Response

September 16, 2014

1 Note

We would like to thank reviewers for their comments. Below we propose several improvements based on these comments. We hope we can incorporate these into a new version if the paper is accepted for GMD.

2 Review #1

1. p. 4086: Our text was poorly worded. We did not mean to imply these operators were equal, but instead we replace $\nu\Delta^2$ with a new operator, $(\nabla \cdot \tau\nabla)\Delta$ with a symmetric tensor τ. In case $\tau = \nu I$ where I is an identity, both operators are equal.

2. We address the comment about how equations (2) and (3) are derived from (1) above. Instead of a weak form for $Q_t = -\nu\Delta^2$, in (2) and (3) we formulate a weak form for $Q_t = -(\nabla\cdot\tau\nabla)\Delta Q$. As noted above, this later form is more general and is equal to the original formulation when $\tau = \nu I$.

3. Section 5.3: In this paper we wanted to focus only on the spatial error, and in all cases used timesteps where the time truncation error was negligible as compared to the spatial error. While running simulations for convergence we made sure that temporal errors did not dominate. For test case 2 (TC2), we obtain 4th order convergence when using timesteps near the CFL limit. For the test case 5 (TC5) we reduced time steps so that time truncation errors are of 4th order. For example, for the resolution with spatial scales Δx the time was Δt. For the refined grid with spatial scales $\Delta x/2$ we ran a simulation with $\Delta t/4$.

   We supply exact time steps below:

   For TC2, for uniform resolutions of 3, 1.5, 0.75, and 0.5 degrees and constant-coefficient and tensor hyperviscosities (HVs), we used time steps of 270, 135, 90, 65, and 45 seconds. For refined grids, time steps were defined by smallest grid scales, Δx_{high}, see Table 1.

   For TC5, for uniform resolutions of 3, 1.5, 0.75, and 0.5 degrees and constant-coefficient and tensor hyperviscosities (HVs), we used time steps of 576, 144, 64, and 36 seconds. For refined grids, time steps were defined by smallest grid scales, Δx_{high}, see Table 2.

4. Minor comments will be addressed in a revised manuscript.

grid	Δx_{high}	time step
deg3-x2-lowconn	1.5°	135
deg1.5-x2-lowconn	0.75°	65
deg1-x2-lowconn	0.5°	45
deg0.75-x2-lowconn	0.375°	30
deg0.5-x2-lowconn	0.25°	20
deg3-x4-lowconn	0.75°	40
deg1.5-x4-lowconn	0.375°	30
deg1-x4-lowconn	0.25°	20
deg0.75-x4-lowconn	0.125°	15
deg3-x8-lowconn	0.375°	30
deg1.5-x8-lowconn	0.1875°	15
deg1-x8-lowconn	0.125°	10

Table 1: TC2, convergence studies. Summary of the x2, x4 and x8 families of grids.

grid	Δx_{high}	time step
deg3-x2-lowconn	1.5°	144
deg1.5-x2-lowconn	0.75°	36
deg1-x2-lowconn	0.5°	16
¿ deg0.75-x2-lowconn	0.375°	9
deg0.5-x2-lowconn	0.25°	4
deg3-x4-lowconn	0.75°	36
deg1.5-x4-lowconn	0.375°	9
deg1-x4-lowconn	0.25°	4
deg0.75-x4-lowconn	0.125°	2.25
deg0.5-x4-lowconn	0.125°	1
deg3-x8-lowconn	0.375°	9
deg1.5-x8-lowconn	0.1875°	2.25
deg1-x8-lowconn	0.125°	1

Table 2: TC5, convergence studies. Summary of the x2, x4 and x8 families of grids.

3 Review #2

1. This review suggested that we compare tensor and scalar hyperviscosities on the same grid for the test case 5. In figure 1 below we provide plots for both HVs on a highly distorted grid in (a) and (b) subplots. We notice that the numerical noise at the transition region present in the simulation (a) with scalar HV is practically eliminated when tensor HV is used in (b). If the simulation in (c) for a better quality grid (low-connectivity grid) is considered optimal, then one can conclude that tensor hyperviscosity simulation provides a very-close-to-optimal result even if a low quality grid is used.

