradually and the mixing increases gradually with the deformation rate. This must be

discussed. What is of even more concern is the factor α by which the major parcel axis will be shrunk after mixing has taken place. I cannot understand why α is not a function of the degree of mixing that has actually taken place. I also don't understand what has motivated the actual choise (e.g., $\alpha = 0.05$). If there is no link between α and the mixing one can introduce anything from too excessive or too weak mixing, all depending on the parameter choises.

Aliasing is the misinterpretation of unresolved scales on the resolved scales. In spectral (Eulerian) models aliasing is avoided by having a larger number of degrees of freedom in grid point space (where non-linear advective terms are calculated) than in spectral space. In this way the spectrally unresolved scales are automatically eliminated at each time step. In pseudospectral models this is not the case, and therefore, in such models, one will need to introduce some diffusion (spatial mixing) in order to prevent or reduce the problems with aliasing. In fully Lagrangian models there is no simple parallel to this situation but one may argue that a Lagrangian model without any inter-parcel mixing is equivalent to a pseudospectral Eulerian model. Hence, to avoid the gradual build up of aliasing in a Lagrangian model one must introduce spatial mixing. Considering the specific type of mixing in LASM it is not clear to me how one can be certain to avoid aliasing. In particular, I am here thinking about the situation where the degree of disorder $\mathcal{D}_i$ is low, i.e. the mixing threshold γ_m is very large (up to 100). As far as I can see the mixing in LASM will not eliminate aliasing. So I should like the authors to discuss this issue some depth (at least to convince me). See also comments below.

The paper is logically organised and generally well written. However, there are a number simple gramatical errors including missing "the"s, etc. I have listed some examples below together with specific questions and comments. Note: I have not performed a proof reading, so, language/gramma needs a really careful final check.

Specific comments

Note the page and line numbers below refer to the "printer friendly version" of the manuscript.

- **Page 4830, line 18.** It is suggested to change "model" to "modeling"
- **Page 4830, line 18.** What about cloud water and cloud ice?
- **Page 4830, line 21.** Gramma
- **Page 4831, line 19.** "is disbennifit" $\rightarrow$ "is a problem".
- **Page 4831, line 21-22.** "... when the resolution is coarse". Formally, I think this problem is controllable what ever resolution is used (unless it is comparable to molecular scales).
- **Page 4831, line 24.** "... so least resolution": Wrong gramma plus I cannot see that purely Lagrangian (without explicit interparcel mixing) schemes have any diffusion at all.
- **Page 4832, line 15.** "into needle form": in the real world they are not deformed into needles but rather into irregularly shaped thin filaments (i.e. they are often also strongly bended) Maybe you can find a better word than needles?
- **Page 4833, line 16.** "... aliasing error as much as possible...". I cannot see that this is related to aliasing. Only mixing can be used to reduce aliasing in this type of model. Maybe you can explain this in more detail to bring the thoughts of the reader in the right direction.
- **Page 4834, line 4.** "... of deformation matrix, the ..." $\rightarrow$ "... of the deformation matrix, and the ..."
- **Page 4834, equation (4)**. You write that this equation is an ordinary differential equation. But the divergence operator includes partial derivatives?
- **Page 4835, line 21.** "... in the real ..." $\rightarrow$ "... in a real ..."

- **Page 4836, line 5.** "not" → "no"
- **Page 4836, line 17.** "despite" → "spite"
- **Page 4838, line 1.** "for other" → "for the other"
- **Page 4838, line 25.** "defomration" → "deformation"
- **Page 4839, line 2-3.** "aliasing error without using ... schemes.". I can see that this is the essence of the paper. I am however, not sure I understand how one can avoid introducing a certain amount of mixing to avoid the aliasing. Also, as far as I can see, the mixing will be more or less independent of the particular parcel shape formulation one uses. The mixing only depends on the deformation rate (or equivalently on the Lyapunov exponent) of the flow/problem. Use should discuss this statement in more detail to clarify what the magic is. From a theoretical point of view, I simply don't understand how a special shape formulation of individual parcels can reduce aliasing.
- **Page 4839, line 24.** "In future," → "In the future,"
- **Page 4841, line 27.** A γ_m value of 100 seems to be exceptionally high (then parcels are indeed needles). I can see that γ_m is reduced when the degree of disorder is large, but still it seems that a γ_m value of, e.g., 5 is very large. With such large numbers it is obvious that you will minimize the mixing dramatically and therefore, in the validation tests, obtain results, which are similar to an almost unmixed Lagrangian model. However, I am not convinced that the values used for γ_m, $\mathcal{D}_i$, and α are consistent with the mixing limit that must be introduced to avoid aliasing. The γ_m, $\mathcal{D}_i$, and α values seem to be chosen on an ad hoc basis.
- **Page 4841, line 14.** Another related issue: I would guess that α should depend on the length of the time step, or rather by the actual amount of mixing that has been imposed for the particle in question.
- **Page 4844, line 1.** in future" → in the future"
- **Page 4844, line 3.** are summaries" → can be summarized as"
- **Page 4845, line 13.** be in the real" → be in a real"
- **Page 4845, line 16.** Why not use (semi) analytical trajectories to check the importance of errors in trajectory calculations?
- **Page 4847, text arround Fig. 10.** Please provide mixing parameters γ_m, β_1, β_2,
- **Page 4850, line 12.** What is meant by apparent" in this context?
- **Page 4851, text arround Fig. 15** At a first glance the LASM transport scheme seems to perform quite well. It is of course not surprising that the noise level is reduced when the number of parcels is increased. However, as hinted above, my general fear is that the amount of inter-parcel mixing in LASM is generally to low to avoid the build up of noise over time due to some kind of aliasing. I think it is very important to perform some longer simulations with the model (e.g. 10 days) to see what happens with noise in the long run in a realistic geophysical flow. The same comment applies to the simulations shown in Fig. 17.
- **Page 4852, line 8** chaostic" → chaotic"