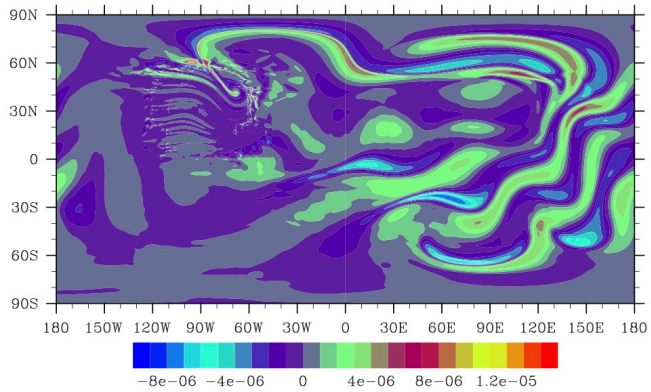
2. page 4086, comment on Δ^2: We used the identity $\Delta = \nabla \cdot \nabla$.

3. page 4086, eqns. 2 and 3, the same as what Reviewer #1 pointed out: We explained this in the reply to Reviewer #1.

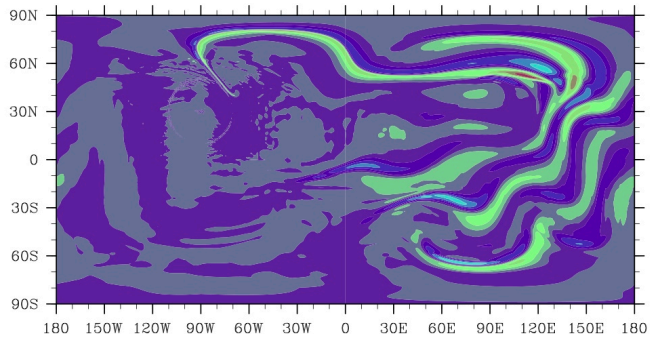
4. Figures 6 and 10: We will address the labels' size in a revised version. In format A4 we expect the labels to be readable.

5. About convergence slope in Figure 12, TC5. We agree with the reviewer. Our speculation that this is due to the use of a reference solution is incorrect. It is more likely due to the fact that the mountain has a cusp in its shape, limiting the convergence to 1st order in the max norm and second order in the l2 norm.

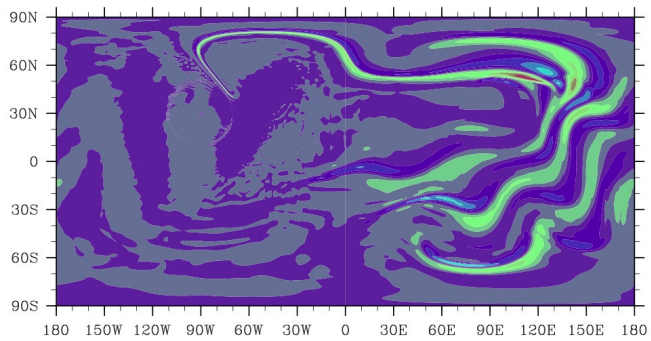
6. Other minor comments will be incorporated into a revised version.



(a) Scalar hyperviscosity using the highly distorted x8 grid shown in Fig. 4(a). Error (15) varies from -8.1×10^{-6} to 1.4×10^{-5}. Normalized l_2 error is 8.44×10^{-4}.



(b) Tensor-based hyperviscosity using the same highly distorted x8 grid. Error (15) varies from -4.7×10^{-6} to 5.75×10^{-6}. Normalized l_2 error is 3.36×10^{-4}.



(c) Tensor-based hyperviscosity using the low-connectivity x8 grid. Error (15) varies from -4.7×10^{-6} to 6.0×10^{-6}. Normalized l_2 error is 2.85×10^{-4}.

Figure 1: The error in the TC5 vorticity field is plotted for the global domain. The color scheme and scales in (a) are the same for all plots.